```
(***Lecture 2 22-10-2013
```

2 ****)

Defining functions

```
?? List
```

$\{e_1, e_2, \ldots\}$ is a list of elements. ≫

```
Attributes[List] = {Locked, Protected}
```

```
?? Count
```

Count[*list*, *pattern*] gives the number of elements in *list* that match *pattern*.
Count[*expr*, *pattern*, *levelspec*] gives the total number of subexpressions matching *pattern* that appear at the levels in *expr* specified by *levelspec*. ≫

```
Attributes[Count] = {Protected}

Options[Count] = {Heads → False}
```

```
Length[{1, 2, 3, 4, 9}]
```

```
5
```

```
, 9
```

```
f[n_] := n^2 + 4
g[x_] := 5 x^3 + 2 x^2 + 4 x + 5
f[Eman]
g[f[amany]]
```

$4 + \text{Eman}^2$

$5 + 4\left(4 + \text{amany}^2\right) + 2\left(4 + \text{amany}^2\right)^2 + 5\left(4 + \text{amany}^2\right)^3$

```
g[1]
f[-2.33]
f[ahmad]
f[x]
f[11]
g[Pi]
N[%]
?? N
```

16

9.4289

$4 + \text{ahmad}^2$

$4 + x^2$

125

$5 + 4\pi + 2\pi^2 + 5\pi^3$

192.337

N[*expr*] gives the numerical value of *expr*.

N[*expr*, *n*] attempts to give a result with *n*-digit precision. ≫

```
Attributes[N] = {Protected}

N /: Default[N, 2] := {MachinePrecision, MachinePrecision}
```

```
N[Pi, 20]
```

3.1415926535897932385

```
f[x_, y_] := x^2 + y^3
f[2, 6]
f[m, n]
f[ssss, dddd]
f[x_, y_, z_] := x^2 + y^3 + z^2
f[1, 2, 3]
```

220

$m^2 + n^3$

$\text{dddd}^3 + \text{ssss}^2$

18

```
f[n_] := n^2 + 4;
g[x_] = 5 x^3 + 2 x^2 + 4 x + 5;
g[f[g[y]]]
```

$$5 + 4\left(4 + \left(5 + 4y + 2y^2 + 5y^3\right)^2\right) + 2\left(4 + \left(5 + 4y + 2y^2 + 5y^3\right)^2\right)^2 + 5\left(4 + \left(5 + 4y + 2y^2 + 5y^3\right)^2\right)^3$$

```
2 d
d2
d 2
```

Problem 2.1

Using *Mathematica*, show that

$$\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+x}}}} = \frac{3+2x}{5+3x}.$$

```
f[x_] := 1/(1 + x)
ff = f[f[f[f[x]]]]
Simplify[%]
```

$$\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+x}}}}$$

$$\frac{3+2x}{5+3x}$$

Problem 2.2

Design a function to check whether for a number n, the formula $n!+1$ generates a prime number.

```
h[n_] := n ! + 1
h[3]
3 // h
h@2
PrimeQ[h[5]]
```

7

7

3

False

```
?? Binomial
```

Binomial[*n*, *m*] gives the binomial coefficient $\binom{n}{m}$. ≫

```
Attributes[Binomial] = {Listable, NumericFunction, Protected}
```

Problem 2.3

Define the function

$$b(n) = 1 + \binom{n}{1} + \binom{n}{2} + \binom{n}{3},$$

```
b[n_] := 1 + Binomial[n, 1] + Binomial[n, 2] + Binomial[n, 3]
b[14]
(*jjhkjkmkmk*)
```

470

```
?? Clear
```

Clear[$symbol_1$, $symbol_2$, …] clears values and definitions for the $symbol_i$.
Clear["$form_1$", "$form_2$", …] clears values and definitions for all symbols whose names match any of the string patterns $form_i$. ≫

Attributes[Clear] = {HoldAll, Protected}

```
?? Mod
```

Mod[m, n] gives the remainder on division of m by n.
Mod[m, n, d] uses an offset d. ≫

Attributes[Mod] = {Listable, NumericFunction, Protected}

```
Mod[18, 5]
Divisible[16, 5]
```

3

False

```
?? #
```

represents the first argument supplied to a pure function.
#n represents the n^{th} argument. ≫

