13 Graphics

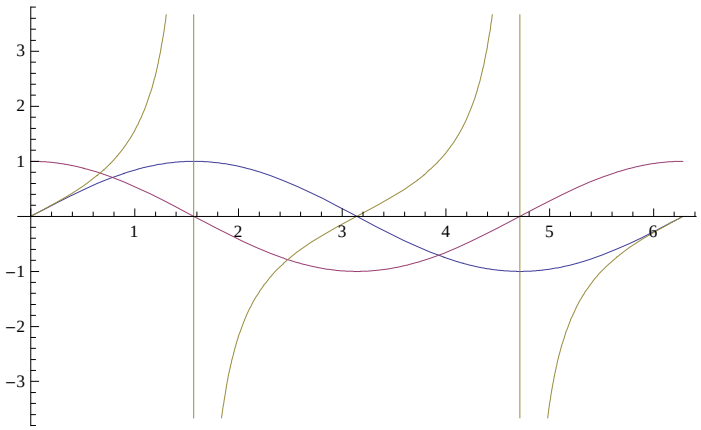
?? Plot

Plot[f, {x, x_{min}, x_{max}}] generates a plot of f as a function of x from x_{min} to x_{max}.
Plot[{f_1, f_2, …}, {x, x_{min}, x_{max}}] plots several functions f_i. ≫

```
Attributes[Plot] = {HoldAll, Protected}
```

```
Options[Plot] = {AlignmentPoint → Center, AspectRatio → 1/GoldenRatio,
  Axes → True, AxesLabel → None, AxesOrigin → Automatic, AxesStyle → {},
  Background → None, BaselinePosition → Automatic, BaseStyle → {},
  ClippingStyle → None, ColorFunction → Automatic, ColorFunctionScaling → True,
  ColorOutput → Automatic, ContentSelectable → Automatic,
  CoordinatesToolOptions → Automatic, DisplayFunction :→ $DisplayFunction,
  Epilog → {}, Evaluated → System'Private'$Evaluated, EvaluationMonitor → None,
  Exclusions → Automatic, ExclusionsStyle → None, Filling → None,
  FillingStyle → Automatic, FormatType :→ TraditionalForm, Frame → False,
  FrameLabel → None, FrameStyle → {}, FrameTicks → Automatic, FrameTicksStyle → {},
  GridLines → None, GridLinesStyle → {}, ImageMargins → 0., ImagePadding → All,
  ImageSize → Automatic, ImageSizeRaw → Automatic, LabelStyle → {},
  MaxRecursion → Automatic, Mesh → None, MeshFunctions → {#1 &}, MeshShading → None,
  MeshStyle → Automatic, Method → Automatic, PerformanceGoal :→ $PerformanceGoal,
  PlotLabel → None, PlotPoints → Automatic, PlotRange → {Full, Automatic},
  PlotRangeClipping → True, PlotRangePadding → Automatic, PlotRegion → Automatic,
  PlotStyle → Automatic, PreserveImageOptions → Automatic, Prolog → {},
  RegionFunction → (True &), RotateLabel → True, Ticks → Automatic,
  TicksStyle → {}, WorkingPrecision → MachinePrecision}
```

```
Plot[{Sin[x], Cos[x], Tan[x]}, {x, 0, 2 Pi}]
```

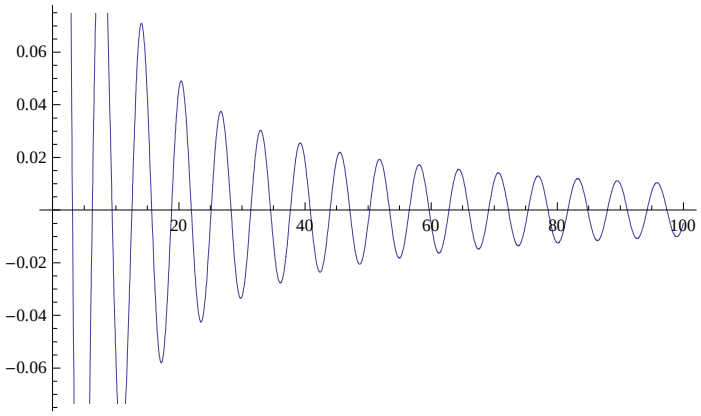


Problem 13.1

Plot the graph of the following functions:

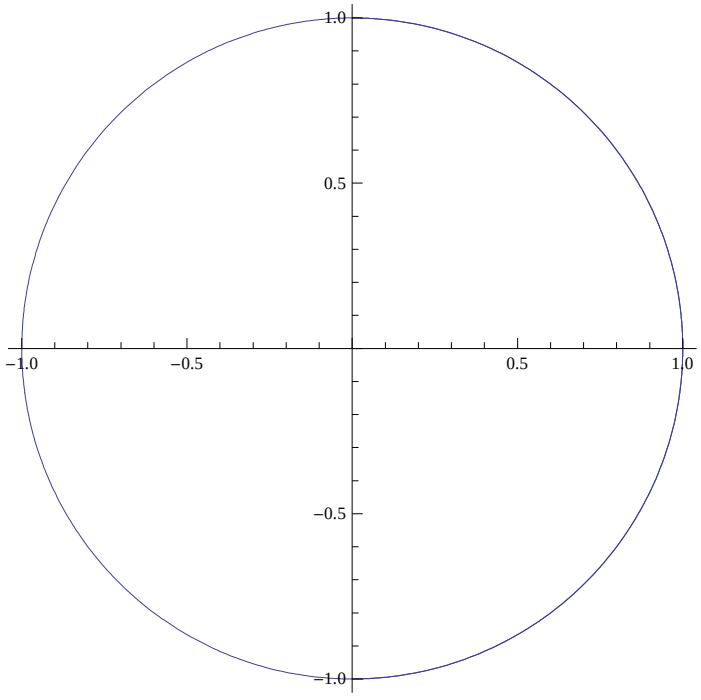
$$f(x) = \sin(x)/x$$
$$x = \sin(3t), y = \cos(4t)$$
$$x^4 - (x^2 - y^2) = 0$$
$$x^4 + (x - 2y^2) > 0$$
$$r = 3\cos(6\theta).$$

```
f[x_] := Sin[x] / x
Plot[f[x], {x, 0, 100}]
```




```
x = Sin[3 t]; y = Cos[3 t];
```

```
ParametricPlot[{x, y}, {t, 0, π}]
```



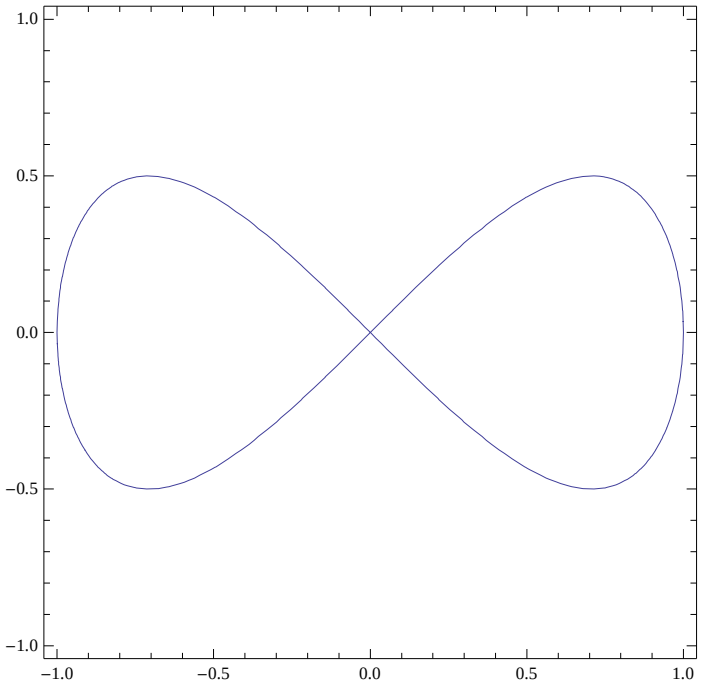
?? ContourPlot

ContourPlot[f, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}] generates a contour plot of f as a function of x and y.
ContourPlot[f == g, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}] plots contour lines for which $f = g$.
ContourPlot[{f_1 == g_1, f_2 == g_2, …}, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}] plots several contour lines. ≫

