```
(***********   Lecture 6 *******)
```

10
Functions with multiple definitions

```
f[n_] := n^2 + 5
f[-4]
```

21

Problem 10.1

Define the function

$$f(x) = \begin{cases} \sqrt{x} & \text{if } x \geq 0 \\ \sqrt{-x} & \text{if } x < 0 \end{cases}$$

and plot the graph of the function for $-1 \leq x \leq 1$.

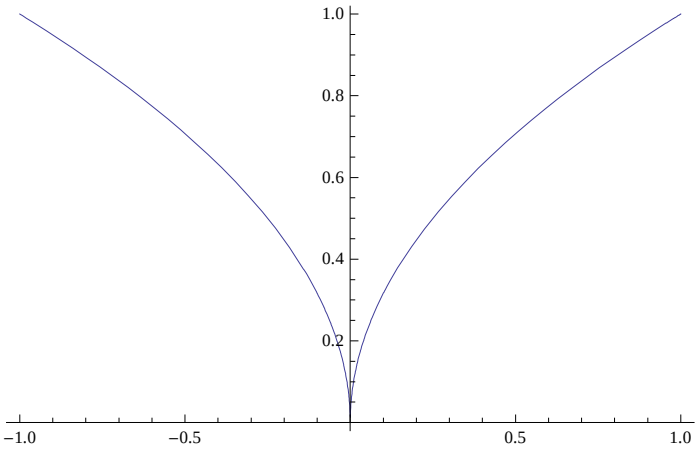
```
g[x_?Positive] = Sqrt[x]
g[x_?Negative] = Sqrt[-x]
g[-1.2]
g[5.]
Plot[g[x], {x, -1, 1}]
```

$\sqrt{x}$

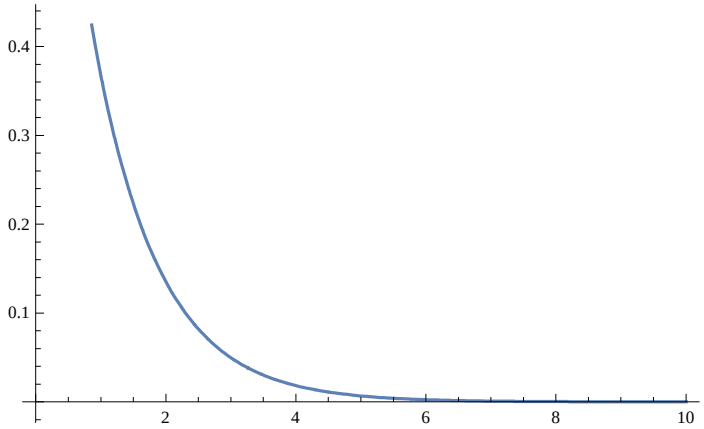
$\sqrt{-x}$

1.09545

2.23607



```
Plot[Exp[-x], {x, 0, 10}]
```



12 Linear algebra

```
a = {x1, y1, z1}; b = {x2, y2, z2};
a - b;
Dot[a, b];
Cross[a, b]
a × b;
```

```
Norm[a]
```

$$\sqrt{\text{Abs}[x1]^2 + \text{Abs}[y1]^2 + \text{Abs}[z1]^2}$$

```
?? Cross
```

Cross[*a*, *b*] gives the vector cross product of *a* and *b*. ≫

```
Attributes[Cross] = {Protected, ReadProtected}
```

```
Cross[{a, b, c}, {x, y, z}]
```

$$\{-c\,y + b\,z,\ c\,x - a\,z,\ -b\,x + a\,y\}$$

```
?? Abs
```

Abs[*z*] gives the absolute value of the real or complex number *z*. ≫

```
Attributes[Abs] = {Listable, NumericFunction, Protected}
```

```
A1 = {{1, 2, 6}, {9, 6, 2}, {8, 11, 10}} // MatrixForm;
A1
```

$$\begin{pmatrix} 1 & 2 & 6 \\ 9 & 6 & 2 \\ 8 & 11 & 10 \end{pmatrix}$$

```
A1 = {{1, 4, 6}, {9, 6, 2}, {8, 11, 10}};
A1 // MatrixForm;
Det[A1];
Inverse[A1] // MatrixForm // N;
Transpose[A1] // MatrixForm;
Dot[A1, A1] // MatrixForm
```