Attributes[Slot] = {NHoldAll, Protected}

2.2 Anonymous functions ;

```
(#^2 + 4) &[2]
```

8

```
Sqrt[#1^2 + #2^2] &[2, 3]
```

$\sqrt{13}$

3 Lists

```
{2, 1, x} == {2, 1, x}
{1, 2} == {1, 2}
{1, 2, 3} == {1, 2}
```

```
True

True

False
```

```
p = {1, 2, 3, r, f, 6};
p[[5]]
p[[-3]]
p[[{-2, 3}]]
First[p]
Last[p]
```

```
f

r

{f, 3}

1

6
```

?? Drop

Drop[*list*, *n*] gives *list* with its first *n* elements dropped.
Drop[*list*, −*n*] gives *list* with its last *n* elements dropped.
Drop[*list*, {*n*}] gives *list* with its n^{th} element dropped.
Drop[*list*, {*m*, *n*}] gives *list* with elements *m* through *n* dropped.
Drop[*list*, {*m*, *n*, *s*}] gives *list* with elements *m* through *n* in steps of *s* dropped.
Drop[*list*, seq_1, seq_2, …] gives a nested list in which elements specified by seq_i have been dropped at level *i* in *list*. ≫

```
Attributes[Drop] = {NHoldRest, Protected}
```

?? Take

Take[*list*, *n*] gives the first *n* elements of *list*.
Take[*list*, −*n*] gives the last *n* elements of *list*.
Take[*list*, {*m*, *n*}] gives elements *m* through *n* of *list*.
Take[*list*, seq_1, seq_2, …] gives a nested list in which elements specified by seq_i are taken at level *i* in *list*. ≫

```
Attributes[Take] = {NHoldRest, Protected}
```

?? Rest

Rest[*expr*] gives *expr* with the first element removed. ≫

```
Attributes[Rest] = {Protected}
```

```
?? Most
```

Most[*expr*] gives *expr* with the last element removed. >>

```
Attributes[Most] = {Protected}
```

Problem 3.1

Let p={a,b,{c,d},e}. From this list produce the list {a,b,c,d,e}.

```
p1 = {a, b, {c, d}, e}
{p1[[1]], p1[[2]], p1[[3, 1]], p1[[3, 2]], p1[[4]]}
```

```
{a, b, {c, d}, e}

{a, b, c, d, e}
```

```
qx = {x1, x2, x3, x4, x5};
qy = {y1, y2, y3, y4, y5};
qx + qy
qx / qy
qx qy
{qx, qy}
```

```
{x1 + y1, x2 + y2, x3 + y3, x4 + y4, x5 + y5}
```

$$\left\{\frac{x1}{y1}, \frac{x2}{y2}, \frac{x3}{y3}, \frac{x4}{y4}, \frac{x5}{y5}\right\}$$

```
{x1 y1, x2 y2, x3 y3, x4 y4, x5 y5}

{{x1, x2, x3, x4, x5}, {y1, y2, y3, y4, y5}}
```

```
Section 1;
Ex1 :
 Expand[(1 + x) ^8]
```

Ex1 : $1 + 8 x + 28 x^2 + 56 x^3 + 70 x^4 + 56 x^5 + 28 x^6 + 8 x^7 + x^8$

```
Factor[%]
```

Factor$\left[1 + 8 x + 28 x^2 + 56 x^3 + 70 x^4 + 56 x^5 + 28 x^6 + 8 x^7 + x^8\right]$

$(1 + x)^8$

$(1 + x)^8$

```
Ex2 : f[x_, y_] := Sin[x]^2 + Cos[x] Tan[y] + Cos[y]^2
f[Pi / 2, Pi]
```

2

```
Ex3 :
    f[x_] := Sin[x] Cos[x];
g[y_] := y^2 + 2;
f[f[f[x]]]
g[f[y]]
```

```
Cos[Cos[Cos[x] Sin[x]] Sin[Cos[x] Sin[x]]]
 Sin[Cos[Cos[x] Sin[x]] Sin[Cos[x] Sin[x]]]
```

```
?? Range
```

Range[i_{max}] generates the list {1, 2, …, i_{max}}.
Range[i_{min}, i_{max}] generates the list {i_{min}, …, i_{max}}.
Range[i_{min}, i_{max}, *di*] uses step *di*. ≫

```
Attributes[Range] = {Listable, Protected}
```

```
Range[15]
Range[3, 20, 4]
```

```
{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}

{3, 7, 11, 15, 19}
```

```
(*****************************************************************29-
 10-2013********************)
?? Table
```

