Damietta University
Faculty of Commerce
English Program

Financial Institutions, Markets, and Money

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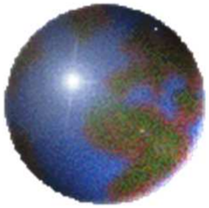

CHAPTER 5

BOND PRICES AND INTEREST RATE RISK

Bond Yields: Set by Market

- In general, a yield on any investment, such as a bond, is the interest rate that equates the market price of an investment with the discounted sum of all cash flows from the investment.

- The ideal yield measure should capture all three potential sources of cash flow from a bond:

(1) coupon payments,
(2) income from reinvesting coupon payments, and
(3) any capital gain or loss.

- We now discuss four yield measures: Yield-To Maturity, Expected Yield, Realized yield, and Total Return.

Yield To Maturity

- Yield-To-Maturity (YTM) or promised yield.
- It is the yield promised the bondholder on the assumption that the bond is held to maturity, all coupon and principal payments are made as promised, and the coupon payments are reinvested at the bond's promised yield for the remaining term-to-maturity.
- Assume, Investor buys 5% percent coupon (semiannual payments) bond for $951.90; bond matures in 3 years. Solve the bond pricing equation for the interest rate (***i***) where that price paid for the bond equals ***PV*** of remaining payments due under the bond.

$$951.90 = \frac{25}{(1 + (i/2))^{1}} + \frac{25}{(1 + (i/2))^{2}} + \ldots \frac{1{,}025}{(1 + (i/2))^{6}}$$

Solving either by trial and error or with a financial calculator results in yield to maturity of 3.4% semiannually, or 6.8% annually.

Problem - YTM

PROBLEM: If a person purchased a 10-year, 5 percent coupon (semiannual payments) bond for $1,050.00, what is the yield-to-maturity?

- Annual coupons will be $50, but the semiannual coupons are worth $25.
- The number of periods is 20 (10 years × 2).
- The current price of the bond is $1,050.00.

SOLUTION:

Semiannual coupon = $C/2$ = $50/2 = $25
Maturity of bond = n = 10 years = 20 semiannual periods
Number of compounding periods = $m = 2$
Number of coupon payments = $m \times n = 10 \times 2 = 20$
Current market price = PB = $1,050.00

The semiannual yield is 2.189 percent, while the annual yield to maturity is 4.377 percent.

Expected yield

The yield-to-maturity tells us what return we will earn on a bond if the borrower makes all cash payments as promised, if interest rates do not change over the bond's maturity, and if the investor holds the bond to maturity. Quite frequently, however, one or more of these events do not occur. For example, an investor may plan to sell a bond before maturity, the bond may be called prior to maturity, or the bond issuer may default. In any event, the return actually earned on a bond is likely different from the promised yield.

Investors and financial institutions that plan to sell their bonds before maturity would like to know the potential impact of interest rate changes on the returns of their bond investments ex ante (*before* the fact). They can use various forecasting techniques to estimate future interest rates based on information about the money supply, inflation rates, economic activity, and the past behavior of interest rates. Once armed with an interest rate forecast, an investor can predict the market price of a bond at the end of a relevant holding period. Given the prediction of the future price, the investor can calculate an **expected yield** that reflects the expected sale price.

You must forecast the below, and Plug forecast results into bond pricing formula

- ❑ Expected interest rate
- ❑ Bond price at end of holding period

Problem - Expected yield

Suppose you purchase a 10-year, 8 percent coupon (annual payments) bond at par and you plan to sell it at the end of 2 years at the prevailing market price.
When you purchase the bond, your investment adviser predicts that similar bonds with 8 years to maturity will yield 6 percent at the end of 2 years.

$$PB = \$1{,}124.20 = \frac{\$80}{(1.06)^1} + \frac{\$80}{(1.06)^2} + \cdots + \frac{\$1{,}080}{(10.6)^8}$$

Your interest rate forecast implies that the bond's expected price $1,124.20

$$\$1{,}000 = \frac{\$80}{(1 + i)^1} + \frac{\$80 + \$1{,}124.20}{(1 + i)^2}$$

Note: To calculate the expected yield over your 2-year holding period, you solve for the interest rate that equates the original purchase price (par, or $1,000, in this example) with the discounted sum of the cash flows you expect to receive (coupon payments and the expected sale price)

Realized yield

The **realized yield** is the return earned on a bond given the cash flows actually received by the investor and assuming that the coupon payments are reinvested at the realized yield. Suppose you purchased the bond in the previous example, a 10-year, 8 percent coupon (annual payments) bond at par. Rather than selling it in 2 years, however, you hold on to it for 3 years and sell it so that you can take a vacation to Cancún. At the time you sell the bond, 7-year bonds with similar characteristics (e.g., default risk) sell at yields of 10 percent.

