اسم المقرر: نظرية الزمرة

الفرقة: الثالثة شعبة فيزياء (تربية عام)

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Examples of subgroups

Def.

For any element a from a group, we let $\langle a \rangle$ denote the set

$$\{a^n \mid n \in \mathbb{Z}\}$$

Theorem

Let G be a group, and let a be any element of G. Then $\langle a \rangle$ is a subgroup of G.

Proof

Since $a \in \langle a \rangle$, $\langle a \rangle$ is not empty. Let $a^n, a^m \in \langle a \rangle$. Then $a^n (a^m)^{-1} = a^{n-m} \in \langle a \rangle$. Hence $\langle a \rangle$ is a subgroup of G

Remarks

(1) The subgroup $\langle a \rangle$ is called the cyclic subgroup of G generated by a

(2) In the case that $G = \langle a \rangle$, we say that G is cyclic and a is a generator of G.

(3) Although the list $\ldots, a^{-2}, a^{-1}, a^{0}, a^{1}, a^{2}, \ldots$ has infinitely many entries, the set $\{a^n \mid n \in \mathbb{Z}\}$ might have only finitely many elements

(4) Note that, since $a^i a^j = a^{i+j} = a^{j+i}$
$= a^j + a^i$,
every cyclic group is Abelian

Example in $\mathbb{Z}_{10}$, $\langle 2 \rangle = \{2, 4, 6, 8, 10\}$.