Table[*expr*, {i_{max}}] generates a list of i_{max} copies of *expr*.
Table[*expr*, {*i*, i_{max}}] generates a list of the values of *expr* when *i* runs from 1 to i_{max}.
Table[*expr*, {*i*, i_{min}, i_{max}}] starts with $i = i_{min}$.
Table[*expr*, {*i*, i_{min}, i_{max}, *di*}] uses steps *di*.
Table[*expr*, {*i*, {i_1, i_2, …}}] uses the successive values i_1, i_2, ….
Table[*expr*, {*i*, i_{min}, i_{max}}, {*j*, j_{min}, j_{max}}, …] gives a nested list. The list associated with *i* is outermost. ≫

```
Attributes[Table] = {HoldAll, Protected}
```

```
Table[x^(n + 1), {n, -5, 5, 2}] // TableForm
```

$\frac{1}{x^4}$

$\frac{1}{x^2}$

1

x^2

x^4

x^6

Problem 3.3

Find all natural numbers n between 1 and 15 for which the polynomial $x^n + 64$ can be written as a product of two nonconstant polynomials with integer coefficients.

```
(******only for n =3k or n=4k************)
```

```
Factor[x^6 + 64];
Table[x^n + 64, {n, 1, 15}];
Table[Factor[x^n + 64], {n, 1, 15}] // TableForm
```

$64+x$
$64+x^2$
$(4+x)\left(16-4x+x^2\right)$
$\left(8-4x+x^2\right)\left(8+4x+x^2\right)$
$64+x^5$
$\left(4+x^2\right)\left(16-4x^2+x^4\right)$
$64+x^7$
$\left(8-4x^2+x^4\right)\left(8+4x^2+x^4\right)$
$\left(4+x^3\right)\left(16-4x^3+x^6\right)$
$64+x^{10}$
$64+x^{11}$
$\left(2-2x+x^2\right)\left(2+2x+x^2\right)\left(4-4x+2x^2-2x^3+x^4\right)\left(4+4x+2x^2+2x^3+x^4\right)$
$64+x^{13}$
$64+x^{14}$
$\left(4+x^5\right)\left(16-4x^5+x^{10}\right)$

```
?? Print
```

Print[*expr*] prints *expr* as output. ≫

```
Attributes[Print] = {Protected}
```

```
Table[Print[n, " - ", Factor[x^n + 64]], {n, 1, 15}];
```

1 - $64+x$

2 - $64+x^2$

3 - $(4+x)\left(16-4x+x^2\right)$

4 - $\left(8-4x+x^2\right)\left(8+4x+x^2\right)$

5 - $64+x^5$

6 - $\left(4+x^2\right)\left(16-4x^2+x^4\right)$

7 - $64+x^7$

8 - $\left(8-4x^2+x^4\right)\left(8+4x^2+x^4\right)$

9 - $\left(4+x^3\right)\left(16-4x^3+x^6\right)$

10 - $64+x^{10}$

11 - $64+x^{11}$

12 - $\left(2-2x+x^2\right)\left(2+2x+x^2\right)\left(4-4x+2x^2-2x^3+x^4\right)\left(4+4x+2x^2+2x^3+x^4\right)$

13 - $64+x^{13}$

14 - $64+x^{14}$

15 - $\left(4+x^5\right)\left(16-4x^5+x^{10}\right)$

Problem 3.4

Determine all the positive integers n between 3 and 50 for which 2^{2008} is divisible by

$$1+\binom{n}{1}+\binom{n}{2}+\binom{n}{3}.$$

```
Mod[15, 4]
```

3

```
b[n_] := 1 + Binomial[n, 1] + Binomial[n, 2] + Binomial[n, 3];
Table[Mod[2^2008, b[n]], {n, 3, 50}]
```

{0, 1, 16, 16, 0, 70, 16, 80, 168, 133, 268, 316, 448, 256, 706, 796, 1096, 723, 1092, 1030, 0, 256, 458, 1240, 2704, 2604, 606, 2922, 640, 1664, 2704, 4076, 2824, 1936, 1024, 8336, 256, 7882, 6974, 4192, 4568, 6061, 1076, 6896, 704, 16 669, 6856, 16 032}

Example 3.5

This example uses BarChart which is a graphic function.

```
?? BarChart
```

BarChart[{y_1, y_2, …}] makes a bar chart with bar lengths y_1, y_2,….
BarChart[{…, w_i[y_i, …], …, w_j[y_j, …], …}] makes a bar chart with bar features defined by the symbolic wrappers w_k.
BarChart[{$data_1$, $data_2$, …}] makes a bar chart from multiple datasets $data_i$. ≫

