Damietta University
Faculty of Commerce
English Program

Financial Institutions, Markets, and Money

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CHAPTER 5

BOND PRICES AND INTEREST RATE RISK

The Time Value of Money:

- Before we can understand how bonds are priced, we must review the concept of the time value of money.
- The time value of money is based on the belief that people have a positive time preference for consumption.
- Thus, the time value of money can be simply stated as a dollar today is worth more than a dollar received at some future date.

ESSENTIALLY, IF YOU HAVE A DOLLAR IN YOUR POCKET TODAY, THAT DOLLAR WILL BE WORTHLESS ONE YEAR FROM TODAY.

- Positive time preference for consumption must be offset by adequate return.

WisdomTimes.com

How to place a value on both dollars today and dollars in the future?

- The time value of money is sometimes referred to as the net present value (NPV) of money.

- Opportunity cost of deferring consumption determines minimum rate of return required on a risk-free investment
  - Present sums are theoretically invested at not less than this rate;
  - Future cash flows are discounted by at least this rate.
- The impact that inflation has on the time value of money is it decreases the value of a dollar over time.
  - Inflation will increase the time value of money to a greater extent.
  - Inflation expectations will increase the used discount rate.

Future Value or Compound Value

$$FV = PV(1 + i)^n \tag{5.1}$$

where:

FV = future value of an investment n periods in the future
PV = present value of an amount of money (the value of money today)
i = interest rate
n = number of interest rate compounding periods

To illustrate, suppose you have \$100 and put it in a savings account at a local bank, expecting to keep it there for 5 years. The bank pays 4 percent interest on savings accounts and compounds interest annually. Applying Equation 5.1, the future value is:

$$\begin{aligned} FV &= \$100(1 + 0.04)^5 \\ &= \$100(1.2167) \\ &= \$121.67 \end{aligned}$$

Thus, at the end of 5 years, the account has \$121.67, which consists of the \$100 original deposit plus \$21.67 of interest.

If the bank decided to pay interest quarterly, the number of compounding periods increases to 20 periods (5 years × 4 quarters) and the annual interest rate converted to a quarterly interest rate is 1.00 percent (4 percent/4 quarters). Applying Equation 5.1 to the new situation, the future value is:

$$
\begin{aligned}
FV &= \$100(1 + 0.01)^{20} \\
&= \$100(1.2202) \\
&= \$122.02
\end{aligned}
$$

Notice that the dollar amount is slightly larger because we have increased the number of compounding periods and are now earning more interest on interest.

Present Value

Present value is the value today of a given sum of money to be received at a given point in the future.

$6209

$$PV = FV\left[\frac{1}{(1+i)^n}\right] \quad (5.2)$$

Going back to our original question, how much would we pay for $121.67 to be received 5 years in the future, if the interest rate on our next best alternative investment (i.e., your **opportunity cost**) were 4 percent? Using Equation 5.2, we compute the present value of $121.67 received in 5 years as:

$$\begin{aligned} PV &= \$121.67\left[\frac{1}{(1+0.04)^5}\right] \\ &= \$121.67(0.8219) \\ &= \$100.00 \end{aligned}$$

- With risk present, a premium may be added to the risk-free rate.
- The higher the discount rate, the lower the present value.

Your rich uncle promises to give you $10,000 when you graduate from college. What is the value of this gift if you plan to graduate in 5 years and interest rates are 10 percent?

PRESENT VALUE TABLE

Present value of $1, that is $(1+r)^{-n}$ where r = interest rate; n = number of periods until payment or receipt.

