CHAPTER

4

Mechanics for ENGINEERS:

STATICS

Equilibrium of Rigid Bodies

Application

Engineers designing this crane will need to determine the forces that act on this body under various conditions.

Reactions at supports

Why there must be a support (or supports)?

Why there must be a reaction (reactions)?

Introduction

- For a rigid body, the condition of static equilibrium means that the *body under study* does not translate or rotate under the given loads that act on the body

- The necessary and sufficient conditions for the static equilibrium of a body are that the forces sum to zero, and the moment about any point sum to zero:

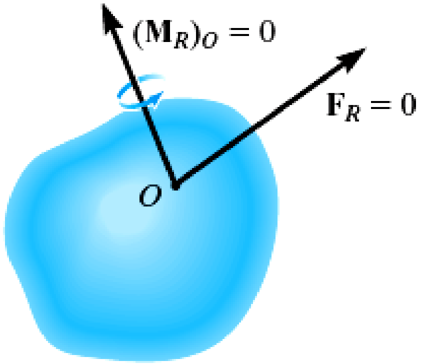
$$\sum \vec{F} = 0 \qquad \sum \vec{M}_O = \sum \left(\vec{r} \times \vec{F}\right) = 0$$

- Equilibrium analysis can be applied to two-dimensional or three-dimensional bodies, but the first step in any analysis is the creation of the *free body diagram*

5.1 Conditions for Rigid-Body Equilibrium

- For equilibrium when considering about O

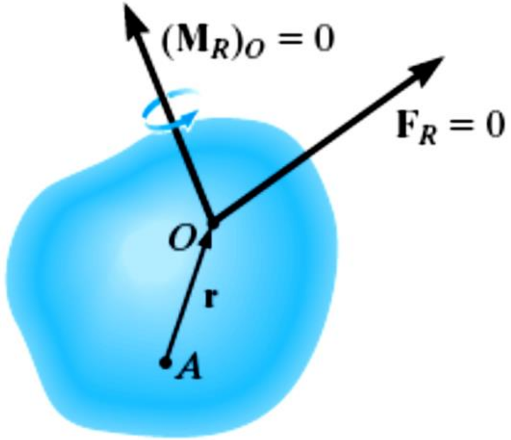
$$F_R = \sum F = 0$$



$$(M_R)_O = \sum M_O = 0$$

- For moment about any point, or in this case, point A
- For any point

$$\sum M_A = r \times F_R + (M_R)_O = 0$$



$$\sum F = 0 \quad \textbf{and} \quad \sum M = 0$$

Equilibrium in 2D

$$\sum F_x = 0 \ , \quad \sum F_y = 0 \ , \quad \sum M_o = 0$$

Where O is a convenient point. Try to choose it so that each equation has only one unknown.

OR

$$\sum M_A = 0 \ , \quad \sum M_B = 0 \ , \quad \sum M_o = 0$$

Equations of Equilibrium

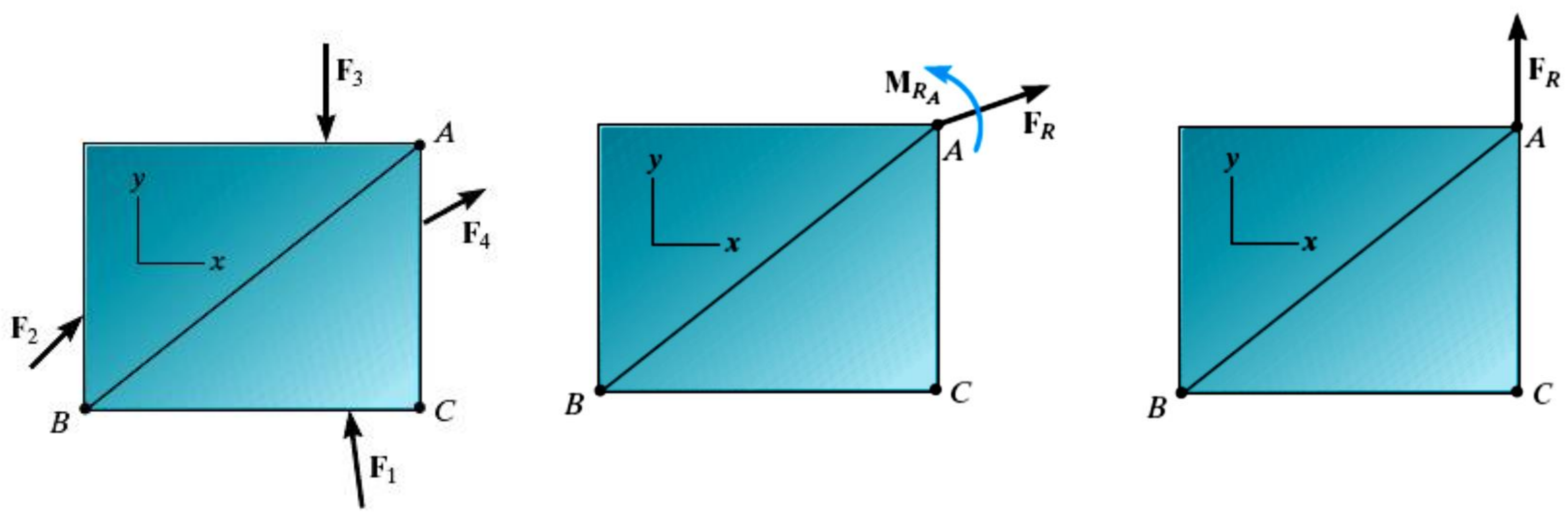
Sets of Equilibrium Equations

- For coplanar equilibrium problems,
  $\sum F_x = 0$; $\sum F_y = 0$; $\sum M_O = 0$

2 alternative sets of 3 independent equilibrium equations,

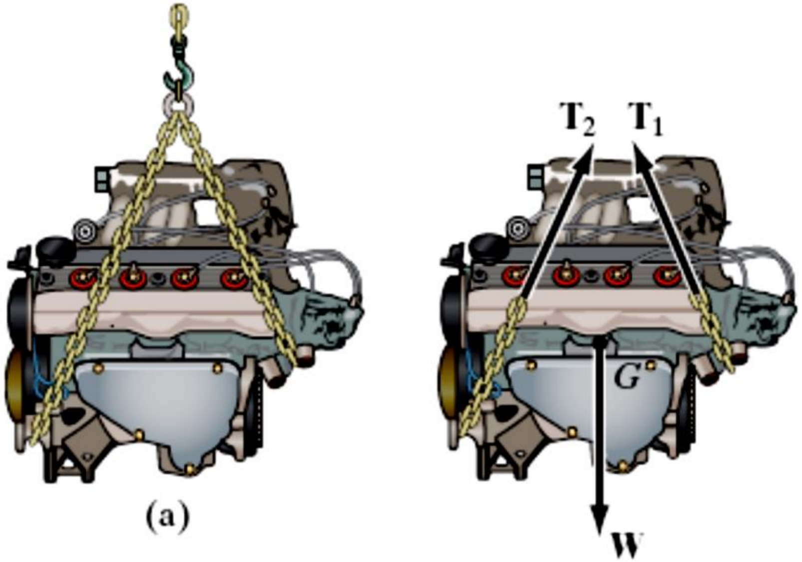
$\sum F_x = 0$; $\sum M_A = 0$; $\sum M_B = 0$ (A, B is not perpendicular to x)

$\sum M_A = 0$; $\sum M_B = 0$; $\sum M_C = 0$ (A, B, C are not on the same line)



Free Body Diagrams

- For a rigid body, there is both external and internal forces
- For FBD, internal forces within the FBD boundary is not needed
- For outside the boundary FBD, it is external forces
- The location of the weight W is at center of gravity



Free Body Diagrams

Procedure for Drawing a FBD

1. Sketch a body

- Consider if the body is free from all the connections

2. Draw all forces and moments

- Draw all external forces and moments acting on the body Include: applied loads, reactions due to supports, and the weight of the object.
- Draw self-weight if considered
- No need to scale arrow size

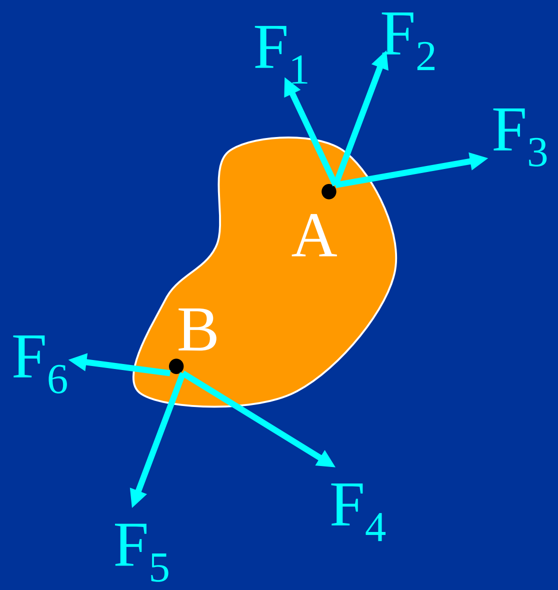
3. Indicate the magnitude and the directions of forces and moments

- Indicate all the magnitude and directions of forces and moments if known
- Split all components in x, y or a convenient reference
- Display all known distance

Two- and Three-Force Members

Two-Force Members

When the member is not subjected to a couple and the forces are applied only at two points, the member is said to be two-force member.

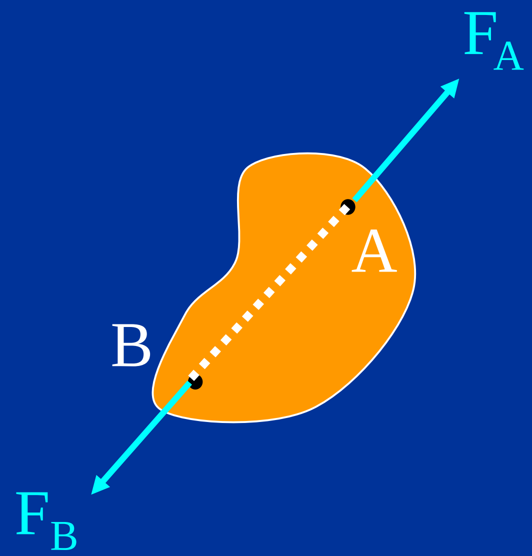


Let:

$$\vec{F}_A = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 \text{ and } \vec{F}_B = \vec{F}_4 + \vec{F}_5 + \vec{F}_6$$

These forces will maintain equilibrium if:

$$\vec{F}_A = -\vec{F}_B$$

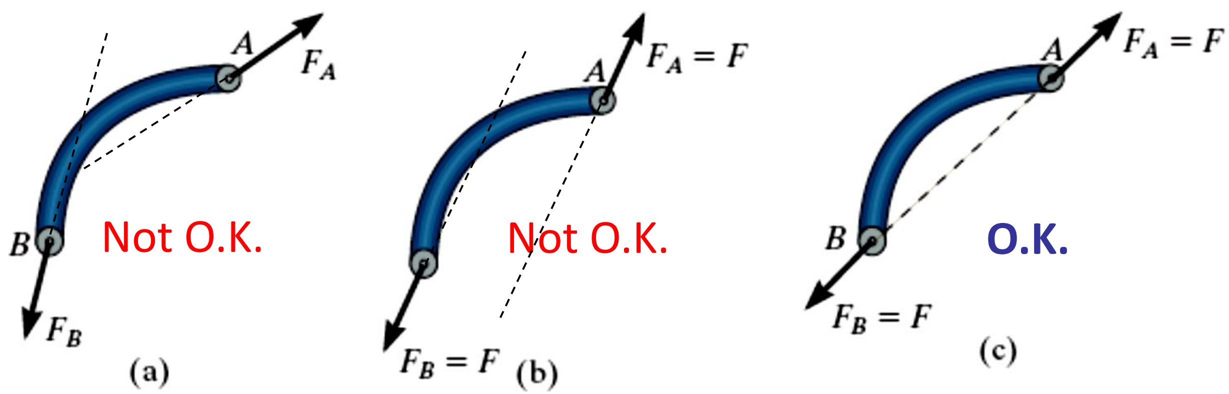


($\mathbf{F_A}$ and $\mathbf{F_B}$ must be collinear)

5.4 Two- and Three-Force Members

Two-Force Members

- is parts that contain on forces acting at two points, no moments
- The two forces are the same in magnitude but opposite directions and lie in the same line of action