Attributes[BarChart] = {Protected, ReadProtected}

```
ChartLabels -> {"a", "b", "c"}
```

ChartLabels → {a, b, c}

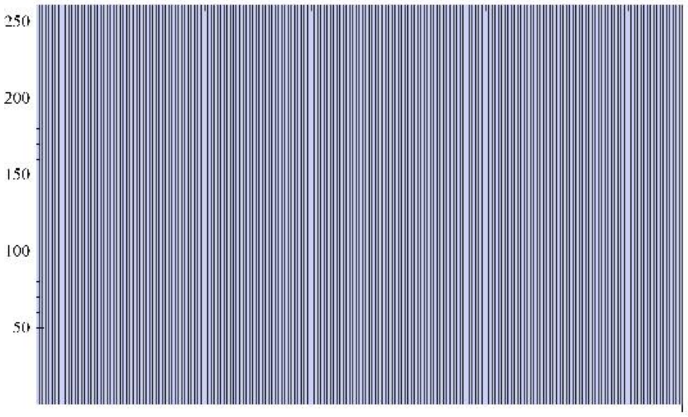
```
?? RandomInteger
```

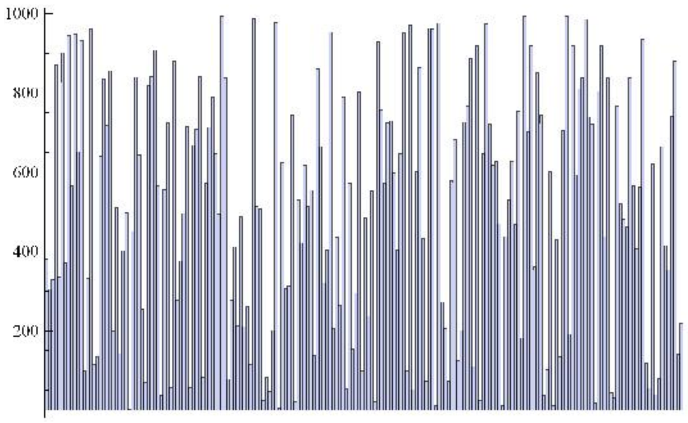
RandomInteger[{i_{min}, i_{max}}] gives a pseudorandom integer in the range {i_{min}, ..., i_{max}}.
RandomInteger[i_{max}] gives a pseudorandom integer in the range {0, ..., i_{max}}.
RandomInteger[] pseudorandomly gives 0 or 1.
RandomInteger[*range*, *n*] gives a list of *n* pseudorandom integers.
RandomInteger[*range*, {n_1, n_2, …}] gives an $n_1 \times n_2 \times \ldots$ array of pseudorandom integers. ≫

Attributes[RandomInteger] = {Protected}

```
xx = RandomInteger[{1, 1000}]
BarChart[Table[xx, {200}]]
yy := RandomInteger[{1, 1000}]
BarChart[Table[yy, {200}]]
```

261





3.2 Listable functions

```
Sin[{Pi / 4, Pi / 3, Pi / 2, Pi}]
cc = {Pi / 4, Pi / 3, Pi / 2, Pi}
cc / 4
cc 2
```

$\left\{\frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}, 1, 0\right\}$

$\left\{\frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi\right\}$

$\left\{\frac{\pi}{16}, \frac{\pi}{12}, \frac{\pi}{8}, \frac{\pi}{4}\right\}$

$\left\{\frac{\pi}{2}, \frac{2\pi}{3}, \pi, 2\pi\right\}$

```
bb = Table[n (n + 1) (n + 2) (n + 3) + 1, {n, 1, 15}]
Sqrt[bb]
```

```
{25, 121, 361, 841, 1681, 3025, 5041, 7921,
 11 881, 17 161, 24 025, 32 761, 43 681, 57 121, 73 441}

{5, 11, 19, 29, 41, 55, 71, 89, 109, 131, 155, 181, 209, 239, 271}
```

```
?? Map
```

Map[*f*, *expr*] or *f* /@ *expr* applies *f* to each element on the first level in *expr*.
Map[*f*, *expr*, *levelspec*] applies *f* to parts of *expr* specified by *levelspec*. »

```
f[x_] := x^3;

Map[f, {1, 2, 3, 4, 5}]
f /@ {1, 2, 3, 4, 5}
```

```
{1, 8, 27, 64, 125}

{1, 8, 27, 64, 125}
```