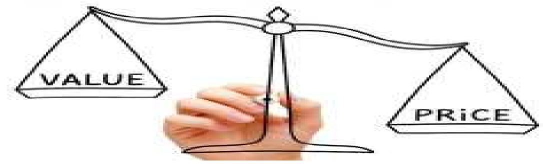
Periods	Interest rates (r)									
(n)	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
1	0.990	0.980	0.971	0.962	0.952	0.943	0.935	0.926	0.917	0.909
2	0.980	0.961	0.943	0.925	0.907	0.890	0.873	0.857	0.842	0.826
3	0.971	0.942	0.915	0.889	0.864	0.840	0.816	0.794	0.772	0.751
4	0.961	0.924	0.888	0.855	0.823	0.792	0.763	0.735	0.708	0.683
5	0.951	0.906	0.863	0.822	0.784	0.747	0.713	0.681	0.650	0.621
6	0.942	0.888	0.837	0.790	0.746	0705	0.666	0.630	0.596	0.564
7	0.933	0.871	0.813	0.760	0.711	0.665	0.623	0.583	0.547	0.513
8	0.923	0.853	0.789	0.731	0.677	0.627	0.582	0.540	0.502	0.467
9	0.914	0.837	0.766	0.703	0.645	0.592	0.544	0.500	0.460	0.424
10	0.905	0.820	0.744	0.676	0.614	0.558	0.508	0.463	0.422	0.386
11	0.896	0.804	0.722	0.650	0.585	0.527	0.475	0.429	0.388	0.350
12	0.887	0.788	0.701	0.625	0.557	0.497	0.444	0.397	0.356	0.319
13	0.879	0.773	0.681	0.601	0.530	0.469	0.415	0.368	0.326	0.290
14	0.870	0.758	0.661	0.577	0.505	0.442	0.388	0.340	0.299	0.263
15	0.861	0.743	0.642	0.555	0.481	0.417	0.362	0.315	0.275	0.239
16	0.853	0.728	0.623	0.534	0.458	0.394	0.339	0.292	0.252	0.218
17	0.844	0.714	0.605	0.513	0.436	0.371	0.317	0.270	0.231	0.198
18	0.836	0.700	0.587	0.494	0.416	0.350	0.296	0.250	0.212	0.180
19	0.828	0.686	0.570	0.475	0.396	0.331	0.277	0.232	0.194	0.164
20	0.820	0.673	0.554	0.456	0.377	0.312	0.258	0.215	0.178	0.149

Periods	Interest rates (r)									
(n)	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%
1	0.901	0.893	0.885	0.877	0.870	0.862	0.855	0.847	0.840	0.833
2	0.812	0.797	0.783	0.769	0.756	0.743	0.731	0.718	0.706	0.694
3	0.731	0.712	0.693	0.675	0.658	0.641	0.624	0.609	0.593	0.579
4	0.659	0.636	0.613	0.592	0.572	0.552	0.534	0.516	0.499	0.482
5	0.593	0.567	0.543	0.519	0.497	0.476	0.456	0.437	0.419	0.402
6	0.535	0.507	0.480	0.456	0.432	0.410	0.390	0.370	0.352	0.335
7	0.482	0.452	0.425	0.400	0.376	0.354	0.333	0.314	0.296	0.279
8	0.434	0.404	0.376	0.351	0.327	0.305	0.285	0.266	0.249	0.233
9	0.391	0.361	0.333	0.308	0.284	0.263	0.243	0.225	0.209	0.194
10	0.352	0.322	0.295	0.270	0.247	0.227	0.208	0.191	0.176	0.162
11	0.317	0.287	0.261	0.237	0.215	0.195	0.178	0.162	0.148	0.135
12	0.286	0.257	0.231	0.208	0.187	0.168	0.152	0.137	0.124	0.112
13	0.258	0.229	0.204	0.182	0.163	0.145	0.130	0.116	0.104	0.093
14	0.232	0.205	0.181	0.160	0.141	0.125	0.111	0.099	0.088	0.078
15	0.209	0.183	0.160	0.140	0.123	0.108	0.095	0.084	0.079	0.065
16	0.188	0.163	0.141	0.123	0.107	0.093	0.081	0.071	0.062	0.054
17	0.170	0.146	0.125	0.108	0.093	0.080	0.069	0.060	0.052	0.045
18	0.153	0.130	0.111	0.095	0.081	0.069	0.059	0.051	0.044	0.038
19	0.138	0.116	0.098	0.083	0.070	0.060	0.051	0.043	0.037	0.031
20	0.124	0.104	0.087	0.073	0.061	0.051	0.043	0.037	0.031	0.026

Bond Pricing: What is a bond?

- A bond is a contractual obligation of a borrower (*issuer*) to make periodic cash payments to a lender over a given number of years.
- A bond constitutes debt, so there is a borrower and a lender. In the parlance of Wall Street, the borrower is referred to as the bond issuer.
- The lender is referred to as the investor, or the bondholder.
- The bond consists of two types of contractual cash flows:
  - At maturity, the lender is paid the original sum borrowed, which is called the principal, face value, or par value, of the bond. Note that these three terms are interchangeable.
  - Periodically before maturity, the issuer must make periodic interest payments to the bondholders. These interest payments are called the coupon payments (C).

It is important to keep in mind that for most bonds the coupon rate, the par value, and the term-to maturity are fixed over the life of the bond contract. Most bonds are first issued in $1,000 or $5,000 denominations.