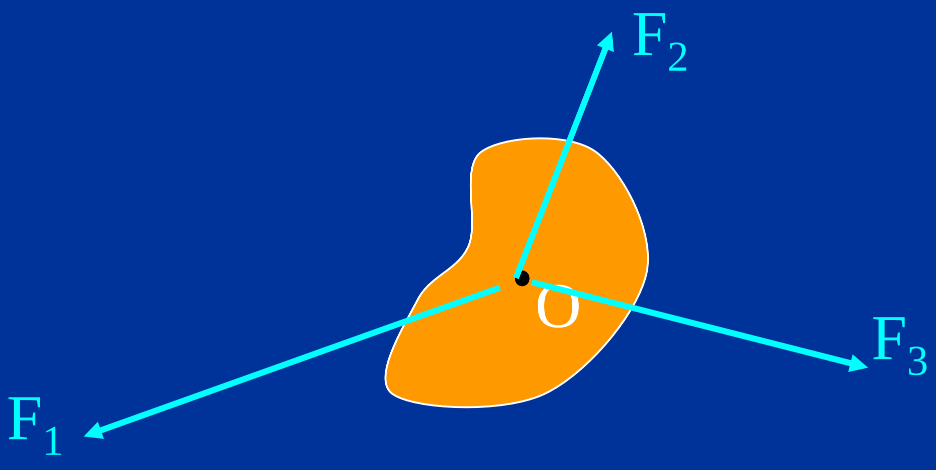
Two-force member

The two forces are in the same line of action

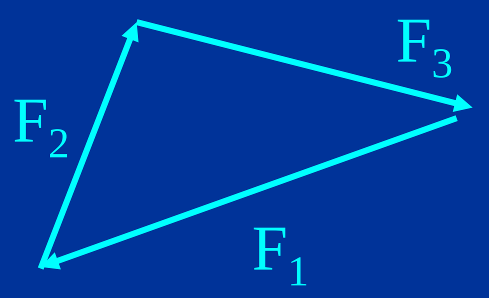
Two- and Three-Force Members

Three-Force Members

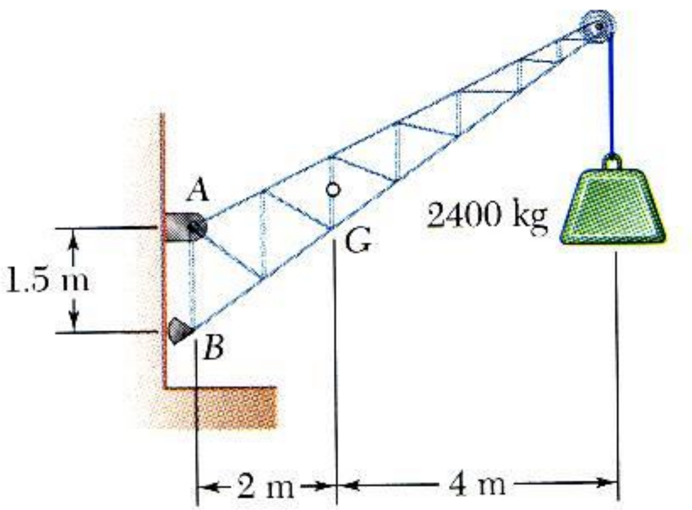
If the member is subjected to three coplanar forces, then it is necessary that the forces are either concurrent or parallel if the member is to be in equilibrium.

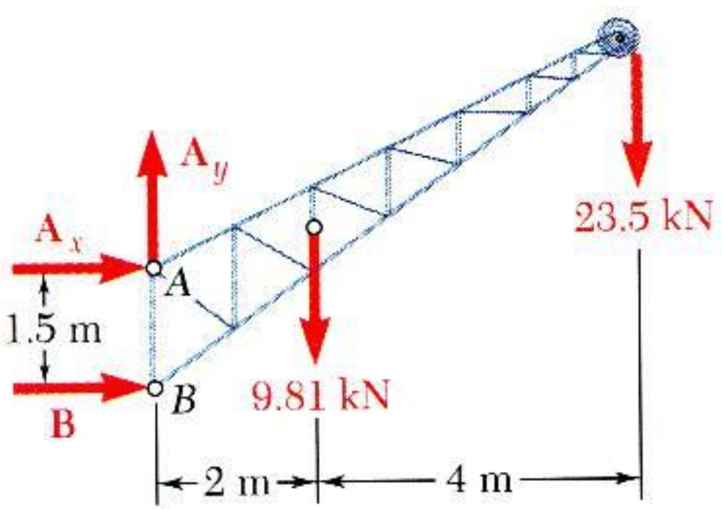


$\Rightarrow$



Free-Body Diagram

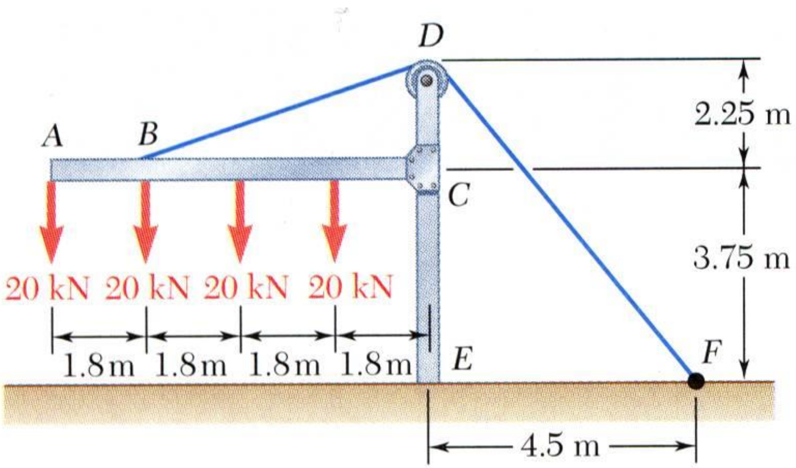




The first step in the static equilibrium analysis of a rigid body is identification of all forces acting on the body with a *free body diagram.*

- Select the body to be analyzed and detach it from the ground and all other bodies and/or supports.
- Indicate point of application, magnitude, and direction of external forces, including the rigid body weight.
- Indicate point of application and assumed direction of unknown forces from reactions of the ground and/or other bodies, such as the supports.
- Include the dimensions, which will be needed to compute the moments of the forces.

Practice

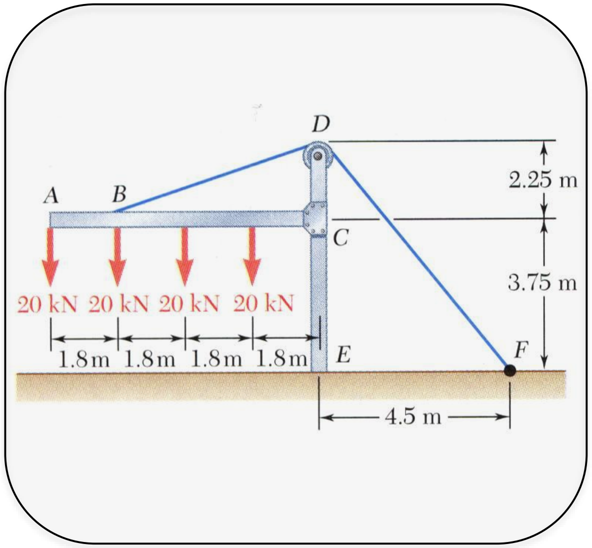


The frame shown supports part of the roof of a small building. Your goal is to draw the free body diagram (FBD) for the frame.

On the following page, you will choose the most correct FBD for this problem.

First, you should draw your own FBD.

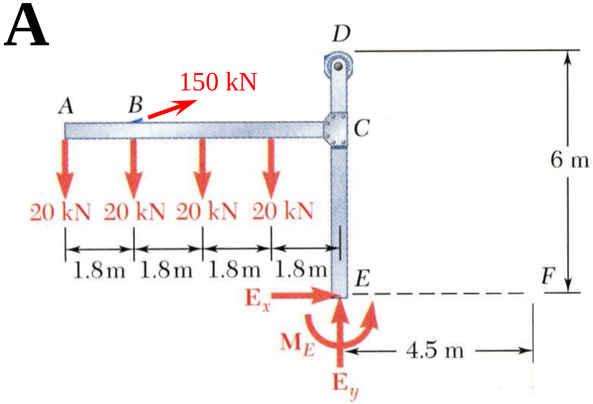
Practice



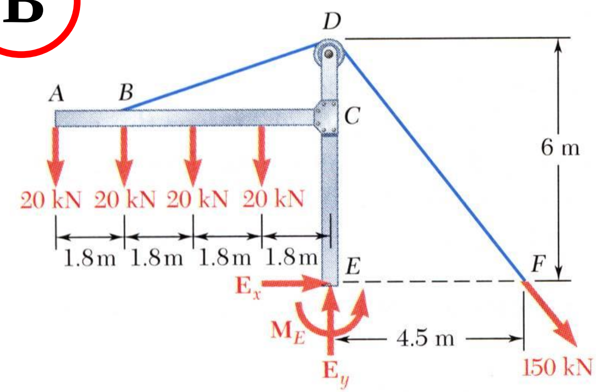
Choose the most correct FBD for the original problem.

Discuss with a neighbor why each choice is correct or incorrect.

A

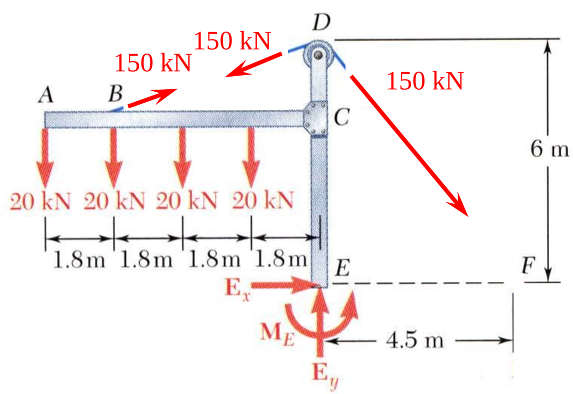


B

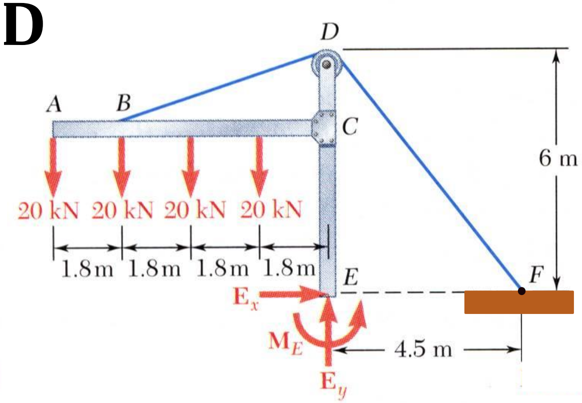


B is the most correct, though C is also correct. A & D are incorrect; why?

C



D



Supports for Rigid Bodies

If a rigid object is subjected to some set of forces but does not move, then its motion could be restrained by a normal force exerted by the ground, a wall or from fixing the object with some support.

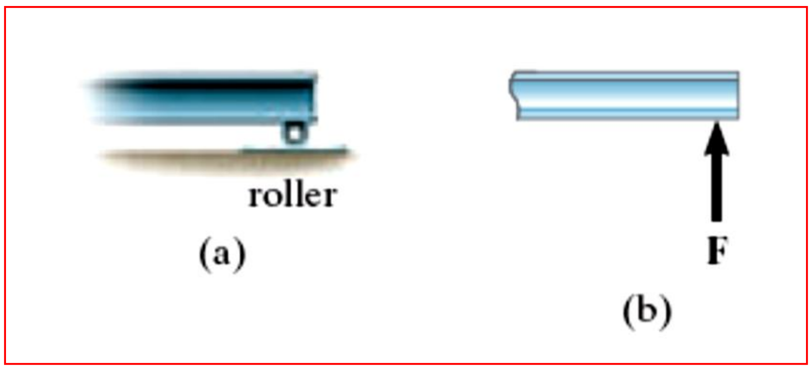
Picture shows the rocker expansion bearing of a plate girder bridge.
The convex surface of the rocker allows the support of the girder to move horizontally.

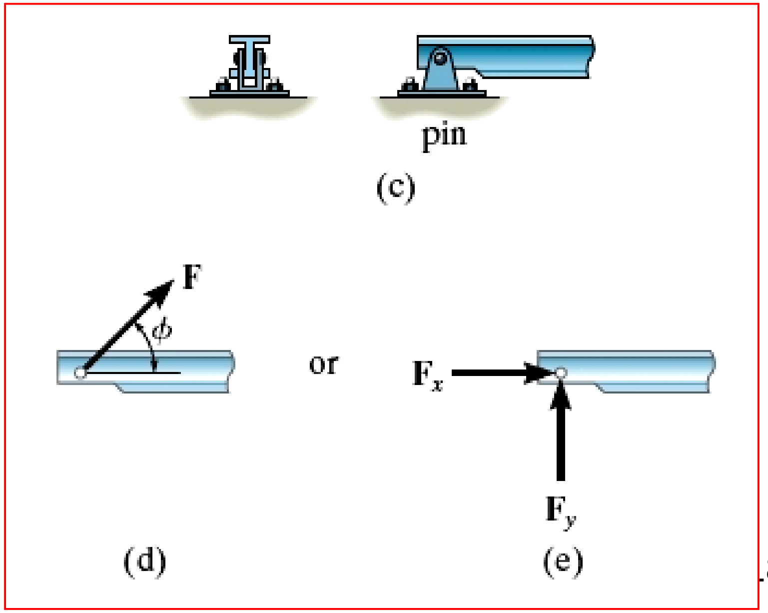
The abutment mounted rocker bearing shown is used to support the roadway of a bridge.