Problem 3.6

The formula $n^2 + n + 41$ has a very interesting property. Observe that this formula produces prime numbers for all n between 0 and 39.

```
Table[n^2 + n + 41, {n, 0, 39}]
PrimeQ[%]
```

```
{41, 43, 47, 53, 61, 71, 83, 97, 113, 131, 151, 173, 197, 223,
 251, 281, 313, 347, 383, 421, 461, 503, 547, 593, 641, 691, 743, 797,
 853, 911, 971, 1033, 1097, 1163, 1231, 1301, 1373, 1447, 1523, 1601}

{True, True, True, True, True, True, True, True, True, True, True, True, True, True,
 True, True, True, True, True, True, True, True, True, True, True, True, True,
 True, True, True, True, True, True, True, True, True, True, True, True, True}
```

```
?? Select
```

Select[*list*, *crit*] picks out all elements e_i of *list* for which *crit*[e_i] is True.
Select[*list*, *crit*, *n*] picks out the first *n* elements for which *crit*[e_i] is True. »

```
Attributes[Select] = {Protected}
```

Problem 3.7

How many numbers of the form $3n^5 + 11$, when n varies from 1 to 2000, are prime?

```
gg = Table[3 n^5 + 11, {n, 1, 2000}];
Select[gg, PrimeQ];
Select[gg, OddQ];
Select[gg, EvenQ]
```

```
{14, 740, 9386, 50 432, 177 158, 483 164, 1 113 890, 2 278 136, 4 259 582, 7 428 308,
 12 252 314, 19 309 040, 29 296 886, 43 046 732, 61 533 458, 85 887 464, 117 406 190,
 157 565 636, 208 031 882, 270 672 608, 347 568 614, 441 025 340, 553 584 386,
```

