7

Loops and repetitions

?? Do

Do[*expr*, {i_{max}}] evaluates *expr* i_{max} times.
Do[*expr*, {*i*, i_{max}}] evaluates *expr* with the variable *i* successively taking on the values 1 through i_{max} (in steps of 1).
Do[*expr*, {*i*, i_{min}, i_{max}}] starts with $i = i_{min}$.
Do[*expr*, {*i*, i_{min}, i_{max}, *di*}] uses steps *di*.
Do[*expr*, {*i*, {i_1, i_2, …}}] uses the successive values i_1, i_2, ….
Do[*expr*, {*i*, i_{min}, i_{max}}, {*j*, j_{min}, j_{max}}, …] evaluates *expr* looping over different values of *j*, etc. for each *i*. ≫

```
Attributes[Do] = {HoldAll, Protected}
```

```
Do[Print[n " --- " x^n], {n, 10}]
```

```
 --- x
2 --- x^2
3 --- x^3
4 --- x^4
5 --- x^5
6 --- x^6
7 --- x^7
8 --- x^8
9 --- x^9
10 --- x^10
```

```
Do[Print[ x^(n - m)], {n, 1, 3}, {m, 1, 5}]
```

1

$\frac{1}{x}$

$\frac{1}{x^2}$

$\frac{1}{x^3}$

$\frac{1}{x^4}$

x

1

$\frac{1}{x}$

$\frac{1}{x^2}$

$\frac{1}{x^3}$

x^2

x

1

$\frac{1}{x}$

$\frac{1}{x^2}$

?? While

While[*test*, *body*] evaluates *test*, then *body*, repetitively, until *test* first fails to give True. ≫

```
Attributes[While] = {HoldAll, Protected}
```

```
n = 0; While[n < 10, Print[n]; n = n + 1]
```

0

1

2

3

4

5

6

7

8

9

?? For

For[*start*, *test*, *incr*, *body*] executes *start*, then repeatedly evaluates *body* and *incr* until *test* fails to give True. ≫

```
Attributes[For] = {HoldAll, Protected}
```

```
For[i = 3, i < 14, i++, Print[i]]
```

```
3
4
5
6
7
8
9
10
11
12
13
```

Problem 7.5

Find the smallest positive integer m such that $529^3 + 132^3 m$ is divisible by 262417.

```
Clear[m]
m = 1;
While[Mod[529^3 + 132^3 m, 262 417] != 0, m++]
m
```

```
1984
```

```
Mod[529^3 + 132^3 × 19, 262 417]
```

```
170 871
```

```
For[m = 1, Mod[529^3 + 132^3 m, 262 417] != 0, m++, m]
m
```

```
1984
```

Problem 7.9

Find the sum of the sequence

$$\frac{1}{1+2} \mid \frac{2}{2+3} \mid \cdots \mid \frac{10}{10+11}.$$

```
Sum[i / (i + i + 1), {i, 1, 10}]
```

$$\sum_{i=1}^{10} i / (i + i + 1.)$$

```
N[%]
```

$$\frac{64\,157\,087}{14\,549\,535}$$

```
4.40956
```

```
4.40956
```

```
For[{i = 1, b = 0}, i < 11, i++, b = b + i / (i + i + 1)]
b
N[b]
```

$$\frac{64\,157\,087}{14\,549\,535}$$

```
4.40956
```

7.2 Nested loops

```
Do[Do[Print[f[i, j]], {i, 1, 5}], {j, 1, 5}]
Do[Print[f[i, j]], {i, 1, 5}, {j, 1, 5}];
```

```
f[1, 1]
f[2, 1]
f[3, 1]
f[4, 1]
f[5, 1]
f[1, 2]
f[2, 2]
f[3, 2]
f[4, 2]
f[5, 2]
f[1, 3]
f[2, 3]
f[3, 3]
f[4, 3]
f[5, 3]
f[1, 4]
f[2, 4]
f[3, 4]
f[4, 4]
f[5, 4]
f[1, 5]
```

```
f[2, 5]
f[3, 5]
f[4, 5]
f[5, 5]
f[1, 1]
f[1, 2]
f[1, 3]
f[1, 4]
f[1, 5]
f[2, 1]
f[2, 2]
f[2, 3]
f[2, 4]
f[2, 5]
f[3, 1]
f[3, 2]
f[3, 3]
f[3, 4]
f[3, 5]
f[4, 1]
f[4, 2]
f[4, 3]
f[4, 4]
f[4, 5]
f[5, 1]
f[5, 2]
f[5, 3]
f[5, 4]
f[5, 5]
```