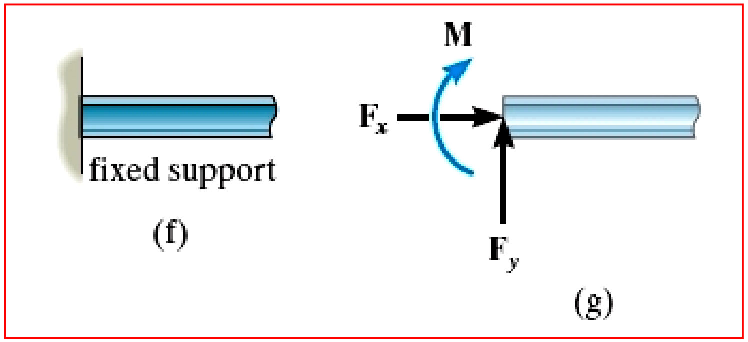
Support of Connection			Reaction	Number of Unknowns
Rollers	Rocker	Frictionless surface	Force with known line of action	1

Support Reactions

- Support reaction is when there is contact and the reaction is coming from the floor or the wall
- The moment at the support is due to the wall
- Different types of supports will have different types of forces







Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2)  weightless link	 or 	One unknown. The reaction is a force which acts along the axis of the link.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

(4)

roller or pin in confined smooth slot

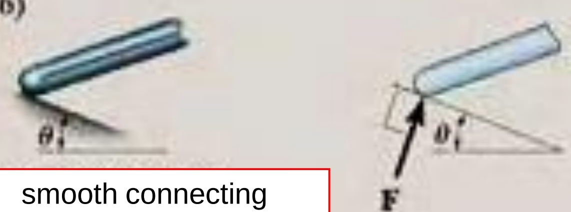
One unknown. The reaction is a force which acts perpendicular to the slot.

(5)

rocker

One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

(6)



smooth connecting surface

One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

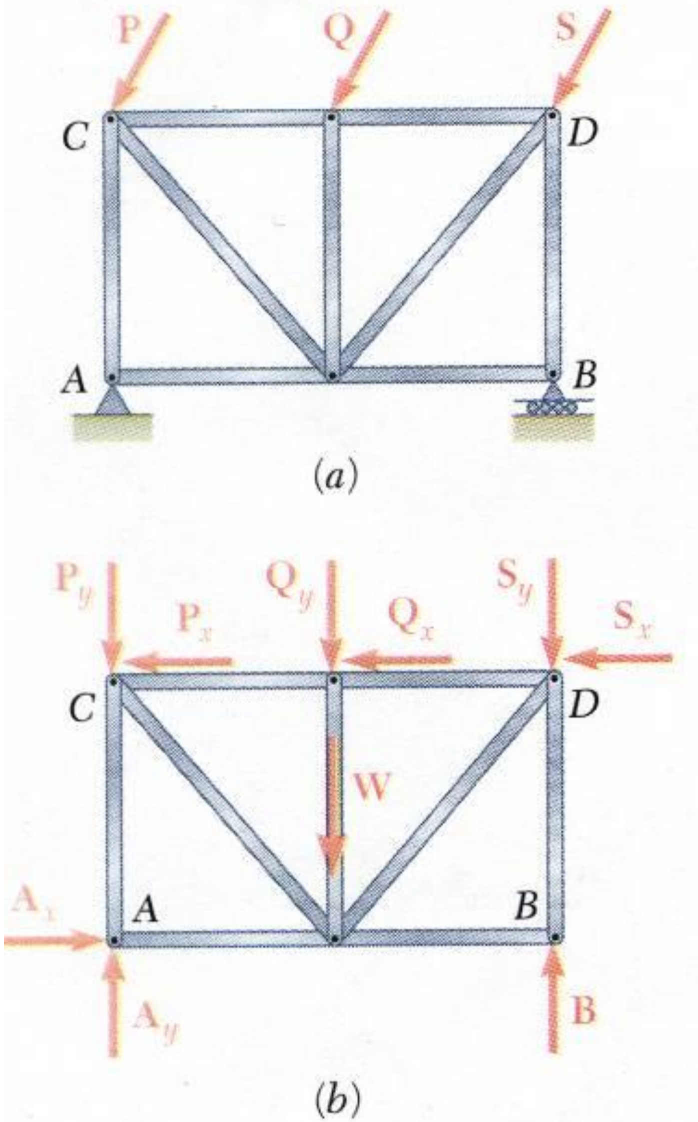
(7)

member pin connected To collar on smooth rod

One unknown. The reaction is a force which acts perpendicular to the rod.

Types of Connection	Reaction	Number of Unknowns
(8)  smooth pin or hinge		Two unknowns. The reactions are two components of force, or the magnitude and direction ϕ of the resultant force. Note that ϕ and θ are not necessarily equal [usually not, unless the rod shown is a link as in (2)].
(9)  member fixed connected To collar on smooth rod		Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.
(10)  fixed support		Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction ϕ of the resultant force.

Equilibrium of a Rigid Body in Two Dimensions



- For known forces and moments that act on a two-dimensional structure, the following are true:

$$F_z = 0 \quad M_x = M_y = 0 \quad M_z = M_O$$

- Equations of equilibrium become

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_A = 0$$

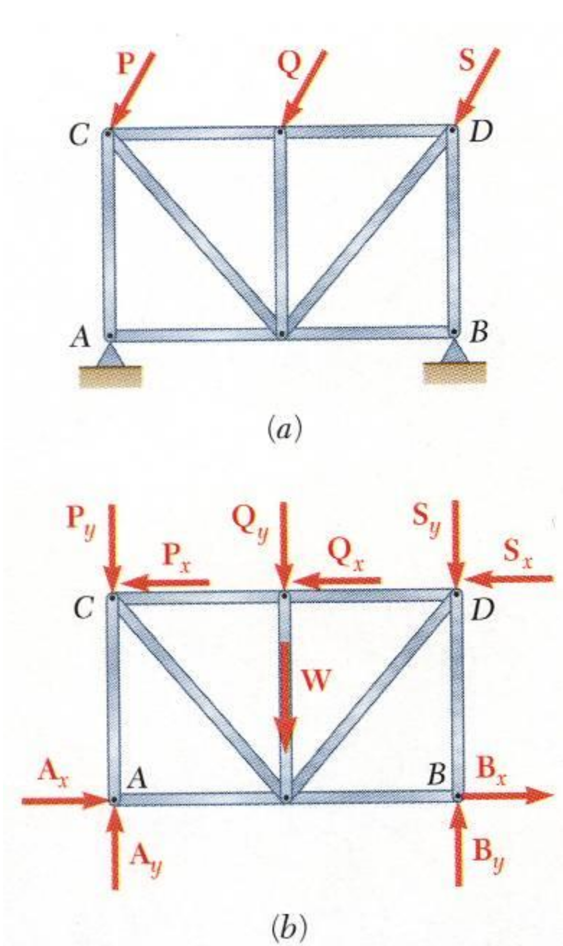
where A can be any point in the plane of the body.

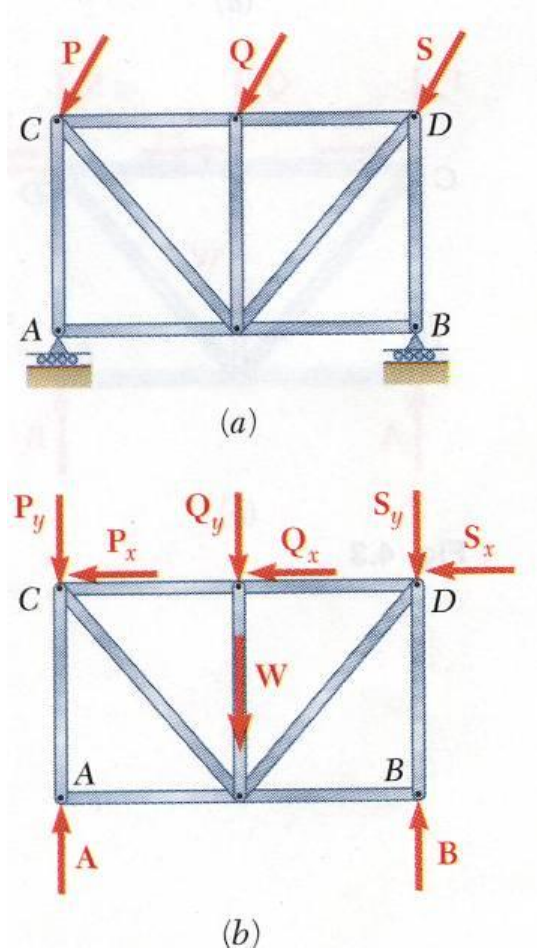
- The 3 equations can be solved for no more than 3 unknowns.

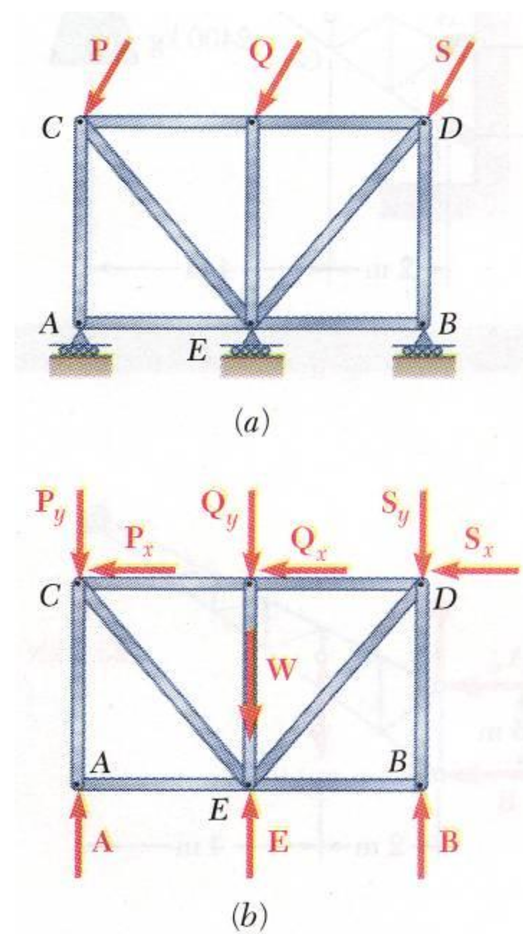
- The 3 equations cannot be augmented with additional equations, but they can be replaced