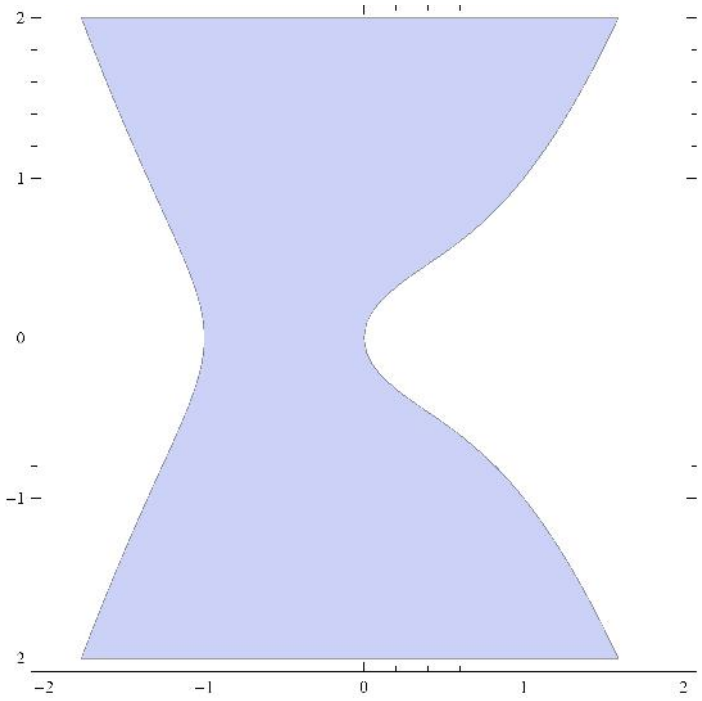
```
Attributes[ContourPlot] = {HoldAll, Protected}
```

```
Options[ContourPlot] = {AlignmentPoint → Center, AspectRatio → 1,
  Axes → False, AxesLabel → None, AxesOrigin → Automatic, AxesStyle → {},
  Background → None, BaselinePosition → Automatic, BaseStyle → {},
  BoundaryStyle → None, BoxRatios → Automatic, ClippingStyle → None,
  ColorFunction → Automatic, ColorFunctionScaling → True, ColorOutput → Automatic,
  ContentSelectable → Automatic, ContourLabels → Automatic, ContourLines → True,
  Contours → Automatic, ContourShading → Automatic, ContourStyle → Automatic,
  CoordinatesToolOptions → Automatic, DisplayFunction :→ $DisplayFunction,
  Epilog → {}, Evaluated → System`Private`$Evaluated, EvaluationMonitor → None,
  Exclusions → Automatic, ExclusionsStyle → None, FormatType :→ TraditionalForm,
  Frame → True, FrameLabel → None, FrameStyle → {}, FrameTicks → Automatic,
  FrameTicksStyle → {}, GridLines → None, GridLinesStyle → {}, ImageMargins → 0.,
  ImagePadding → All, ImageSize → Automatic, ImageSizeRaw → Automatic, LabelStyle → {},
  LightingAngle → None, MaxRecursion → Automatic, Mesh → None, MeshFunctions → {},
  MeshStyle → Automatic, Method → Automatic, PerformanceGoal :→ $PerformanceGoal,
  PlotLabel → None, PlotPoints → Automatic, PlotRange → {Full, Full, Automatic},
  PlotRangeClipping → True, PlotRangePadding → Automatic,
  PlotRegion → Automatic, PreserveImageOptions → Automatic,
  Prolog → {}, RegionFunction → (True &), RotateLabel → True,
  Ticks → Automatic, TicksStyle → {}, WorkingPrecision → MachinePrecision}
```

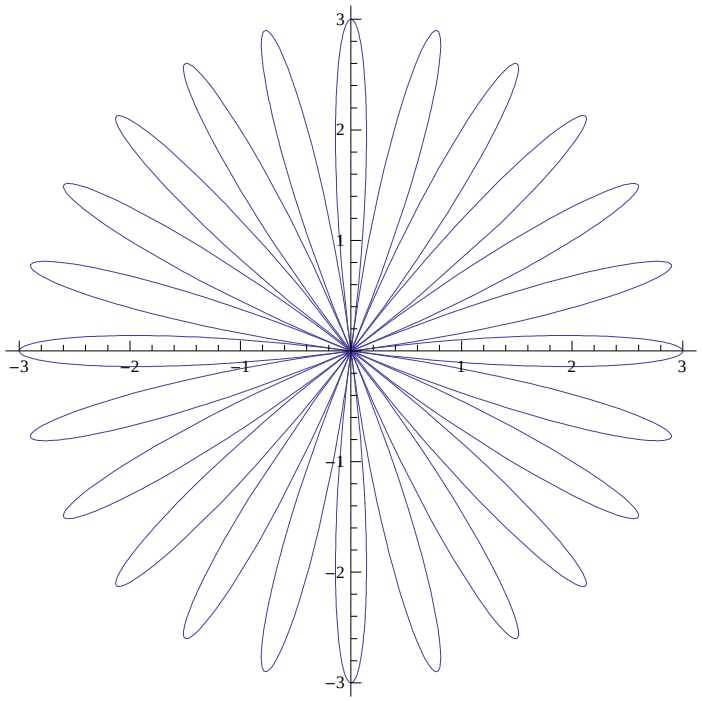
```
ContourPlot[x^4 - (x^2 - y^2) == 0, {x, -1, 1}, {y, -1, 1}]
```



```
RegionPlot[x^4 + (x - 2 y^2) <= 0, {x, -2, 2}, {y, -2, 2}]
```




```
PolarPlot[3 Cos[12 x], {x, 0, 2 Pi}]
```