In this case, the realized yield is different from the promised yield of 8 percent (or the expected yield of 13.81 percent calculated previously) because market yields on similar bonds increased to 10 percent. Similar to the calculation of expected yield, we calculate the yield actually earned on the investment by solving for the interest rate that equates the price you originally paid for the bond with the discounted sum of the cash flows you actually received.

$$PB = \$902.63 = \frac{\$80}{(1.10)^1} + \frac{\$80}{(1.10)^2} + \cdots + \frac{\$1{,}080}{(1.10)^7}$$

$$\$1{,}000 = \frac{\$80}{(1+i)^1} + \frac{\$80}{(1+i)^2} + \cdots + \frac{\$80 + \$902.63}{(1+i)^3}$$

Solving the preceding equation either by trial and error or with a financial calculator results in a realized yield of 4.91 percent annually. The difference between the realized yield and the promised yield in this case is accounted for by the capital loss of \$97.37 (\$1,000 − \$902.63) suffered when the bond was sold before maturity. In sum, realized yield is useful because it allows an individual investor or financial institution to evaluate the return on a bond ex post (after the end of the holding period or investment horizon).

- Investor's *ex post* or actual rate of return, given the cash flows actually received and their timing may differ from YTM or Expected Yield due to—
  - change in the amount or timing of promised payments (e.g. default).
  - change in market interest rates affecting premium or discount.

Total Return

- Unfortunately, both the expected yield and realized yield calculations assume that we will be able to reinvest the coupon payments at the calculated yield.
- But, if we know (or if we are willing to make an explicit assumption about) the actual reinvestment rate, we can calculate something called the total return on a bond.
- It is the return we receive on a bond that considers capital gains or losses and changes in the reinvestment rate.

- Yield metric that considers capital gains or losses as well as changes in reinvestment rate.
- Find interest rate that compounds initial purchase price to sum of terminal value of bond plus future value of all coupon payments received (based on known or assumed reinvestment rate).

Problem - Total Return

To calculate the total return for a bond, we must first determine two things: (1) the terminal value of the bond (the selling price if we sell the bond, the call price if the bond is called prior to maturity, or the par or face value of the bond if we hold it until maturity), and (2) the accumulated future value of all the coupon payments we received based on a known (or assumed) reinvestment rate. Once we know these two values, we determine the interest rate that equates our initial purchase price to the sum of these two values over the number of periods in our holding period.

Consider the previous example in which we purchased a 10-year, 8 percent coupon (annual payments) bond at par. We sold the bond after 3 years for $902.63 (the bond's terminal value).

Assuming that he reinvested the coupon payments at the initial promised yield of 8 percent, we would calculate the accumulated future value as follows:

You need:

Terminal Value of Bond

+ Future Value of Reinvested Interest Coupons

= Total Accumulated

Computing Total Return

$$FV\ of\ Annuity = P\left[\frac{(1+r)^n - 1}{r}\right]$$

$P = Periodic\ Payment$
$r = rate\ per\ period$
$n = number\ of\ periods$

$$\begin{aligned} FV_c &= C\,[(1+i)^n - 1]/i \\ &= \$80\,[(1+0.08)^3 - 1]/0.08 \\ &= \$259.71 \end{aligned}$$

Solve for *(i)*

Purchase Price of Bond = Total Accumlated / $(1+i)^n$

$$\$1{,}000 = \frac{\$902.63 + \$259.71}{(1+i)^3}$$

Solving for ***i***, we learn that the total return for this investment is 5.14 percent. This is higher than the 4.91 percent we calculated earlier because our assumed reinvestment rate of 8 percent is higher than the implicitly assumed reinvestment rate of 4.91 percent in the realized yield calculation.

Bond theorems

- Bond prices are inversely related to bond yields.
- The price volatility of a long-term bond is greater than that of a short-term bond, holding the coupon rate constant.
- The price volatility of a low-coupon bond is greater than that of a high-coupon, bond, holding maturity constant

Thank you