$$\sum F_x = 0 \quad \sum M_A = 0 \quad \sum M_B = 0$$

Statically Indeterminate Reactions



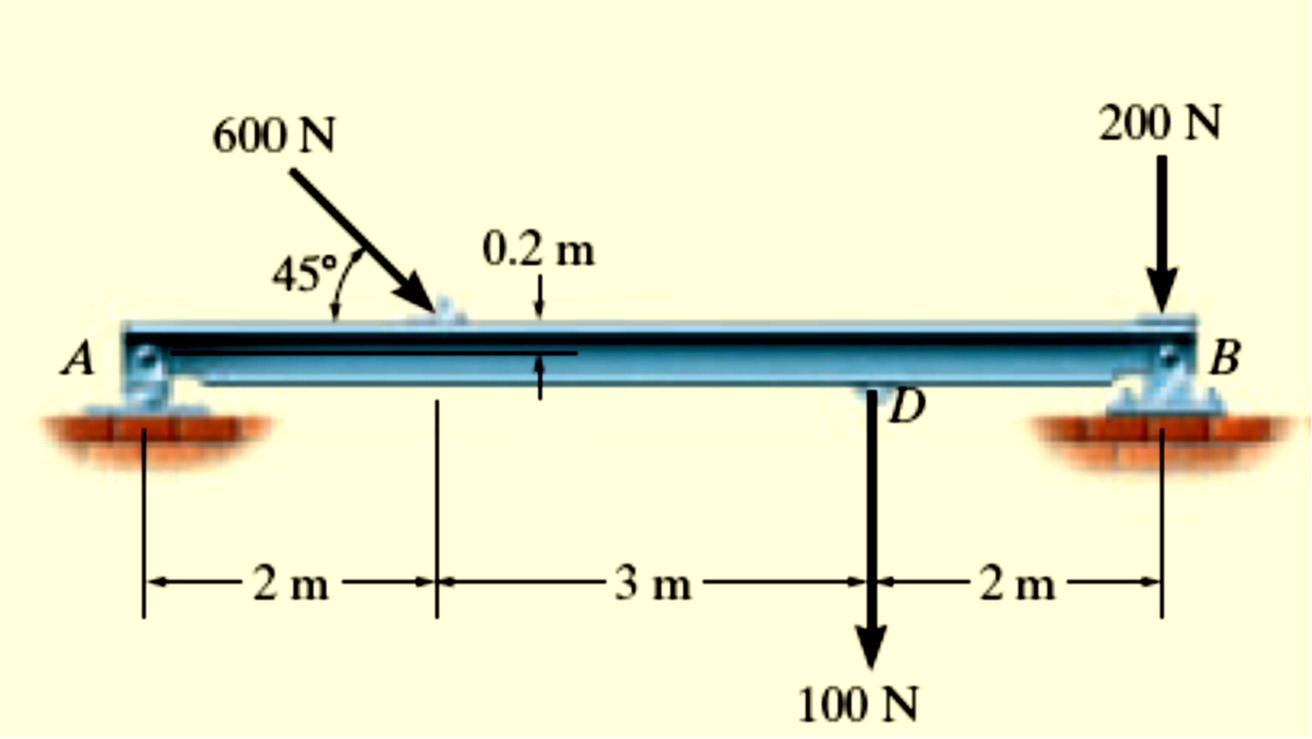




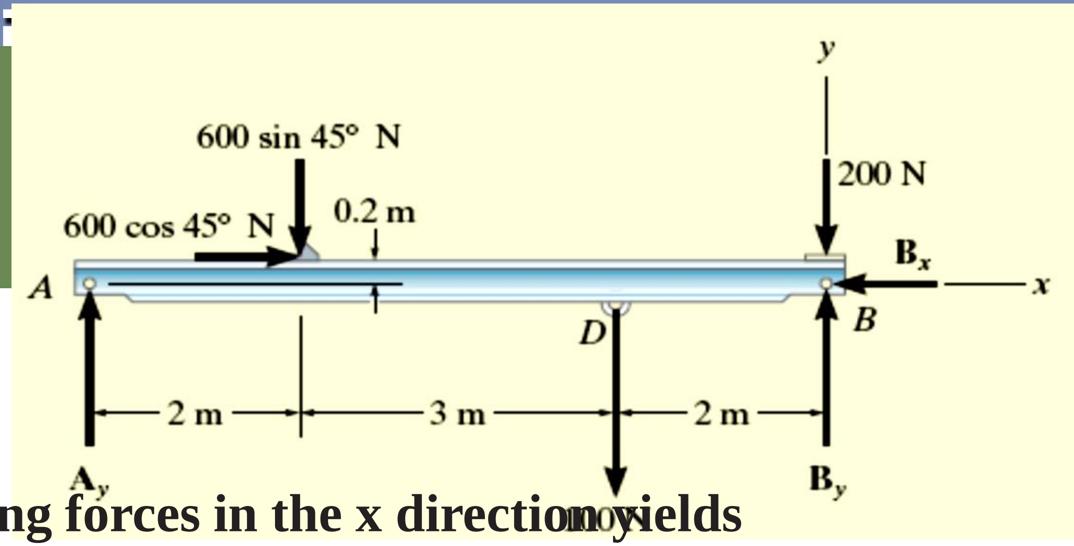
- More unknowns than equations
- Fewer unknowns than equations, partially constrained
- Equal number unknowns and equations but improperly constrained

Example

Determine the horizontal and vertical components of reaction for the beam loaded. Neglect the weight of the beam in the calculations. (A is a roller and B is a hinge support.)



Solution



Equations of Equilibrium. **Summing forces in the x direction yields**

$$\overset{+}{\rightarrow}\sum F_x = 0, \qquad 600\cos 45 - B_x = 0, \qquad B_x = 424\ N$$

A direct solution for A_y can be obtained by applying the moment equation $\sum M_B = 0$ about point B:

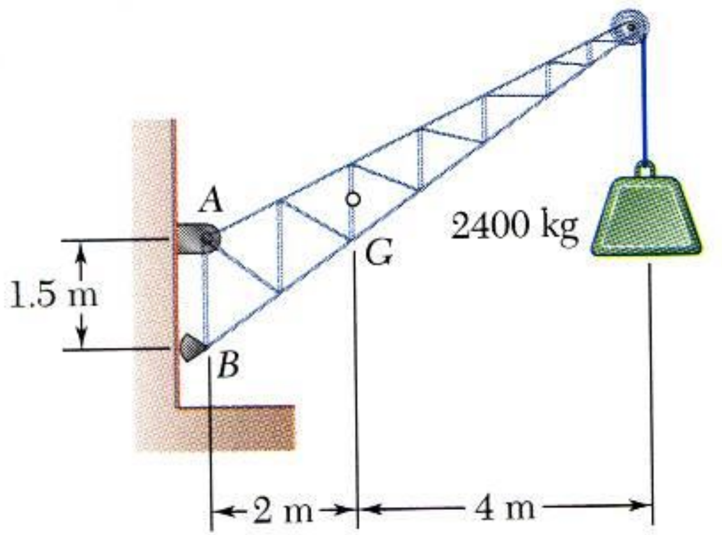
$$\curvearrowleft + \sum M_B = 0; \;\; 100 \times 2 + (600 sin45) \times 5 - (600 cos45) \times 0.2 - A_y \times 7 = 0$$

$$A_y = 319\ N$$

Summing forces in the *y* direction, using this result, gives

$$+\uparrow \sum F_x = 0, \qquad 319 - 600\sin 45 - 100 - 200 + B_y = 0, \qquad B_y = 405\ N$$

Sample Problem 4.1



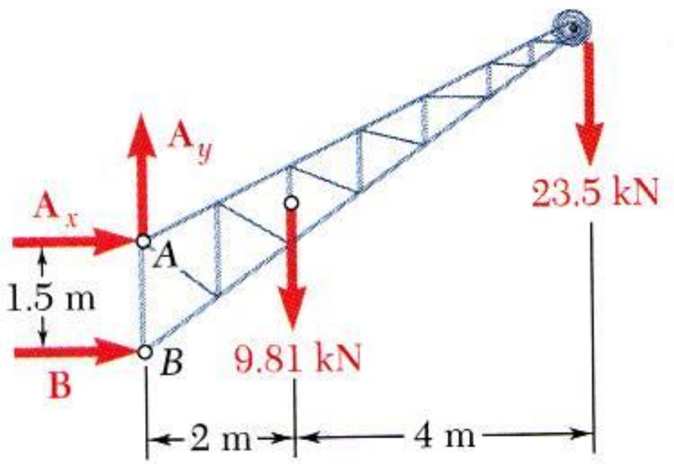
A fixed crane has a mass of 1000 kg and is used to lift a 2400 kg crate. It is held in place by a pin at *A* and a rocker at *B*. The center of gravity of the crane is located at *G*.

Determine the components of the reactions at *A* and *B*.

SOLUTION:

- Create a free-body diagram for the crane.
- Determine *B* by solving the equation for the sum of the moments of all forces about *A*. Note there will be no contribution from the unknown reactions at *A*.
- Determine the reactions at *A* by solving the equations for the sum of all horizontal force components and all vertical force components.
- Check the values obtained for the reactions by verifying that the sum of the moments about *B* of all forces is zero.

Sample Problem 4.1



- Create the free-body diagram.

- Determine B by solving the equation for the sum of the moments of all forces about A.

$$\sum M_A = 0: \quad +B(1.5\text{m}) - 9.81\,\text{kN}(2\text{m}) - 23.5\,\text{kN}(6\text{m}) = 0$$

$$\boxed{B = +107.1\,\text{kN}}$$

- Determine the reactions at A by solving the equations for the sum of all horizontal forces and all vertical forces.

$$\sum F_x = 0: \quad A_x + B = 0$$

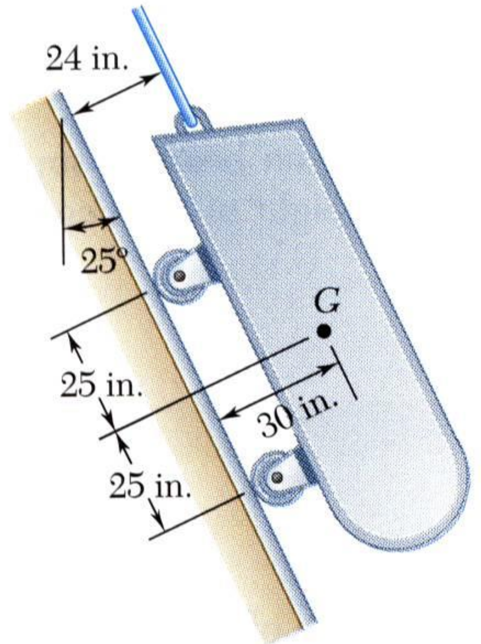
$$\boxed{A_x = -107.1\,\text{kN}}$$

$$\sum F_y = 0: \quad A_y - 9.81\,\text{kN} - 23.5\,\text{kN} = 0$$

$$\boxed{A_y = +33.3\,\text{kN}}$$

- Check the values obtained.

Sample Problem 4.3



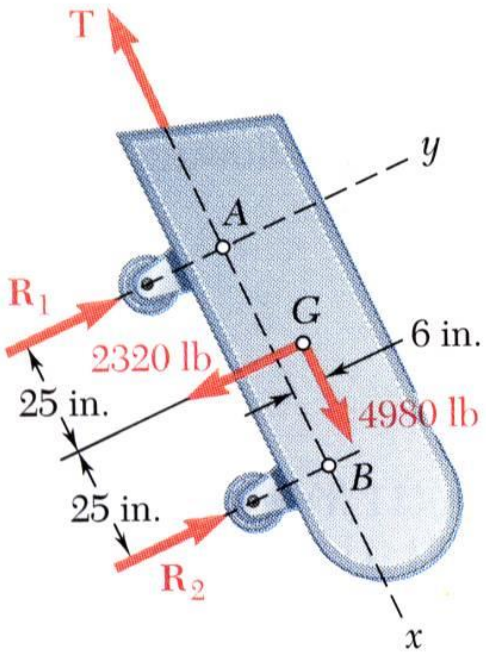
A loading car is at rest on an inclined track. The gross weight of the car and its load is 5500 lb, and it is applied at at *G*. The cart is held in position by the cable.

Determine the tension in the

SOLUTION:

- Create a free-body diagram for the car with the coordinate system aligned with the track.
- Determine the reactions at the wheels by solving equations for the sum of moments about points above each axle.
- Determine the cable tension by solving the equation for the sum of force components parallel to the track.
- Check the values obtained by verifying that the sum of force components perpendicular to the track are zero.

Sample Problem 4.3



- **Create a free-body diagram**

$$W_x = +(5500\text{ lb})\cos 25^\circ$$
$$= +4980\text{ lb}$$

$$W_y = -(5500\text{ lb})\sin 25^\circ$$
$$= -2320\text{ lb}$$

- **Determine the reactions at the wheels.**

$$\sum M_A = 0: \quad -(2320\text{ lb})25\text{in.} - (4980\text{ lb})6\text{in.}$$
$$+ R_2(50\text{in.}) = 0$$

$$\boxed{R_2 = 1758\text{ lb}}$$

$$\sum M_B = 0: \quad +(2320\text{ lb})25\text{in.} - (4980\text{ lb})6\text{in.}$$
$$- R_1(50\text{in.}) = 0$$

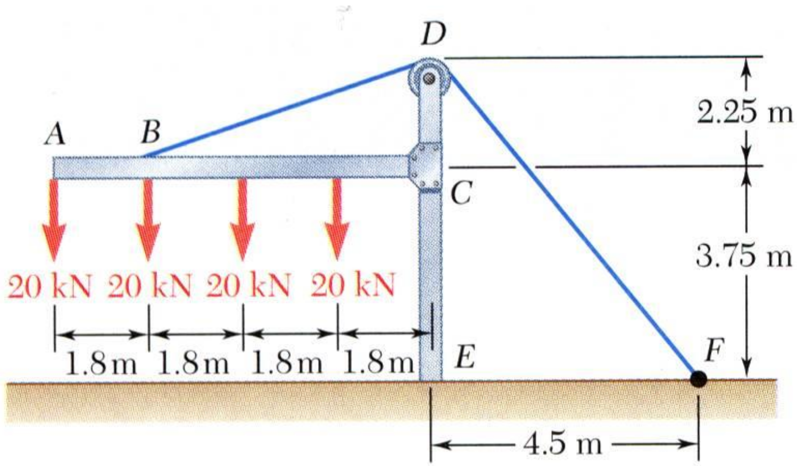
$$\boxed{R_1 = 562\text{ lb}}$$

- **Determine the cable tension.**

$$\sum F_x = 0: \quad +4980\text{ lb} - \mathrm{T} = 0$$

$$\boxed{T = +4980\text{ lb}}$$

Sample Problem 4.4



The frame supports part of the roof of a small building. The tension in the cable is 150 kN.

Determine the reaction at the fixed end *E*.

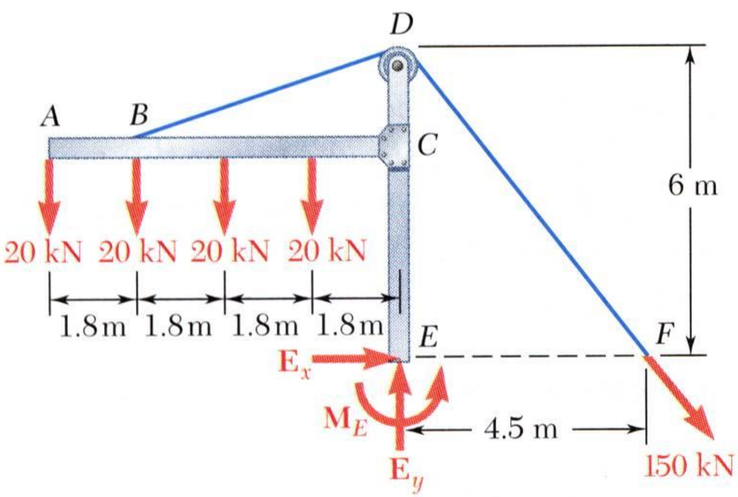
SOLUTION:

- Discuss with a neighbor the steps for solving this problem

- Create a free-body diagram for the frame and cable.