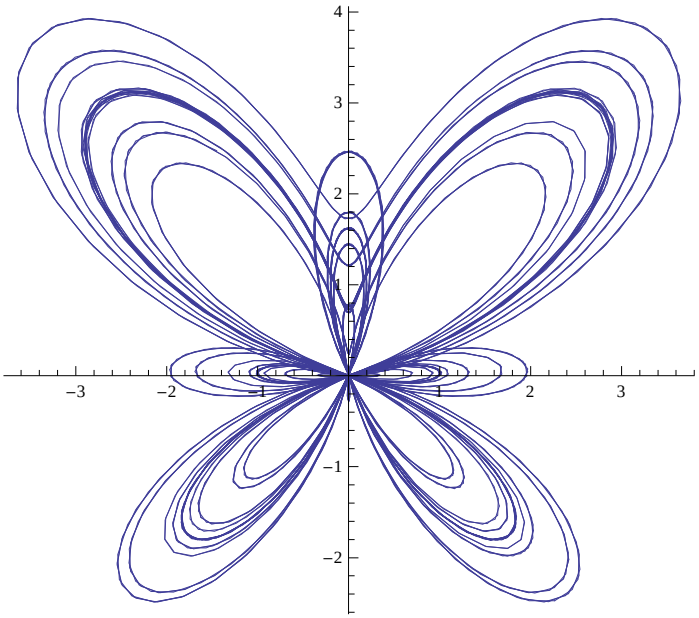


Problem 13.4

Draw the butterfly curve, discovered by Temple H. Fay, given by

$$x(t) = \sin(t)\left(e^{\cos(t)} - 2\cos(4t) - \sin^5(t/12)\right)$$
$$x(t) = \cos(t)\left(e^{\cos(t)} - 2\cos(4t) - \sin^5(t/12)\right)$$

```
x = Sin[t] (Exp[Cos[t]] - 2 Cos[4 t] - Sin[t / 12]^5);
y = Cos[t] (Exp[Cos[t]] - 2 Cos[4 t] - Sin[t / 12]^5);
ParametricPlot[{x, y}, {t, -100, 100}]
```




```
?? PlotStyle
```

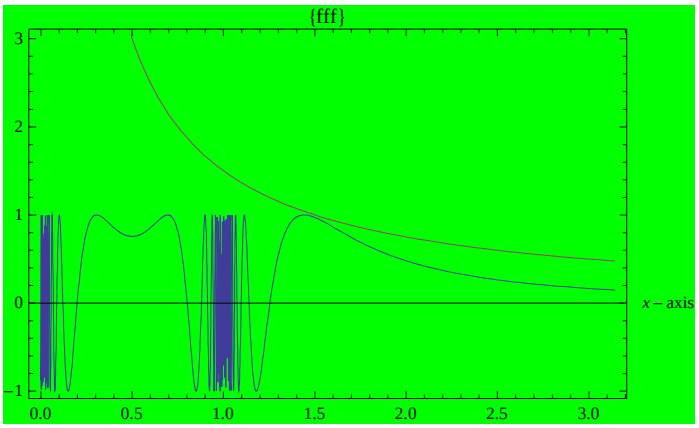
PlotStyle is an option for plotting and related functions that specifies styles in which objects are to be drawn. ≫

```
Attributes[PlotStyle] = {Protected}
```

Problem 13.5

Plot the graphs of the functions $\sin(\frac{1}{x^2-x})$ and $\frac{1.5}{x}$ in the range $[0, \pi]$.

```
Clear[x, y]
Plot[{Sin[1 / (x^2 - x)], 1.5 / x}, {x, 0, Pi}, AxesLabel → {x - axis, y - axis},
 PlotLabel → {fff}, Frame → True, Frame → Thick, Background → Green]
```