Homework

Problem 7.11

Find all the pairs (n, m) for $n, m \leq 10$ such that $n^2 + m^2$ is a squared number (e.g., $(3,4)$ as $3^2 + 4^2 = 5^2$).

```
Clear[m, n]
Table[Sqrt[m^2 + n^2], {m, 1, 10}, {n, 1, 10}]
Select[Table[Sqrt[m^2 + n^2], {m, 1, 10}, {n, 1, 10}], IntegerQ]
```

```
Select[{Sqrt[2], Sqrt[5], Sqrt[10], Sqrt[17], Sqrt[26], Sqrt[37], 5 Sqrt[2], Sqrt[65], Sqrt[82], Sqrt[101], Sqrt[5], 2 Sqrt[2],
  Sqrt[13], 2 Sqrt[5], Sqrt[29], 2 Sqrt[10], Sqrt[53], 2 Sqrt[17], Sqrt[85], 2 Sqrt[26], Sqrt[10], Sqrt[13], 3 Sqrt[2], 5,
  Sqrt[34], 3 Sqrt[5], Sqrt[58], Sqrt[73], 3 Sqrt[10], Sqrt[109], Sqrt[17], 2 Sqrt[5], 5, 4 Sqrt[2], Sqrt[41], 2 Sqrt[13],
  Sqrt[65], 4 Sqrt[5], Sqrt[97], 2 Sqrt[29], Sqrt[26], Sqrt[29], Sqrt[34], Sqrt[41], 5 Sqrt[2], Sqrt[61], Sqrt[74], Sqrt[89],
  Sqrt[106], 5 Sqrt[5], Sqrt[37], 2 Sqrt[10], 3 Sqrt[5], 2 Sqrt[13], Sqrt[61], 6 Sqrt[2], Sqrt[85], 10, 3 Sqrt[13],
  2 Sqrt[34], 5 Sqrt[2], Sqrt[53], Sqrt[58], Sqrt[65], Sqrt[74], Sqrt[85], 7 Sqrt[2], Sqrt[113], Sqrt[130], Sqrt[149],
  Sqrt[65], 2 Sqrt[17], Sqrt[73], 4 Sqrt[5], Sqrt[89], 10, Sqrt[113], 8 Sqrt[2], Sqrt[145], 2 Sqrt[41], Sqrt[82], Sqrt[85],
  3 Sqrt[10], Sqrt[97], Sqrt[106], 3 Sqrt[13], Sqrt[130], Sqrt[145], 9 Sqrt[2], Sqrt[181], Sqrt[101], 2 Sqrt[26],
  Sqrt[109], 2 Sqrt[29], 5 Sqrt[5], 2 Sqrt[34], Sqrt[149], 2 Sqrt[41], Sqrt[181], 10 Sqrt[2]}, IntegerQ]
```

```
{5, 5, 10, 10}
```

```
?? Select
```

Select[*list*, *crit*] picks out all elements e_i of *list* for which *crit*[e_i] is True.
Select[*list*, *crit*, *n*] picks out the first *n* elements for which *crit*[e_i] is True. ≫

```
Attributes[Select] = {Protected}
```

```
?? Nest
```

Nest[*f*, *expr*, *n*] gives an expression with *f* applied *n* times to *expr*. ≫

```
Attributes[Nest] = {Protected}
```

```
f[x_] := 1 / (1 + x)
Nest[f, x, 3]
```

$$\frac{1}{1+\frac{1}{1+\frac{1}{1+x}}}$$

```
?? NestList
```

NestList[*f*, *expr*, *n*] gives a list of the results of applying *f* to *expr* 0 through *n* times. ≫

```
Attributes[NestList] = {Protected}
```

```
NestList[f, x, 4]
```