- Apply the equilibrium equations for the reaction force components and couple at *E*.

Sample Problem 4.4



- The free-body diagram was created in an earlier exercise.

- Apply one of the three equilibrium equations. Try using the condition that the sum of forces in the x-direction must sum to zero.

- Which equation is correct?

A. $$\sum F_x = 0: \quad E_x + \frac{4.5}{7.5}(150\,\text{kN}) = 0$$
$$\boxed{E_x = -90.0\,\text{kN}}$$

B. $$\sum F_x = 0: \quad E_x + \cos 36.9^o (150\,\text{kN}) = 0$$

C. $$\sum F_x = 0: \quad E_x + \sin 36.9^o (150\,\text{kN}) = 0$$
$$\boxed{E_x = -90.0\,\text{kN}}$$

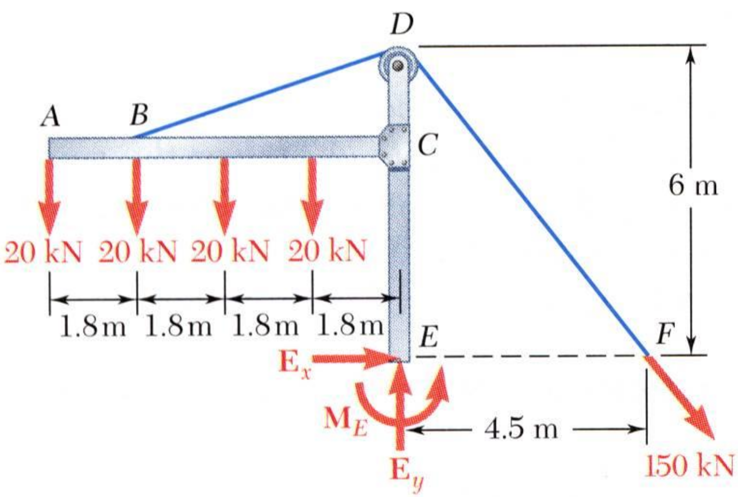
D. $$\sum F_x = 0: \quad E_x + \frac{6}{7.5}(150\,\text{kN}) = 0$$

E. $$\sum F_x = 0: \quad E_x - \sin 36.9^o (150\,\text{kN}) = 0$$

- What does the negative sign signify?

- Discuss why the others are incorrect.

Sample Problem 4.4



- Now apply the condition that the sum of forces in the y-direction must sum to zero.

- Which equation is correct?

A. $\sum F_y = 0: E_y - 4(20\,\text{kN}) - \sin 36.9^o(150\,\text{kN}) = 0$

B. $\sum F_y = 0: \quad E_y - 4(20\,\text{kN}) + \frac{6}{7.5}(150\,\text{kN}) = 0$

C. $\sum F_y = 0: E_y - 4(20\,\text{kN}) - \cos 36.9^o(150\,\text{kN}) = 0$

$\boxed{E_y = +200\ \text{kN}}$

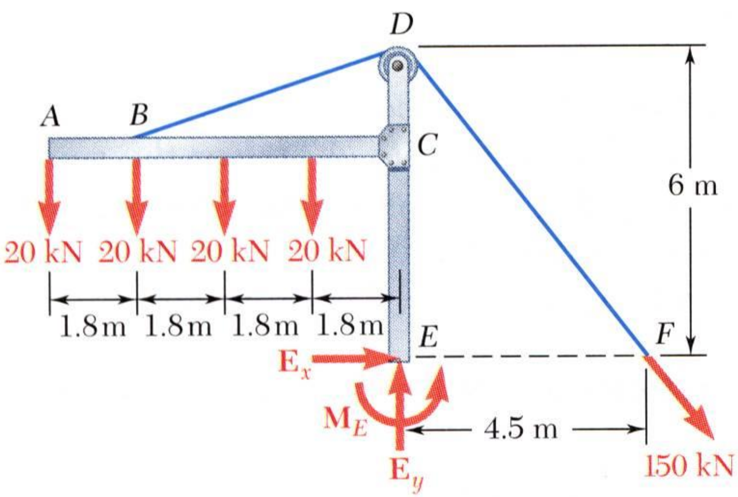
D. $\sum F_y = 0: \quad E_y - 4(20\,\text{kN}) - \frac{6}{7.5}(150\,\text{kN}) = 0$

$\boxed{E_y = +200\ \text{kN}}$

E. $\sum F_y = 0: \quad E_y + 4(20\,\text{kN}) - \frac{6}{7.5}(150\,\text{kN}) = 0$

- What does the positive sign signify?
- Discuss why the others are incorrect.

Sample Problem 4.4



- Finally, apply the condition that the sum of moments about *any point* must equal zero.
- Discuss with a neighbor which point is the best for applying this equilibrium condition, and why.
- Three good points are D, E, and F. Discuss what advantage each point has over the others, or perhaps why each is equally good.
- Assume that you choose point E to apply the sum-of-moments condition. Write the equation and compare your answer with a neighbor.

$$\sum M_E = 0: \quad +20\,\mathrm{kN}(7.2\,\mathrm{m}) + 20\,\mathrm{kN}(5.4\,\mathrm{m})$$

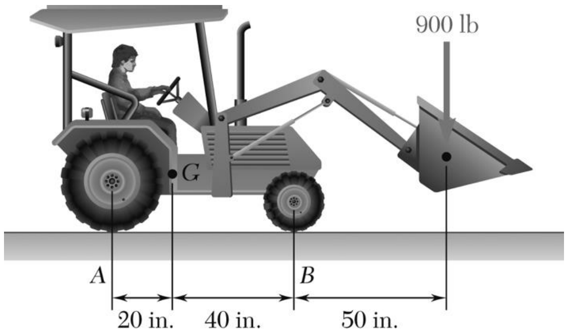
$$+20\,\mathrm{kN}(3.6\,\mathrm{m}) + 20\,\mathrm{kN}(1.8\,\mathrm{m})$$

$$-\frac{6}{7.5}(150\,\mathrm{kN})4.5\,\mathrm{m} + M_E = 0$$

$$\boxed{M_E = 180.0\,\mathrm{kN}\cdot\mathrm{m}}$$

- Discuss with a neighbor the origin of each term in the above equation and what the positive value of M_E means.

Practice



A 2100-lb tractor is used to lift 900 lb of gravel. Determine the reaction at each of the two rear wheels and two front wheels

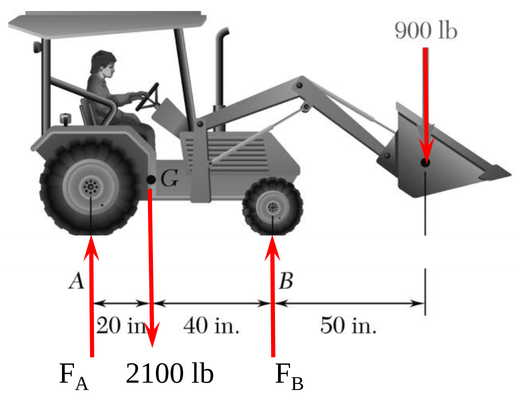
Discuss with a neighbor what steps to take to solve this problem.

- First, create a *free body diagram*.
- Second, apply the *equilibrium conditions* to generate the three equations, and use these to solve for the desired quantities.

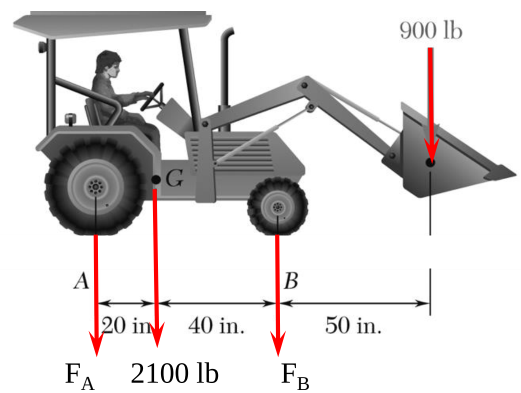
Practice

- Draw the free body diagram of the tractor (on your own first).
- From among the choices, choose the best FBD, and discuss the problem(s) with the other FBDs.

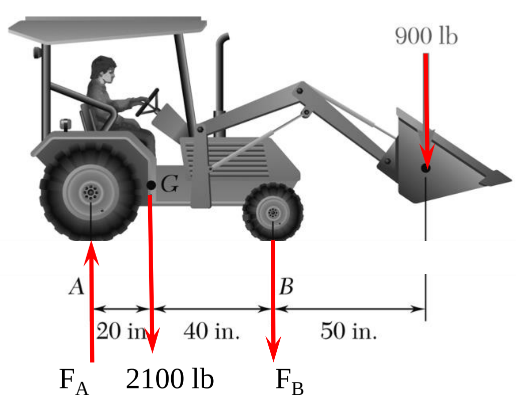
A.



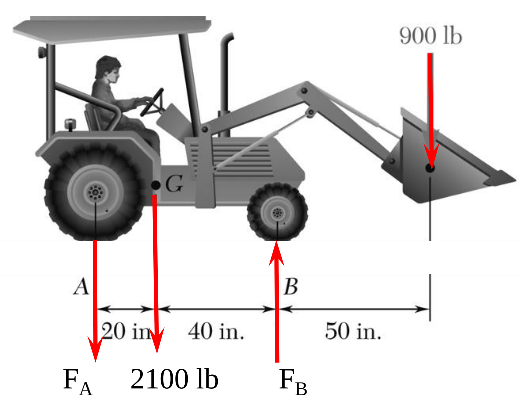
B.



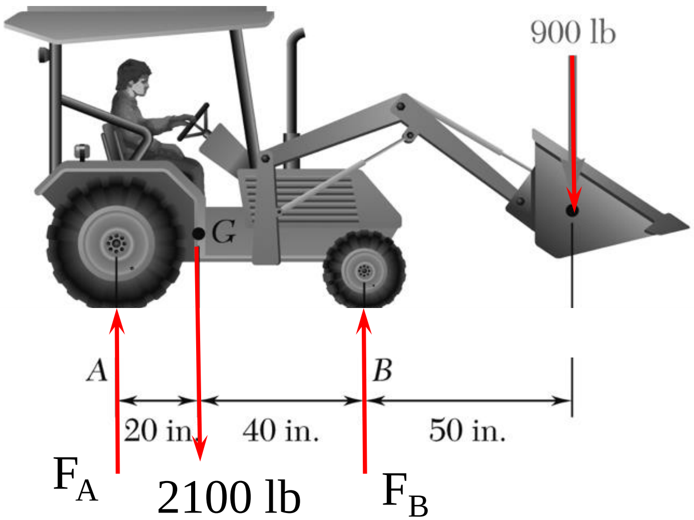
C.



D.



Practice



Now let's apply the equilibrium conditions to this FBD.

- Start with the moment equation:

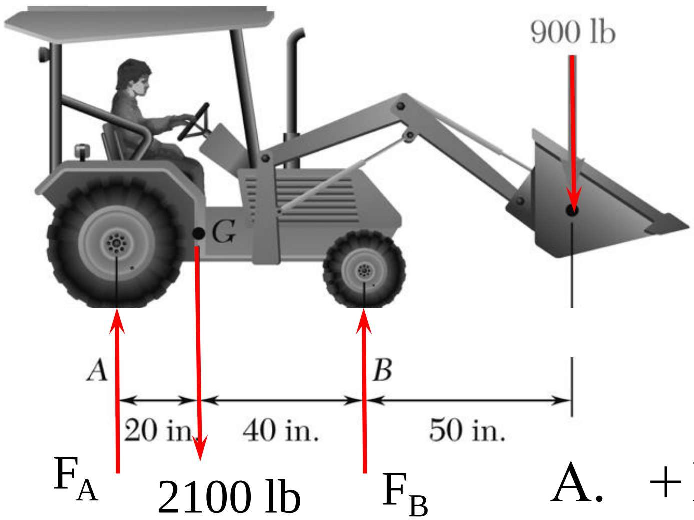
$$\sum M_{pt} = 0$$

Discuss with a neighbor:

- What's the advantage to starting with this instead of the other conditions?
- About what point should we sum moments, and why?

Points A or B are equally good because each results in an equation with only one unknown.

Practice



Assume we chose to use point B.