HomeWork

Problem 13.6

Plot the graph of the function

$$f(x) = \begin{cases} -x, & \text{if } |x| < 1 \\ \sin(x), & \text{if } 1 \le |x| < 2 \\ \cos(x), & \text{otherwise.} \end{cases}$$

13.2 Three-dimensional graphs

```
?? Plot3D
```

Plot3D[f, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}] generates a three–dimensional plot of f as a function of x and y.
Plot3D[{f_1, f_2, …}, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}] plots several functions. ≫

```
Attributes[Plot3D] = {HoldAll, Protected}

Options[Plot3D] =
 {AlignmentPoint → Center, AspectRatio → Automatic, AutomaticImageSize → False,
  Axes → True, AxesEdge → Automatic, AxesLabel → None, AxesOrigin → Automatic,
  AxesStyle → {}, Background → None, BaselinePosition → Automatic, BaseStyle → {},
  BoundaryStyle → GrayLevel[0], Boxed → True, BoxRatios → {1, 1, 0.4}, BoxStyle → {},
  ClippingStyle → Automatic, ColorFunction → Automatic, ColorFunctionScaling → True,
  ColorOutput → Automatic, ContentSelectable → Automatic, ControllerLinking → Automatic,
  ControllerMethod → Automatic, ControllerPath → Automatic,
  CoordinatesToolOptions → Automatic, DisplayFunction :→ $DisplayFunction,
  Epilog → {}, Evaluated → System‘Private‘$Evaluated, EvaluationMonitor → None,
  Exclusions → Automatic, ExclusionsStyle → None, FaceGrids → None, FaceGridsStyle → {},
  Filling → None, FillingStyle → Opacity[0.5], FormatType :→ TraditionalForm,
  ImageMargins → 0., ImagePadding → All, ImageSize → Automatic, LabelStyle → {},
  Lighting → Automatic, MaxRecursion → Automatic, Mesh → Automatic,
  MeshFunctions → {#1 &, #2 &}, MeshShading → None, MeshStyle → Automatic,
  Method → Automatic, NormalsFunction → Automatic, PerformanceGoal :→ $PerformanceGoal,
  PlotLabel → None, PlotPoints → Automatic, PlotRange → {Full, Full, Automatic},
  PlotRangePadding → Automatic, PlotRegion → Automatic, PlotStyle → Automatic,
  PreserveImageOptions → Automatic, Prolog → {}, RegionFunction → (True &),
  RotationAction → Fit, SphericalRegion → False, TextureCoordinateFunction → Automatic,
  TextureCoordinateScaling → Automatic, Ticks → Automatic, TicksStyle → {},
  ViewAngle → Automatic, ViewCenter → Automatic, ViewMatrix → Automatic,
  ViewPoint → {1.3, -2.4, 2.}, ViewRange → All, ViewVector → Automatic,
  ViewVertical → {0, 0, 1}, WorkingPrecision → MachinePrecision}
```

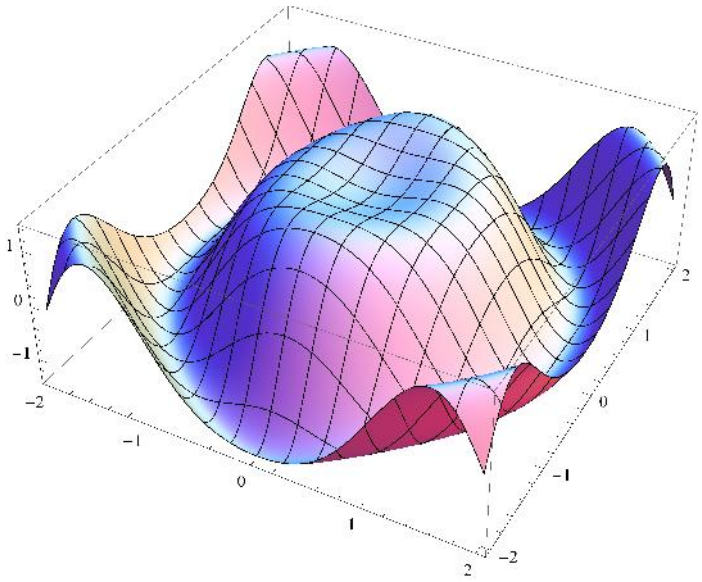
Problem 13.11

Plot the graph of the “cowboy hat” equation

$$\sin(x^2+y^2)e^{-x^2}+\cos(x^2+y^2)$$

as both x and y range from -2 to 2.