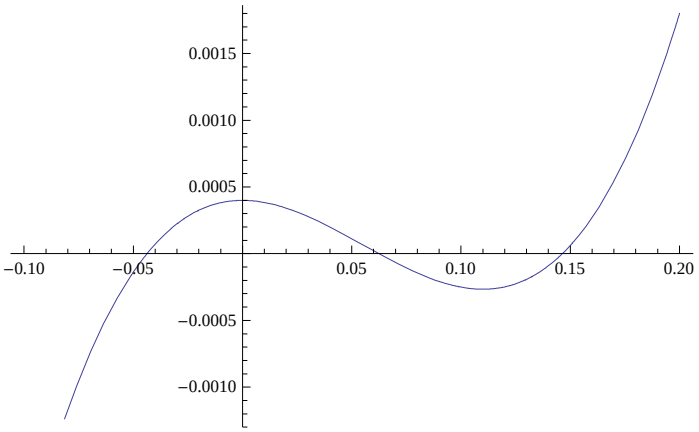
$$\left\{x, \frac{1}{1+x}, \frac{1}{1+\frac{1}{1+x}}, \frac{1}{1+\frac{1}{1+\frac{1}{1+x}}}, \frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+x}}}}\right\}$$

(*Bisection method $x^3 - 0.165x^2 + 3.993\times10^{-4} = 0$ *)

```
eq = x^3 - 0.165 x^2 + 3.993 × 10^-4;
FindRoot[eq == 0, {x, 0.2}]
NSolve[eq == 0, x]
Plot[eq, {x, -0.1, 0.2}]
```

```
{x → 0.14636}
```

```
{{x → -0.0437371}, {x → 0.0623776}, {x → 0.14636}}
```




```
f[x_] := x^3 - 0.165 x^2 + 3.993 × 10^-4;
xl = 0; xu = 0.11;
xm = (xl + xu) / 2;
f[xm];
f[xm] f[xu]
f[xm] f[xl]
```

```
-1.77156 × 10^-8
```

```
2.65734 × 10^-8
```

```
xl = xm
xm = (xl + xu) / 2
f[xm];
f[xm] f[xu]
f[xm] f[xl]
```

```
0.055
```

```
0.0825
```

```
4.31818 × 10^-8
```

```
-1.07954 × 10^-8
```

```
xu = xm
xm = (xl + xu) / 2
f[xm];
f[xm] f[xu]
f[xm] f[xl]
```

0.0825

0.06875

9.02432×10^{-9}

-3.70229×10^{-9}

```
xu = xm
xm = (xl + xu) / 2
f[xm];
f[xm] f[xu]
f[xm] f[xl]
```

0.06875

0.061875

-2.4947×10^{-10}

2.98432×10^{-10}

```
xl = xm
xm = (xl + xu) / 2
f[xm];
f[xm] f[xu]
f[xm] f[xl]
```

0.061875

0.0653125

1.44304×10^{-9}

-1.1632×10^{-10}

```
xu = xm
xm = (xl + xu) / 2
f[xm];
f[xm] f[xu]
f[xm] f[xl]
```

0.0653125

0.0635938

2.80237×10^{-10}

-4.84469×10^{-11}

```
xu = xm
xm = (xl + xu) / 2
f[xm];
f[xm] f[xu]
f[xm] f[xl]
```

```
0.0635938
```

```
0.0627344
```

$$3.43207 \times 10^{-11}$$

$$-1.42457 \times 10^{-11}$$

```
xu = xm
xm = (xl + xu) / 2
f[xm];
f[xm] f[xu]
f[xm] f[xl]
```

```
0.0627344
```

```
0.0623047
```

$$-2.06404 \times 10^{-12}$$

$$2.91359 \times 10^{-12}$$

```
xl = xm
xm = (xl + xu) / 2
f[xm];
f[xm] f[xu]
f[xm] f[xl]
```

```
0.0623047
```

```
0.0625195
```

$$4.01725 \times 10^{-12}$$

$$-8.21624 \times 10^{-13}$$

```
xl = xm
xm = (xl + xu) / 2
f[xm];
f[xm] f[xu]
f[xm] f[xl]
```

```
0.0625195
```

```
0.062627
```

$$7.05542 \times 10^{-12}$$

$$2.80852 \times 10^{-12}$$

Home work use Do or While

```
?? While
```

While[*test*, *body*] evaluates *test*, then *body*, repetitively, until *test* first fails to give True. ≫

```
Attributes[While] = {HoldAll, Protected}
```