Choose the correct equation for

$$\sum M_B = 0.$$

A. $+F_A(60\text{ in.}) - 2100\text{lb}\,(40\text{ in.}) - 900\text{ lb}\,(50\text{ in.}) = 0$

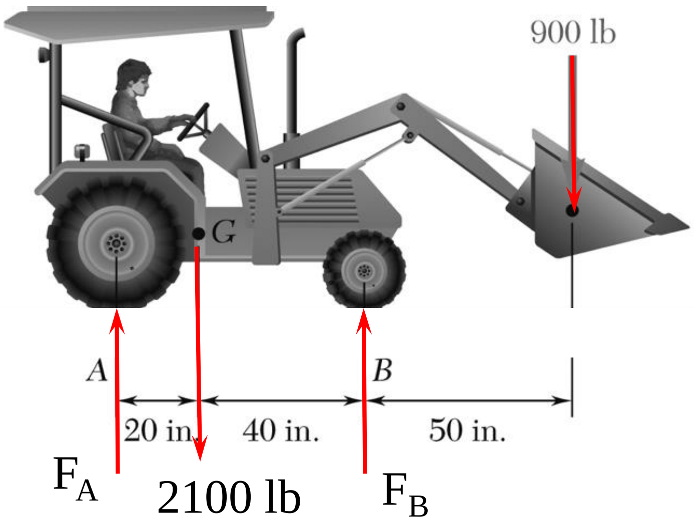
B. $+F_A(20\text{ in.}) - 2100\text{lb}\,(40\text{ in.}) - 900\text{ lb}\,(50\text{ in.}) = 0$

C. $-F_A(60\text{ in.}) - 2100\text{lb}\,(40\text{ in.}) + 900\text{ lb}\,(50\text{ in.}) = 0$

D. $-F_A(60\text{ in.}) + 2100\text{lb}\,(40\text{ in.}) - 900\text{ lb}\,(50\text{ in.}) = 0$

F_A=650 lb, so the reaction *at each wheel* is 325 lb

Practice



Now apply the final equilibrium condition, $\Sigma F_y = 0$.

$$F_A - 2100 \text{ lb} + F_B - 900 \text{ lb} = 0$$

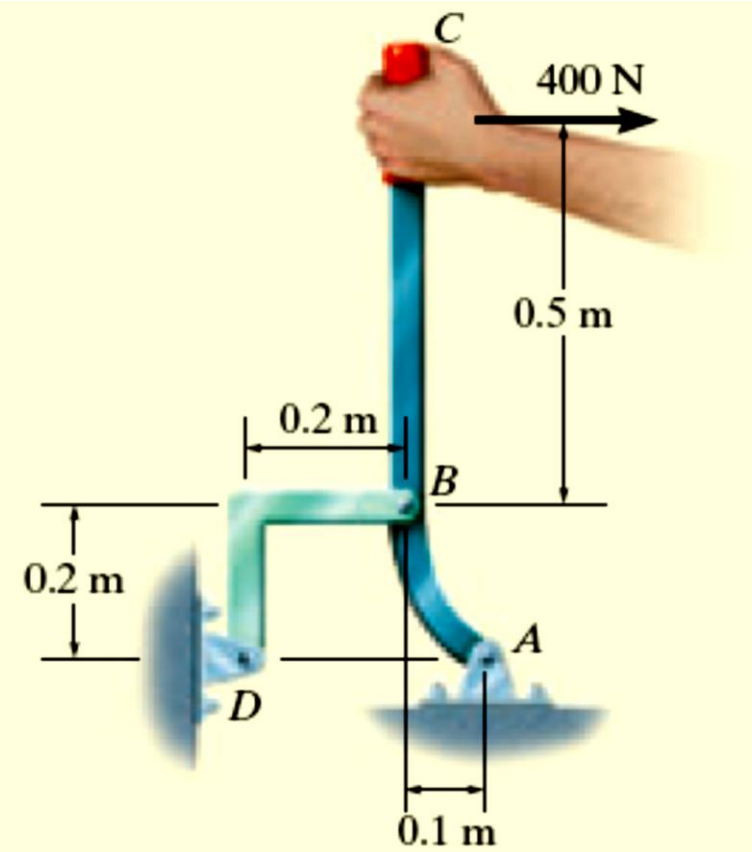
$$\text{or} \quad +650 \text{ lb} - 2100 \text{ lb} + F_B - 900 \text{ lb} = 0$$

$$\Rightarrow \boxed{F_B = 2350 \text{ lb}, \text{ or } 1175 \text{ lb at each front wheel}}$$

Why was the third equilibrium condition, $\Sigma F_x = 0$ not used?

Example

The lever ABC is pin-supported at A and connected to a short link BD. If the weight of the members are negligible, determine the force of the pin on the lever at A.



Equations of Equilibrium.

By requiring the force system to be concurrent at O, since $\sum M_O = 0$, the angle θ which defines the line of action of $\mathbf{F}_A$ can be determined from trigonometry,

$$\theta = \tan^{-1}\left(\frac{0.7}{0.4}\right) = 60.3^{o}$$

Using the x, y axes and applying the force equilibrium equations,

$$\overset{+}{\rightarrow}\sum F_x = 0, \qquad F_A \cos 63.3 - F\cos 45 + 400 = 0,$$

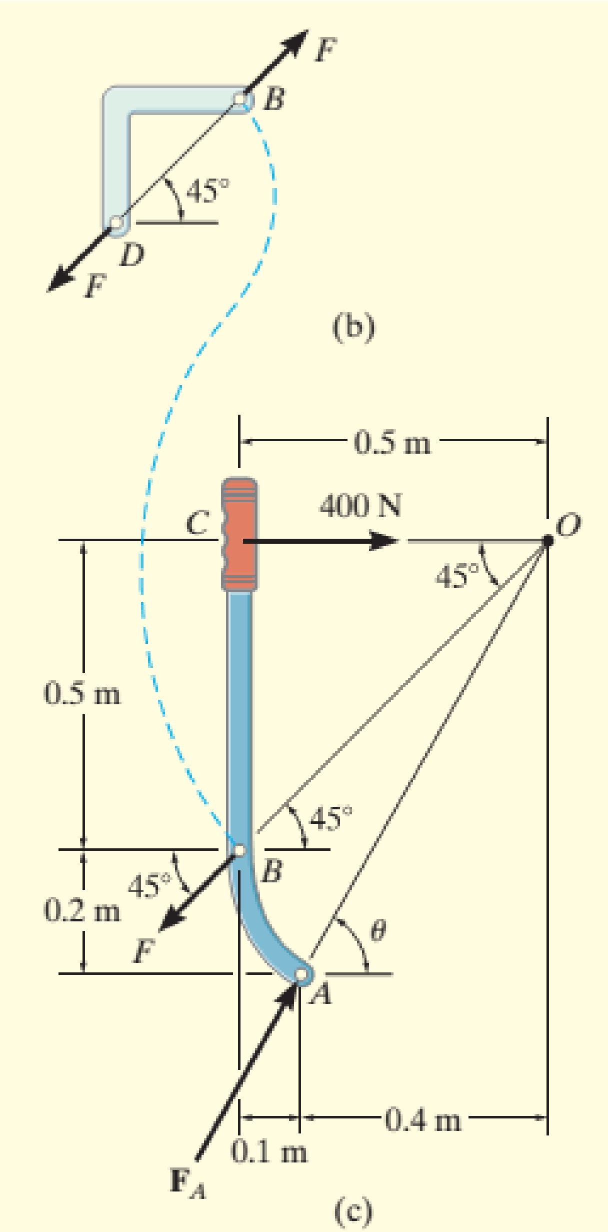
$$+\uparrow \sum F_y = 0, \qquad F_A \sin 60.3 - F\sin 45 = 0$$

-Solving, we get

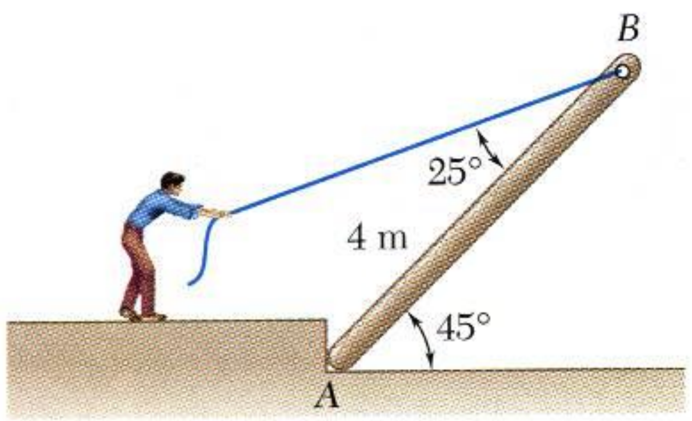
$$F_A = 1.07\text{kN}$$

$$F = 1.32 \text{ kN}$$

NOTE: We can also solve this problem by representing the force at A by its two components $\mathbf{A}_x$ and $\mathbf{A}_y$ and applying $\sum M_A = 0$, $\sum F_x = 0$, $\sum F_y = 0$ to the lever. Once A_x and A_y are determined, we can get FA and θ.



Sample Problem 4.4



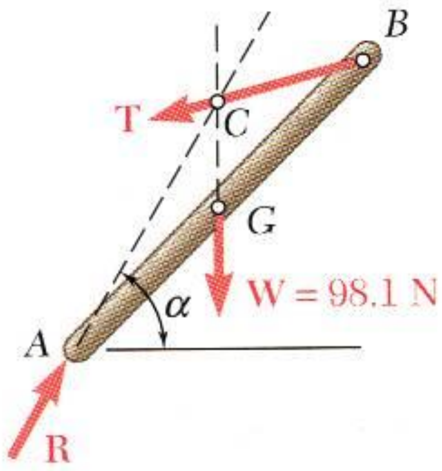
A man raises a 10 kg joist, of length 4 m, by pulling on a rope.

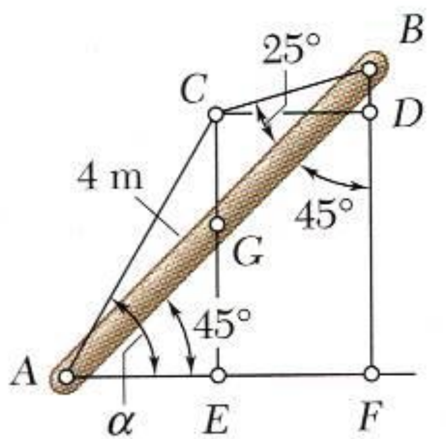
Find the tension in the rope and the reaction at *A*.

SOLUTION:

- Create a free-body diagram of the joist. Note that the joist is a 3 force body acted upon by the rope, its weight, and the reaction at *A*.
- The three forces must be concurrent for static equilibrium. Therefore, the reaction ***R*** must pass through the intersection of the lines of action of the weight and rope forces. Determine the direction of the reaction force ***R***.
- Utilize a force triangle to determine the magnitude of the reaction force ***R***.

Sample Problem 4.4





- Create a free-body diagram of the joist.
- Determine the direction of the reaction force ***R***.

$$AF = AB\cos 45 = (4\,\text{m})\cos 45 = 2.828\,\text{m}$$

$$CD = AE = \tfrac{1}{2}\,AF = 1.414\,\text{m}$$

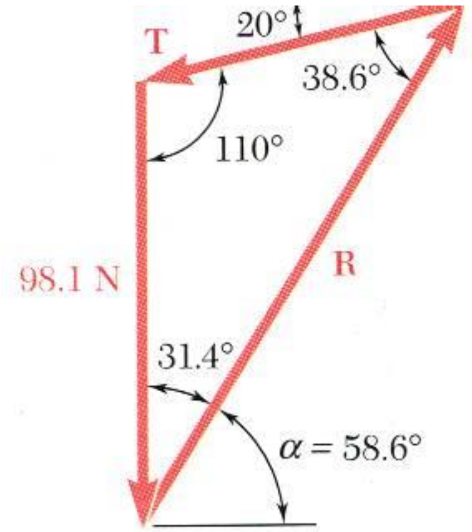
$$BD = CD\cot(45 + 25) = (1.414\,\text{m})\tan 20 = 0.515\,\text{m}$$

$$CE = BF - BD = (2.828 - 0.515)\,\text{m} = 2.313\,\text{m}$$

$$\tan\alpha = \frac{CE}{AE} = \frac{2.313}{1.414} = 1.636$$

$$\boxed{\alpha = 58.6^\circ}$$

Sample Problem 4.4



- Determine the magnitude of the reaction force ***R***.

$$\frac{T}{\sin 31.4^\circ} = \frac{R}{\sin 110^\circ} = \frac{98.1\,\mathrm{N}}{\sin 38.6^\circ}$$

$$\boxed{\begin{aligned} T &= 81.9\,\mathrm{N} \\ R &= 147.8\,\mathrm{N} \end{aligned}}$$

Equilibrium of a Rigid Body in Three Dimensions

- Six scalar equations are required to express the conditions for the equilibrium of a rigid body in the general three dimensional case.

$$\sum \boldsymbol{F} = \sum \boldsymbol{F_x i} + \sum \boldsymbol{F_y j} + \sum \boldsymbol{F_z k} = \boldsymbol{0}$$

$$\boldsymbol{M_O} = \sum \boldsymbol{M_x i} + \sum \boldsymbol{M_y j} + \sum \boldsymbol{M_z k} = \boldsymbol{0}$$

- The scalar equations are conveniently obtained by applying the vector forms of the conditions for equilibrium,

$$\sum \vec{F} = 0, \qquad \sum \vec{M}_O = \sum (\vec{r} \times \vec{F}) = 0$$

Equilibrium Equations

When the force system is replaced by a resultant force and moment that are zero, the rigid body is in equilibrium.