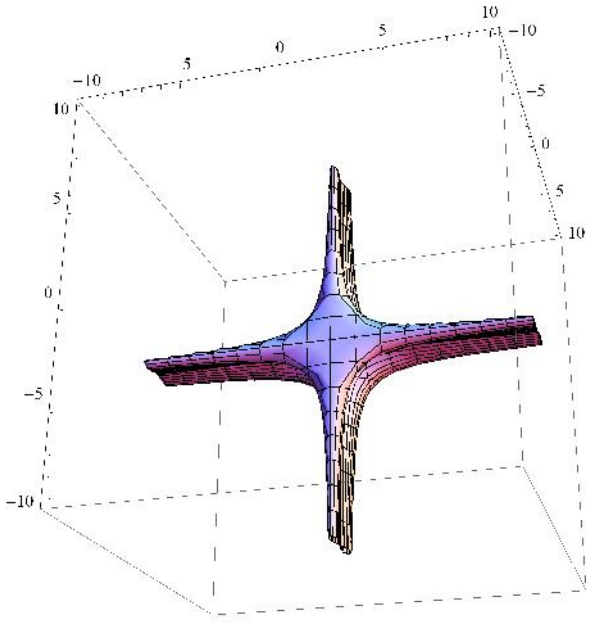
```
f[x_, y_] := Exp[-x^2] Sin[(x^2 + y^2)] + Cos[x^2 + y^2]
Plot3D[f[x, y], {x, -2, 2}, {y, -2, 2}, PlotPoints → 150]
```



Problem 13.13

Plot the graph of $6x^2 - 2x^4 - y^2z^2 = 0$.

```
ContourPlot3D[6 x^2 - 2 x^4 - y^2 z^2 == 0, {x, -10, 10}, {y, -10, 10}, {z, -10, 10}]
```

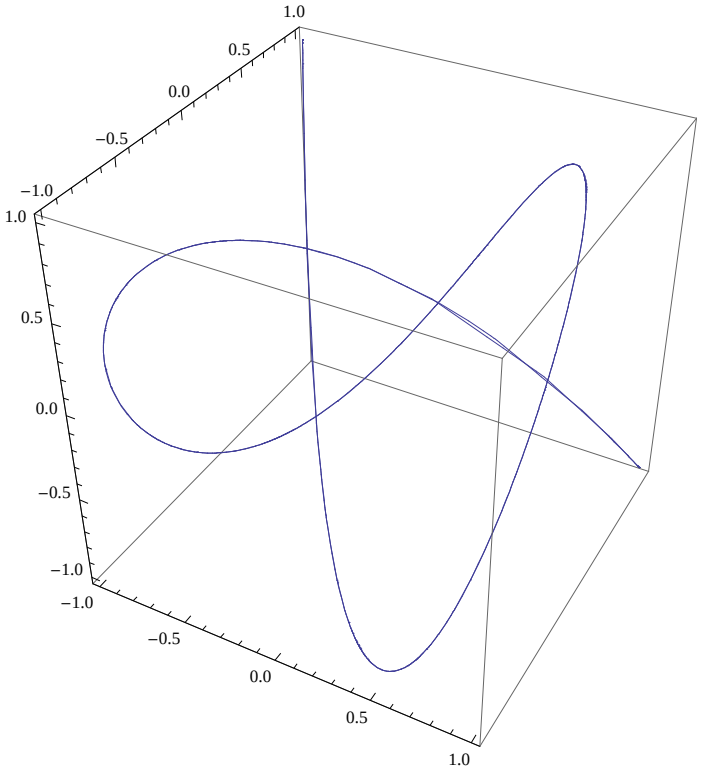


Homework

Problem 13.14

Plot the graph of $x = \sin(3t), y = \cos(4t), z = \sin(5t)$, for $-\pi \leq t \leq \pi$. Then create a dynamic setting and plot the graph of $x = \sin(nt), y = \cos(mt), z = \sin(5t)$ for $1 \leq n, m \leq 10$.

```
ParametricPlot3D[{x = Sin[3 t], y = Cos[4 t], z = Sin[5 t]}, {t, -Pi, Pi}]
```