What is a bond? Example

For example, if a bond pays $80 of coupon interest annually for 10 years and the face value is $1,000:

Par Value	$1000
Scheduled Payments	$80 per year for 10 years
	$1000 at the end of 10 years.
Coupon Rate	80/1000 = %8
Future Value	$2159

The price of a bond is the present value of the future cash flows (coupon payments and principal amount) discounted by the interest rate (the required rate of return on this risk class in today's market), which represents the time value of money.

PV of bond cash flows

$$PB = \frac{C_1}{(1+i)^1} + \frac{C_2}{(1+i)^2} + \dots \frac{C_N + F_N}{(1+i)^N} \quad (5.3)$$

where:

PB = the price of the bond or present value of the stream of cash payments
C = the periodic coupon payment
F = par value or face value (principal amount) to be paid at maturity
i = market interest rate (discount rate or market yield)
n = number of periods to maturity

Consider a 3-year bond with a face value of $1,000 and a coupon rate of 8 percent. The coupon payments are $80 per year. If coupon payments are made annually and the current market rate of interest on similar bonds is 10 percent, the price of the bond, using Equation 5.3, is:

$$\begin{aligned} PB &= \frac{\$80}{(1.10)^1} + \frac{\$80}{(1.10)^2} + \frac{\$1{,}080}{(1.10)^3} \\ &= \$73.73 + \$66.12 + \$811.42 \\ &= \$950.27 \end{aligned}$$

Par, Premium, and Discount Bonds

- Cash flows are assumed to flow at end of the period and to be reinvested at ***i***.
- Increasing ***i*** decreases price (PB); decreasing ***i*** increases price; thus bond prices and interest rates move inversely.
- If market rate equals coupon rate, bond trades at **par** (Par Bond)
- If coupon rate exceeds market rate, the bond trades above par—at a **premium** (Premium Bond)
- If market rate exceeds coupon rate, bond trades below par—at a **discount** (Discount Bond).
- Bonds typically pay interest **semiannually**.

Semiannual Compounding

If coupon payments are made more than once a year, we modify Equation 5.3 as follows:

$$PB = \frac{C/m}{(1 + i/m)^1} + \frac{C/m}{(1 + i/m)^2} + \frac{C/m}{(1 + i/m)^3} + \cdots + \frac{(C/m) + F}{(1 + i/m)^{nm}}, \qquad (5.4)$$

where m is the number of times coupon payments are made each year and the other terms are as previously defined. In the case of a bond with semiannual coupon payments (i.e., twice per year), $m = 2$. For example, if our 3-year, 5 percent coupon bond pays interest semiannually and the current market yield is 6 percent, the price of the bond would be:

$$PB = \frac{\$25}{(1.03)^1} + \frac{\$25}{(1.03)^2} + \cdots + \frac{\$1{,}025}{(1.03)^6}$$
$$= \$972.91$$

Zero Coupon Bonds

- No periodic coupon payments.
- Issued at discount from par.
- Single payment of par value at maturity.
- PB is simply PV of FV represented by par value, discounted at market rate.

The price (or yield) of a zero coupon bond is simply a special case of Equation 5.4 in that all the coupon payments are set equal to zero. Hence the pricing equation is:

$$PB = \frac{F}{\left(1 + \frac{i}{m}\right)^{mn}} \tag{5.5}$$

where:

PB = the price of the bond
F = the amount of cash payments at maturity (face value)
i = the interest rate (yield) for n periods
n = number of years until the payment is due
m = number of times interest is compounded each year

For example, the price of a zero coupon bond with a \$1,000 face value and 10-year maturity, and assuming semiannual compounding, when the market interest rate is 12 percent is calculated as follows:

$$PB = \frac{\$1{,}000}{(1.06)^{20}} = \$311.80$$

Notice that our calculation is based on semiannual compounding because most U.S. bonds pay coupon interest semiannually.

Bond Yields: Risks and Rewards

Yield should rewards investor for at least three risks:

- **Credit or default risk:** chance that issuer may be unable or unwilling to pay as agreed.

Interest rate risk is the risk related to changes in interest rates that cause a bond's total return to differ from the promised yield or yield-to-maturity. Interest rate risk comprises two different but closely related risks: (1) price risk and (2) reinvestment risk.

- **Reinvestment risk:** potential effect of variability of market interest rates on return at which payments can be reinvested when received.
- **Price risk:** Inverse relationship between bond prices and interest rates.

Bond prices and yields vary inversely. Specifically, as the market rate of interest (or yield) rises, a bond's market price declines; or as the market rate of interest (or yield) declines, a bond's market price rises. This inverse relationship exists because the coupon rate or interest rate on a bond is fixed at the time the bond is issued.

Thank you