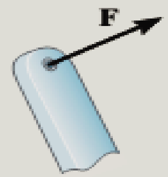
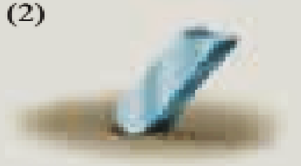
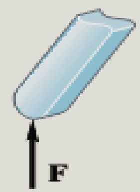
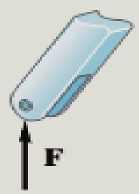
$$\sum \underset{\sim}{F} = 0 \quad \text{and} \quad \sum \underset{\sim}{M} = 0$$

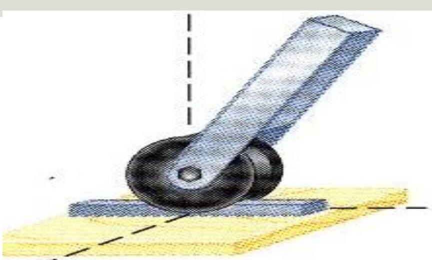
$$\sum F_x = 0, \qquad \sum M_x = 0$$

$$\sum F_y = 0, \qquad \sum M_y = 0$$

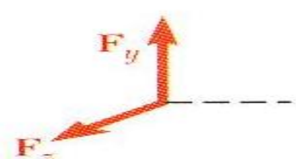
$$\sum F_z = 0, \qquad \sum M_z = 0$$

Mechanics for Engineers: Statics

Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a force which acts away from the member in the known direction of the cable.
(2)  smooth surface support		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

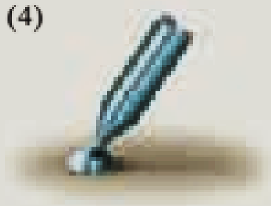


Wheel on rail

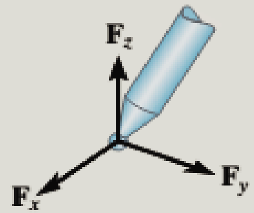


Two force components

(4)



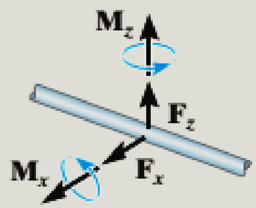
ball and socket



Three unknowns. The reactions are three rectangular force components.

(5)

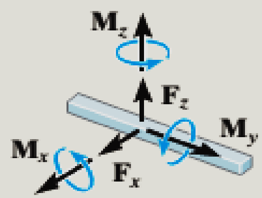
single journal bearing



Four unknowns. The reactions are two force and two couple-moment components which act perpendicular to the shaft. Note: The couple moments are *generally not applied* if the body is supported elsewhere. See the examples.

(6)

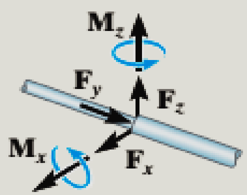
single journal bearing with square shaft



Five unknowns. The reactions are two force and three couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.

(7)

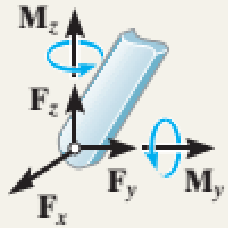
single thrust bearing



Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.

(8)

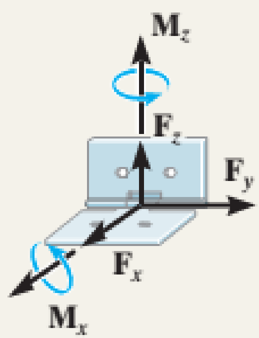
single smooth pin



Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.

(9)

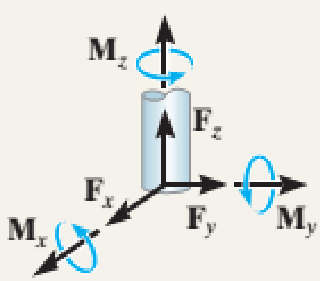
single hinge



Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.

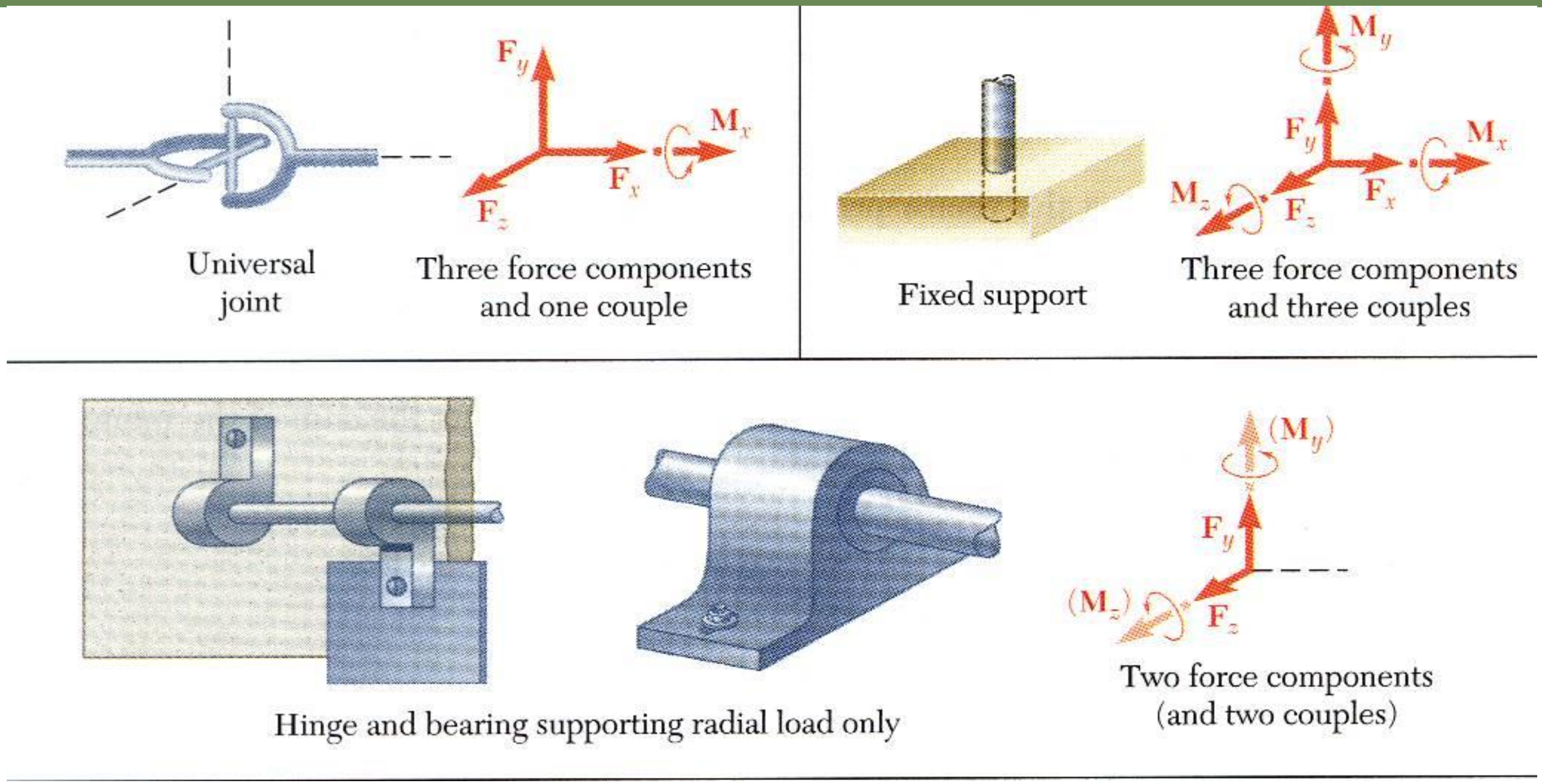
(10)

fixed support



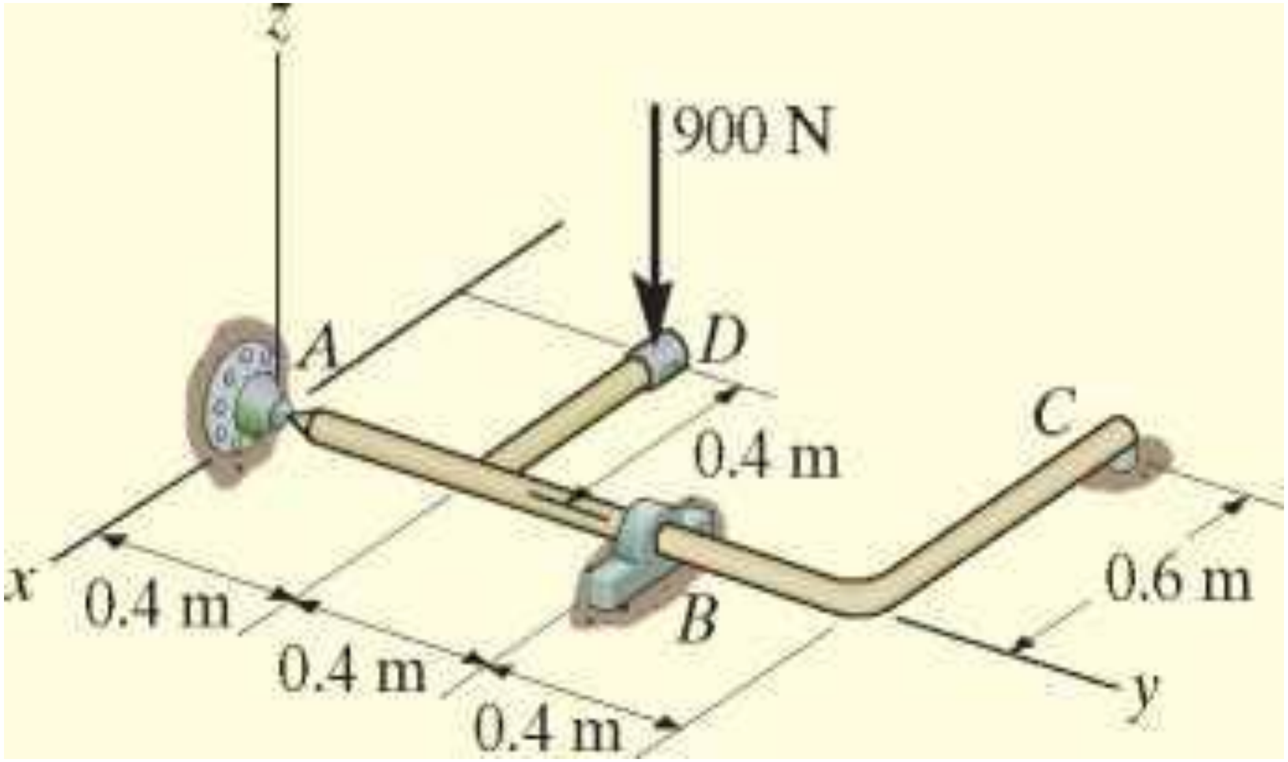
Six unknowns. The reactions are three force and three couple-moment components.

Reactions at Supports and Connections for a Three-Dimensional Structure

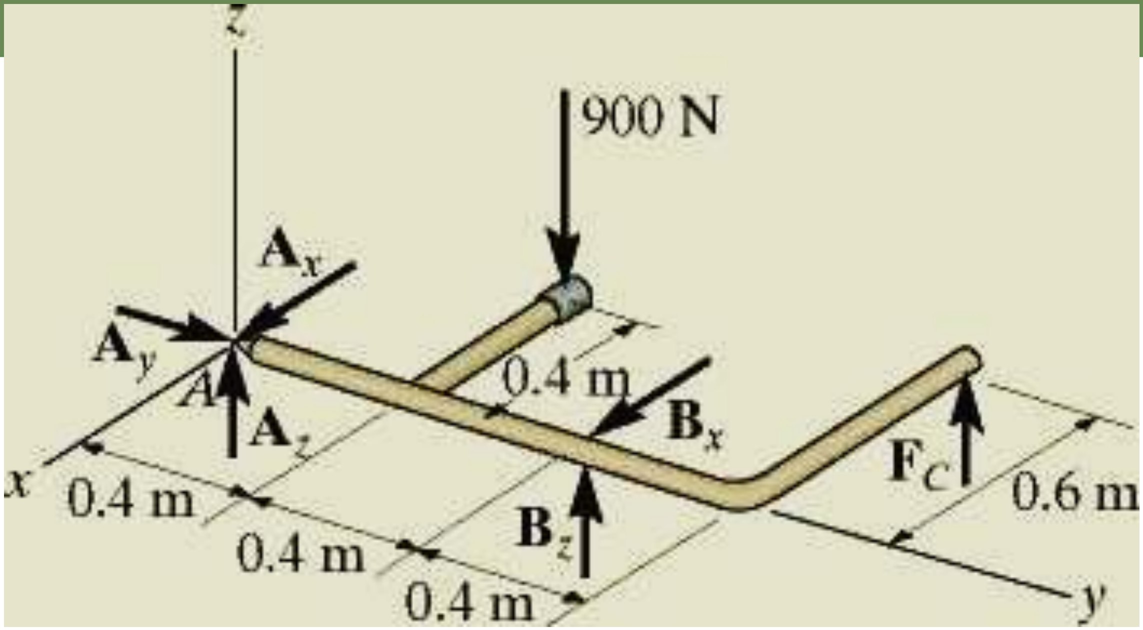


Example

Determine the components of reaction that the ball-and-socket joint at A, the smooth journal bearing at B, and the roller support at C exert on the rod assembly in Fig.