688 035 032, 847 425 758, 1 035 075 764, 1 254 586 490, 1 509 853 136, 1 805 076 182,
2 144 772 908, 2 533 788 914, 2 977 309 640, 3 480 871 886, 4 050 375 332, 4 692 094 058,
5 412 688 064, 6 219 214 790, 7 119 140 636, 8 120 352 482, 9 231 169 208, 10 460 353 214,
11 817 121 940, 13 311 159 386, 14 952 627 632, 16 752 178 358, 18 720 964 364,
20 870 651 090, 23 213 428 136, 25 762 020 782, 28 529 701 508, 31 530 301 514,
34 778 222 240, 38 288 446 886, 42 076 551 932, 46 158 718 658, 50 551 744 664,
55 273 055 390, 60 340 715 636, 65 773 441 082, 71 590 609 808, 77 812 273 814,
84 459 170 540, 91 552 734 386, 99 115 108 232, 107 169 154 958, 115 738 468 964,
124 847 387 690, 134 521 003 136, 144 785 173 382, 155 666 534 108, 167 192 510 114,
179 391 326 840, 192 292 021 886, 205 924 456 532, 220 319 327 258, 235 508 177 264,
251 523 407 990, 268 398 290 636, 286 166 977 682, 304 864 514 408, 324 526 850 414,
345 190 851 140, 366 894 309 386, 389 675 956 832, 413 575 475 558, 438 633 509 564,
464 891 676 290, 492 392 578 136, 521 179 813 982, 551 297 990 708, 582 792 734 714,
615 710 703 440, 650 099 596 886, 686 008 169 132, 723 486 239 858, 762 584 705 864,
803 355 552 590, 845 851 865 636, 890 127 842 282, 936 238 803 008, 984 241 203 014,
1 034 192 643 740, 1 086 151 884 386, 1 140 178 853 432, 1 196 334 660 158,
1 254 681 606 164, 1 315 283 196 890, 1 378 204 153 136, 1 443 510 422 582,
1 511 269 191 308, 1 581 548 895 314, 1 654 419 232 040, 1 729 951 171 886,
1 808 216 969 732, 1 889 290 176 458, 1 973 245 650 464, 2 060 159 569 190,
2 150 109 440 636, 2 243 174 114 882, 2 339 433 795 608, 2 438 970 051 614,
2 541 865 828 340, 2 648 205 459 386, 2 758 074 678 032, 2 871 560 628 758,
2 988 751 878 764, 3 109 738 429 490, 3 234 611 728 136, 3 363 464 679 182,
3 496 391 655 908, 3 633 488 511 914, 3 774 852 592 640, 3 920 582 746 886,
4 070 779 338 332, 4 225 544 257 058, 4 384 980 931 064, 4 549 194 337 790,
4 718 291 015 636, 4 892 379 075 482, 5 071 568 212 208, 5 255 969 716 214,
5 445 696 484 940, 5 640 863 034 386, 5 841 585 510 632, 6 047 981 701 358,
6 260 171 047 364, 6 478 274 654 090, 6 702 415 303 136, 6 932 717 463 782,
7 169 307 304 508, 7 412 312 704 514, 7 661 863 265 240, 7 918 090 321 886,
8 181 126 954 932, 8 451 108 001 658, 8 728 170 067 664, 9 012 451 538 390,
9 304 092 590 636, 9 603 235 204 082, 9 910 023 172 808, 10 224 602 116 814,
10 547 119 493 540, 10 877 724 609 386, 11 216 568 631 232, 11 563 804 597 958,
11 919 587 431 964, 12 284 073 950 690, 12 657 422 878 136, 13 039 794 856 382,
13 431 352 457 108, 13 832 260 193 114, 14 242 684 529 840, 14 662 793 896 886,
15 092 758 699 532, 15 532 751 330 258, 15 982 946 180 264, 16 443 519 650 990,
16 914 650 165 636, 17 396 518 180 682, 17 889 306 197 408, 18 393 198 773 414,
18 908 382 534 140, 19 435 046 184 386, 19 973 380 519 832, 20 523 578 438 558,
21 085 834 952 564, 21 660 347 199 290, 22 247 314 453 136, 22 846 938 136 982,
23 459 421 833 708, 24 084 971 297 714, 24 723 794 466 440, 25 376 101 471 886,
26 042 104 652 132, 26 722 018 562 858, 27 416 059 988 864, 28 124 447 955 590,
28 847 403 740 636, 29 585 150 885 282, 30 337 915 206 008, 31 105 924 806 014,
31 889 410 086 740, 32 688 603 759 386, 33 503 740 856 432, 34 335 058 743 158,
35 182 797 129 164, 36 047 198 079 890, 36 928 506 028 136, 37 826 967 785 582,
38 742 832 554 308, 39 676 351 938 314, 40 627 779 955 040, 41 597 373 046 886,
42 585 390 092 732, 43 592 092 419 458, 44 617 743 813 464, 45 662 610 532 190,
46 726 961 315 636, 47 811 067 397 882, 48 915 202 518 608, 50 039 642 934 614,
51 184 667 431 340, 52 350 557 334 386, 53 537 596 521 032, 54 746 071 431 758,
55 976 271 081 764, 57 228 487 072 490, 58 503 013 603 136, 59 800 147 482 182,
61 120 188 138 908, 62 463 437 634 914, 63 830 200 675 640, 65 220 784 621 886,
66 635 499 501 332, 68 074 658 020 058, 69 538 575 574 064, 71 027 570 260 790,
72 541 962 890 636, 74 082 076 998 482, 75 648 238 855 208, 77 240 777 479 214,
78 860 024 647 940, 80 506 314 909 386, 82 179 985 593 632, 83 881 376 824 358,
85 610 831 530 364, 87 368 695 457 090, 89 155 317 178 136, 90 971 048 106 782,
92 816 242 507 508, 94 691 257 507 514, 96 596 453 108 240, 98 532 192 196 886,
100 498 840 557 932, 102 496 766 884 658, 104 526 342 790 664, 106 587 942 821 390,
108 681 944 465 636, 110 808 728 167 082, 112 968 677 335 808, 115 162 178 359 814,
117 389 620 616 540, 119 651 396 484 386, 121 947 901 354 232, 124 279 533 640 958,

126 646 694 794 964, 129 049 789 313 690, 131 489 224 753 136, 133 965 411 739 382,
136 478 763 980 108, 139 029 698 276 114, 141 618 634 532 840, 144 245 995 771 886,
146 912 208 142 532, 149 617 700 933 258, 152 362 906 583 264, 155 148 260 693 990,
157 974 202 040 636, 160 841 172 583 682, 163 749 617 480 408, 166 699 985 096 414,
169 692 727 017 140, 172 728 298 059 386, 175 807 156 282 832, 178 929 763 001 558,
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