14 Calculus and equations

Example	Commands to solve an equation
$x^4 - 3x^3 + 2x + 10 = 0$	`Solve`
$x^6 - 4x^3 + 12x + 10 = 0$	`NSolve`
$x^3 - 3x^2 + 5 < 0$	`Reduce`
$x^{x-10} = x^2$ and $x \in \mathbb{N}$	`FindInstance`
$\sin(x) = x - 1$	`FindRoot`

```
Solve[x^4 - 3 x^3 + 2 x + 10 == 0, x];
N[%]
NSolve[x^4 - 3 x^3 + 2 x + 10 == 0, x]
```

```
{{x → -0.822108 - 1.00134 i}, {x → -0.822108 + 1.00134 i},
 {x → 2.32211 - 0.75191 i}, {x → 2.32211 + 0.75191 i}}
```


```
{{x → -0.822108 - 1.00134 i}, {x → -0.822108 + 1.00134 i},
 {x → 2.32211 - 0.75191 i}, {x → 2.32211 + 0.75191 i}}
```


```
Reduce[x^3 - 3 x^2 + 5 < 0, x]
```

```
x < Root[5 - 3 #1^2 + #1^3 &, 1]
N[%]
```

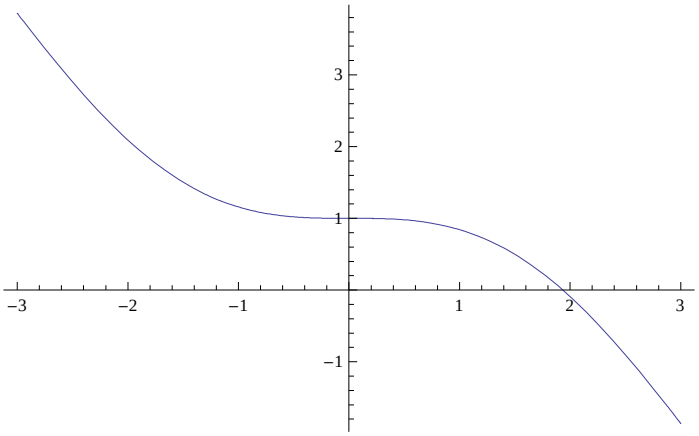
```
x < Root[5 - 3 #1^2 + #1^3 &, 1]
```

```
x < -1.1038
```

```
FindRoot[Sin[x] == x - 1, {x, 1}]
```

```
{x → 1.93456}
```

```
Plot[Sin[x] - x + 1 == 0, {x, -3, 3}]
```




```
Solve[{6 x + 2 y + z == 1, x + 2 y - 3 z == 0, x - y + z == 0}, {x, y, z}]
```

$$\left\{\left\{x \to \frac{1}{17},\ y \to \frac{4}{17},\ z \to \frac{3}{17}\right\}\right\}$$

```
?? D
```

D[f, x] gives the partial derivative $\partial f/\partial x$.
D[f, {x, n}] gives the multiple derivative $\partial^n f/\partial x^n$.
D[f, x, y, ...] differentiates f successively with respect to x, y, ...
D[f, {{x_1, x_2, ...}}] for a scalar f gives the vector derivative $(\partial f/\partial x_1, \partial f/\partial x_2, ...)$.
D[f, {*array*}] gives a tensor derivative. ≫

```
Attributes[D] = {Protected, ReadProtected}
```

```
Options[D] := {NonConstants → {}}
```

```
?? Integrate
```

Integrate[f, x] gives the indefinite integral $\int f\, dx$.

Integrate[f, {x, x_{min}, x_{max}}] gives the definite integral $\int_{x_{min}}^{x_{max}} f\, dx$.

Integrate[f, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}, ...] gives the multiple integral $\int_{x_{min}}^{x_{max}} dx \int_{y_{min}}^{y_{max}} dy \ldots f$. ≫

```
Attributes[Integrate] = {Protected, ReadProtected}
```

```
Options[Integrate] :=
 {Assumptions :→ $Assumptions, GenerateConditions → Automatic, PrincipalValue → False}
```

```
∫x^2 Cos[x] dx
```

```
∫_0^10. x^6 dx
```

$$2\,x\,\text{Cos}[x] + \left(-2 + x^2\right)\,\text{Sin}[x]$$

$$1.42857 \times 10^6$$

Problem 14.5

Evaluate the following:

$$\frac{\partial f}{\partial x}, \text{ when } f = \sin(x)/x,$$

$$\frac{\partial^2 f}{\partial x^2}, \text{ when } f = \sin(x)/x,$$

$$\frac{\partial^3 f}{\partial x^2 \partial y}, \text{ when } f = e^{xy},$$

$$\int \left(\cos(x)/x - \sin(x)/x^2\right) dx$$

$$\int_{-1}^{1}\int_{-1}^{1} \cos(x^2 + y^2 + xy)\, dx\, dy.$$

```
D[Sin[x] / x, x]
```