$$\begin{pmatrix} 85 & 94 & 74 \\ 79 & 94 & 86 \\ 187 & 208 & 170 \end{pmatrix}$$

```
A = {{11, 65, 51, 84, 10}, {12, 22, 11, 12, 19}, {3, 32, 33, 23, 25}, {4, 45, 87, 98, 13}, {6, 65, 66, 34, 26}}
```

```
{{11, 65, 51, 84, 10}, {12, 22, 11, 12, 19},
 {3, 32, 33, 23, 25}, {4, 45, 87, 98, 13}, {6, 65, 66, 34, 26}}
```

```
A // MatrixForm
```

$$\begin{pmatrix} 11 & 65 & 51 & 84 & 10 \\ 12 & 22 & 11 & 12 & 19 \\ 3 & 32 & 33 & 23 & 25 \\ 4 & 45 & 87 & 98 & 13 \\ 6 & 65 & 66 & 34 & 26 \end{pmatrix}$$

```
A.A // MatrixForm
```

$$\begin{pmatrix} 1450 & 8207 & 10\,927 & 11\,449 & 3972 \\ 591 & 3391 & 3515 & 3347 & 1463 \\ 758 & 4615 & 5245 & 4499 & 2412 \\ 1315 & 9289 & 12\,954 & 12\,923 & 4682 \\ 1336 & 7152 & 7873 & 7018 & 4063 \end{pmatrix}$$

```
Det[A]
```

```
24 800 152
```

```
Inverse[A] // MatrixForm
```

$$\begin{pmatrix} -\frac{281\,187}{12\,400\,076} & \frac{2\,792\,025}{24\,800\,152} & -\frac{706\,805}{6\,200\,038} & \frac{596\,911}{24\,800\,152} & \frac{595\,997}{24\,800\,152} \\ \frac{69\,111}{3\,100\,019} & -\frac{128\,997}{6\,200\,038} & \frac{14\,648}{3\,100\,019} & -\frac{137\,979}{6\,200\,038} & \frac{81\,925}{6\,200\,038} \\ -\frac{23\,311}{953\,852} & \frac{24\,257}{1\,907\,704} & -\frac{16\,593}{476\,926} & \frac{36\,727}{1\,907\,704} & \frac{45\,661}{1\,907\,704} \\ \frac{162\,839}{12\,400\,076} & -\frac{165\,069}{24\,800\,152} & \frac{148\,011}{6\,200\,038} & \frac{65\,677}{24\,800\,152} & -\frac{606\,745}{24\,800\,152} \\ -\frac{5377}{953\,852} & \frac{4695}{1\,907\,704} & \frac{34\,145}{476\,926} & -\frac{4295}{1\,907\,704} & -\frac{55\,101}{1\,907\,704} \end{pmatrix}$$

Generate a 3×2 array:

```
?? Dot
```

a.b.c or Dot[*a*, *b*, *c*] gives products of vectors, matrices, and tensors. ≫

```
Attributes[Dot] = {Flat, OneIdentity, Protected}
```

```
A3 = {{2, 0, 4}, {3, 4, 5}, {4, 5, 6}};
Det[A3]
A2 = Inverse[A3] // MatrixForm
Dot[A2, A2] // MatrixForm // N
Det[A3];
A3 A3;
```

$$-6$$

$$\begin{pmatrix} \frac{1}{6} & -\frac{10}{3} & \frac{8}{3} \\ -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{6} & \frac{5}{3} & -\frac{4}{3} \end{pmatrix}$$

$$\begin{pmatrix} 0.166667 & -3.33333 & 2.66667 \\ -0.333333 & 0.666667 & -0.333333 \\ 0.166667 & 1.66667 & -1.33333 \end{pmatrix} . \begin{pmatrix} 0.166667 & -3.33333 & 2.66667 \\ -0.333333 & 0.666667 & -0.333333 \\ 0.166667 & 1.66667 & -1.33333 \end{pmatrix}$$

```
Eigenvalues[A3] // N
```

$$\{12.4807, -0.480741, 0.\}$$

```
Eigenvectors[A1] // N // MatrixForm
```

$$\begin{pmatrix} 0.444492 & 0.471074 & 1. \\ 0.964777 - 0.776783\,i & -1.68974 + 0.687128\,i & 1. \\ 0.964777 + 0.776783\,i & -1.68974 - 0.687128\,i & 1. \end{pmatrix}$$

```
Integrate[x^2, {x, 0, 5}]
Integrate[Cos[x] Tan[x], x]
```

$$\frac{125}{3}$$

$$-\text{Cos}[x]$$