Solution



Equations of Equilibrium.

$\sum F_y = 0;$ $A_y = 0$ **Ans.**

The force F_C can be determined directly by summing moments about the y axis.

$\sum M_y = 0;$ $F_C\,(0.6\text{ m}) - 900\text{ N}(0.4\text{ m}) = 0$

$F_C = 600\text{ N}$ **Ans.**

Using this result, B_z can be determined by summing moments about the x axis.

$\sum M_x = 0$; $B_z(0.8\ m) + 600\ N(1.2\ m) - 900\ N(0.4\ m) = 0$

$B_z = -450\ N$ Ans.

The negative sign indicates that $\mathbf{B_z}$ acts downward. The force B_x can be found by summing moments about the z axis.

$\sum M_z = 0$; $-B_x(0.8\ m) = 0 \Longrightarrow B_x = 0$

Thus,

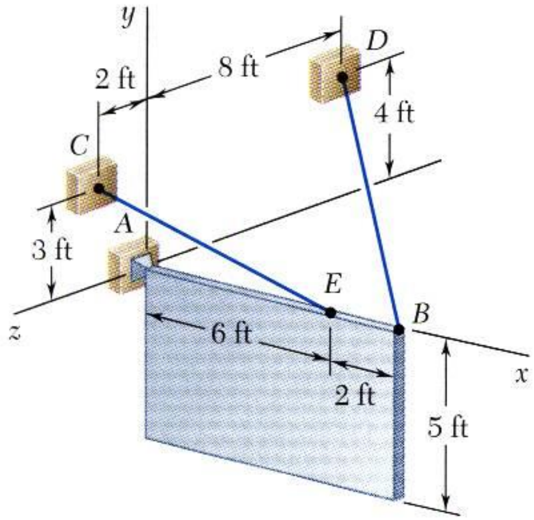
$\sum F_x = 0$; $A_x + 0 = 0 \Longrightarrow A_x = 0$ Ans.

Finally, using the results of Bz and FC.

$\sum F_z = 0$; $A_z + (-450\ N) + 600\ N - 900\ N = 0$

$A_z = 750\ N$ Ans.

Sample Problem 4.8



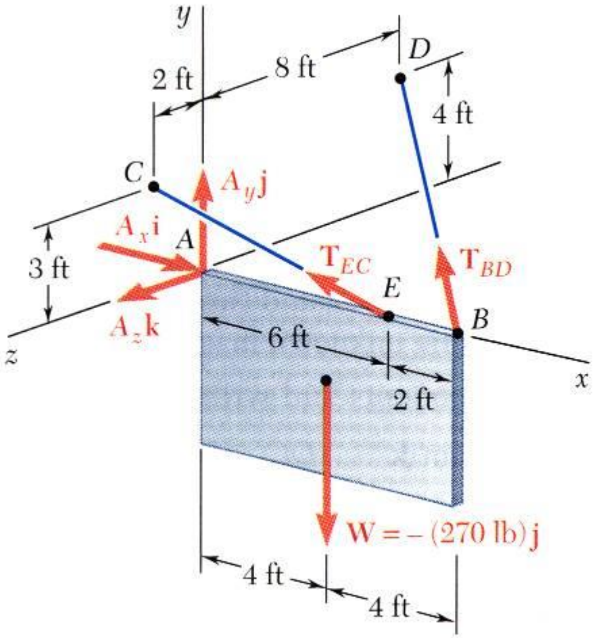
A sign of uniform density weighs 270 lb and is supported by a ball-and-socket joint at *A* and by two cables.

Determine the tension in each cable and the reaction at *A*.

SOLUTION:

- Create a free-body diagram for the sign.
- Apply the conditions for static equilibrium to develop equations for the unknown reactions.

Sample Problem 4.8



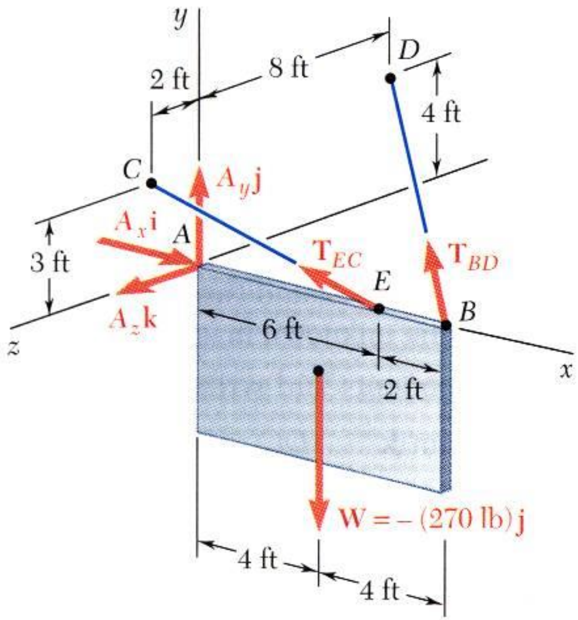
- Create a free-body diagram for the sign.

  Since there are only 5 unknowns, the sign is partially constrained. All forces intersect with the x-axis, so ΣM_X=0, so this equation is not useful to the solution.

$$\vec{T}_{BD} = T_{BD}\frac{\vec{r}_D - \vec{r}_B}{\left|\vec{r}_D - \vec{r}_B\right|} = T_{BD}\frac{-8\vec{i} + 4\vec{j} - 8\vec{k}}{12} = T_{BD}\left(-\tfrac{2}{3}\vec{i} + \tfrac{1}{3}\vec{j} - \tfrac{2}{3}\vec{k}\right)$$

$$\vec{T}_{EC} = T_{EC}\frac{\vec{r}_C - \vec{r}_E}{\left|\vec{r}_C - \vec{r}_E\right|} = T_{EC}\frac{-6\vec{i} + 3\vec{j} + 2\vec{k}}{7} = T_{EC}\left(-\tfrac{6}{7}\vec{i} + \tfrac{3}{7}\vec{j} + \tfrac{2}{7}\vec{k}\right)$$

Sample Problem 4.8



- Apply the conditions for static equilibrium to develop equations for the unknown reactions.

$$\sum \vec{F} = \vec{A} + \vec{T}_{BD} + \vec{T}_{EC} - (270\text{ lb})\vec{j} = 0$$

$$\vec{i}: \quad A_x - \tfrac{2}{3}T_{BD} - \tfrac{6}{7}T_{EC} = 0$$

$$\vec{j}: \quad A_y + \tfrac{1}{3}T_{BD} + \tfrac{3}{7}T_{EC} - 270\text{ lb} = 0$$

$$\vec{k}: \quad A_z - \tfrac{2}{3}T_{BD} + \tfrac{2}{7}T_{EC} = 0$$

$$\sum \vec{M}_A = \vec{r}_B \times \vec{T}_{BD} + \vec{r}_E \times \vec{T}_{EC} + (4\text{ ft})\vec{i} \times (-270\text{ lb})\vec{j} = 0$$

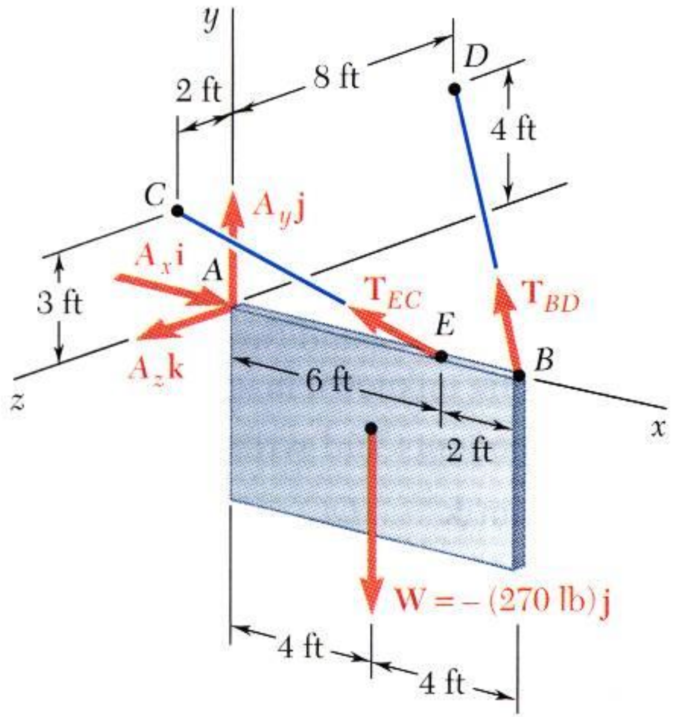
$$\vec{j}: \quad 5.333T_{BD} - 1.714T_{EC} = 0$$

$$\vec{k}: \quad 2.667T_{BD} + 2.571T_{EC} - 1080\text{ lb} = 0$$

Solve the 5 equations for the 5 unknowns,

$$\boxed{\begin{array}{l} T_{BD} = 101.3\text{ lb} \quad T_{EC} = 315\text{ lb} \\ \vec{A} = (338\text{ lb})\vec{i} + (101.2\text{ lb})\vec{j} - (22.5\text{ lb})\vec{k} \end{array}}$$

What if…?



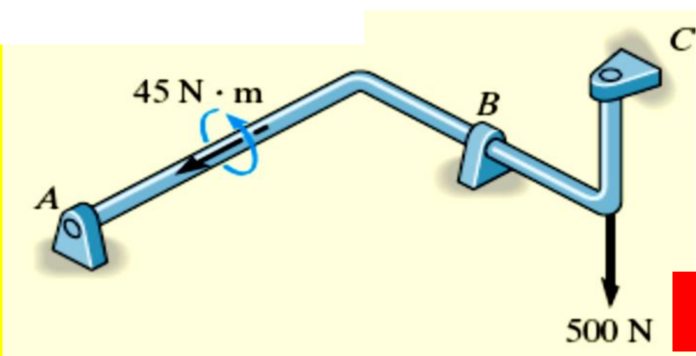
Could this sign be in static equilibrium if cable BD were removved?

Discuss with your neighbor, and be sure to provide the reason(s) for your answer.

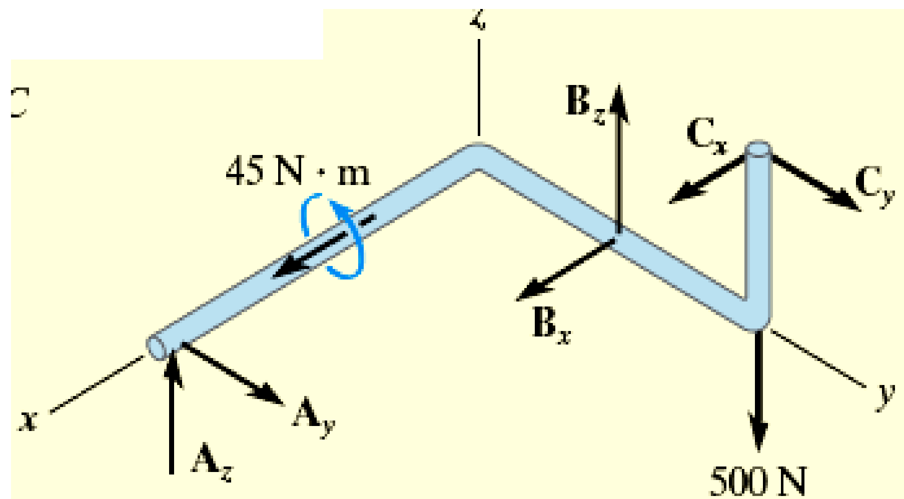
The sign could not be in static equilibrium because T_{EC} causes a moment about the y-axis (due to the existence of $T_{EC,Z}$) which must be countered by an equal and opposite moment. This can only be provided by a cable tension that has a z-component in the negative-z direction, such as what T_{BD} has.

Example 5.14

Several examples of objects along with their associated free-body diagrams are shown. In all cases, the x, y and z axes are established and the unknown reaction components are indicated in the positive sense. The weight of the objects is neglected.



Properly aligned journal bearings at *A*, *B*, *C*.



The force reactions developed by the bearings are sufficient for equilibrium since they prevent the shaft from rotating about each of the coordinate axes.

Constraints for a Rigid Body

Redundant Constraints

- There are more forces and moments from the supports than equilibrium equations
- Statically indeterminate: there are more unknown than equations