$$\frac{\text{Cos}[x]}{x} - \frac{\text{Sin}[x]}{x^2}$$

```
D[Sin[x] / x, {x, 2}]
```

$$-\frac{2\,\text{Cos}[x]}{x^2} + \frac{2\,\text{Sin}[x]}{x^3} - \frac{\text{Sin}[x]}{x}$$

```
D[Exp[x y], {x, 2}, {y, 3}]
```

$$6\,e^{x\,y}\,x + 6\,e^{x\,y}\,x^2\,y + e^{x\,y}\,x^3\,y^2$$

$\int$(Cos[x] / x - Sin[x] / x^2) dx

Integrate[Cos[x] / x - Sin[x] / x^2, x]

$$\frac{\text{Sin}[x]}{x}$$

$$\frac{\text{Sin}[x]}{x}$$

NIntegrate[Cos[x + y^2 + x y], {x, -1, 1}, {y, -1, 1}]

2.92072

Problem 14.6

Consider $f(x,y) = \sin(x+y)\cos(x^2 - y) - \sin(y)$ and generate the graphs of $\frac{\partial^2 f}{\partial x \partial y}$ over the rectangle $-\pi \leq x \leq \pi$ and $-\pi \leq y \leq \pi$. Find the maximum of the function $\frac{\partial^2 f}{\partial x \partial y}$ in this area.

?? D

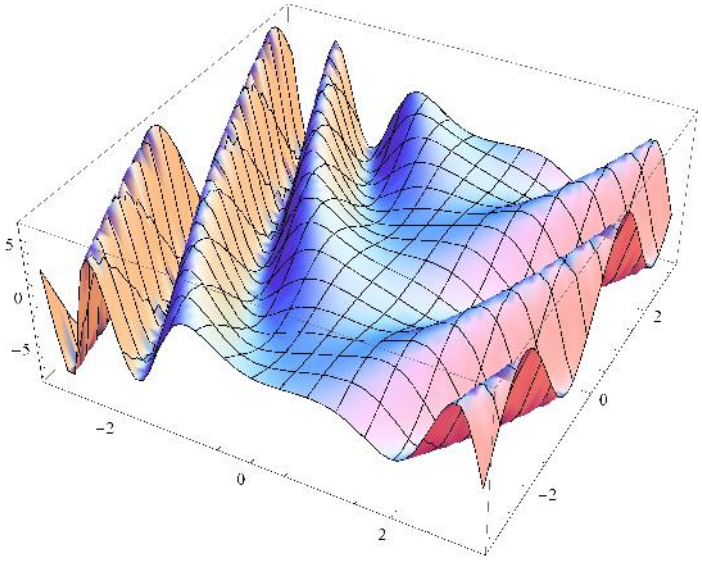
D[*f*, *x*] gives the partial derivative $\partial f / \partial x$.
D[*f*, {*x*, *n*}] gives the multiple derivative $\partial^n f / \partial x^n$.
D[*f*, *x*, *y*, …] differentiates *f* successively with respect to *x*, *y*, ….
D[*f*, {{x_1, x_2, …}}] for a scalar *f* gives the vector derivative $(\partial f / \partial x_1, \partial f / \partial x_2, \ldots)$.
D[*f*, {*array*}] gives a tensor derivative. ≫

Attributes[D] = {Protected, ReadProtected}

Options[D] := {NonConstants → {}}

```
f[x_, y_] := Sin[x + y] Cos[x^2 - y] - Sin[y];
s = D[f[x, y], x, y]
Plot3D[s, {x, -Pi, Pi}, {y, -Pi, Pi}]
NMaximize[{s, -Pi ≤ x ≤ Pi , -Pi ≤ y ≤ Pi }, {x, y}]
```

$$\text{Cos}[x+y]\,\text{Sin}\left[x^2-y\right]-2\,x\,\text{Cos}[x+y]\,\text{Sin}\left[x^2-y\right]-\text{Cos}\left[x^2-y\right]\,\text{Sin}[x+y]+2\,x\,\text{Cos}\left[x^2-y\right]\,\text{Sin}[x+y]$$



$$\{7.28319,\ \{x \to -3.14159,\ y \to 2.57861\}\}$$