Lecture 1

Introduction to Strength of Materials

What is Strength of Materials?

Statics is the study of forces acting in equilibrium on rigid bodies. “Bodies” are solid objects, like steel cables, gear teeth, timber beams, and axle shafts (no liquids or gases); “rigid” means the bodies do not stretch, bend, or twist; and “equilibrium” means the rigid bodies are not accelerating. Most problems in a Statics textbook also assume the rigid bodies are stationary. These assumptions do not match reality perfectly, but they make the math much easier. This model is close enough to reality to be useful for many practical problems.

In *Strength of Materials*, we keep the assumptions of bodies in equilibrium, but we drop the “rigid” assumption. Real cables stretch under tension, real floor joists bend when you walk across a wood floor, and real axle shafts twist under torsional load.

Any material or structure may fail when it is loaded. The successful design of a structure requires detailed structural and stress analysis in order to assess whether or not it can safely support the required loads. Figure 1.1 shows how a structure behaves under applied loads.

To prevent structural failure, a typical design must consider the following three major aspects:

1 Strength – The structure must be strong enough to carry the applied loads.

2 Stiffness – The structure must be stiff enough such that only allowable deformation occurs.

3 Stability – The structure must not collapse through buckling subjected to the applied compressive loads.

The subject of structural and stress analysis provides analytical, numerical and experimental methods for determining the strength, stiffness and stability of load-carrying structural members.

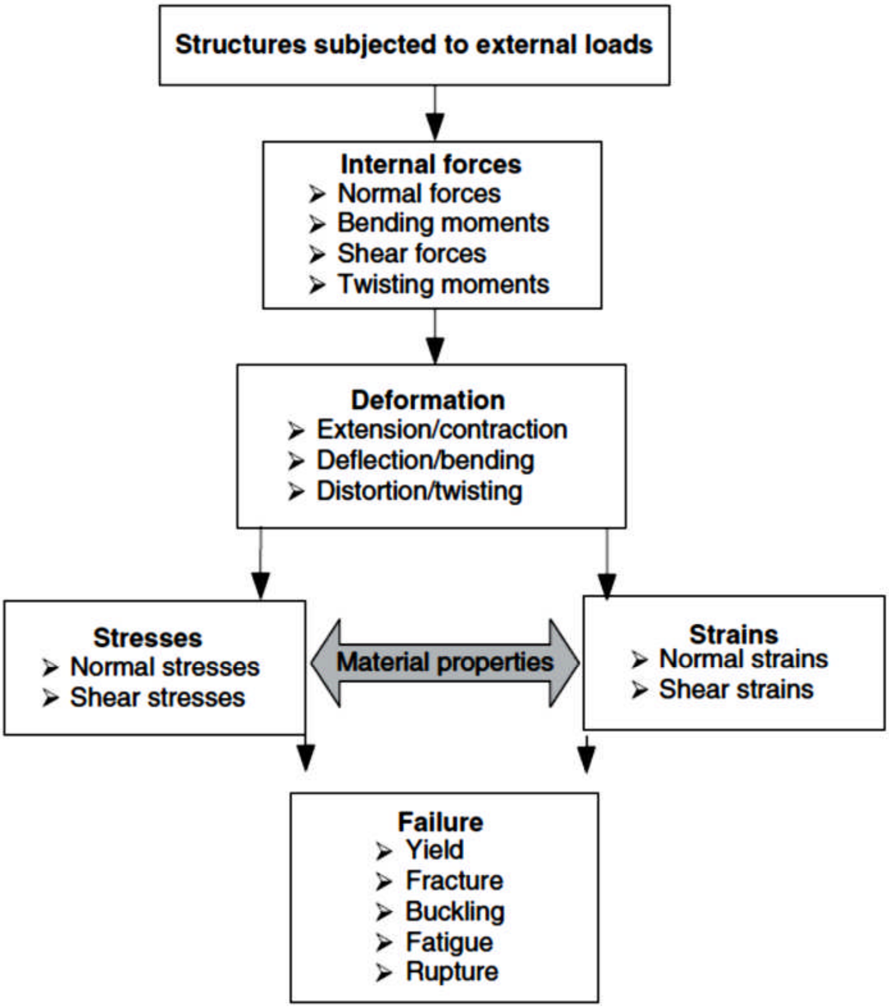
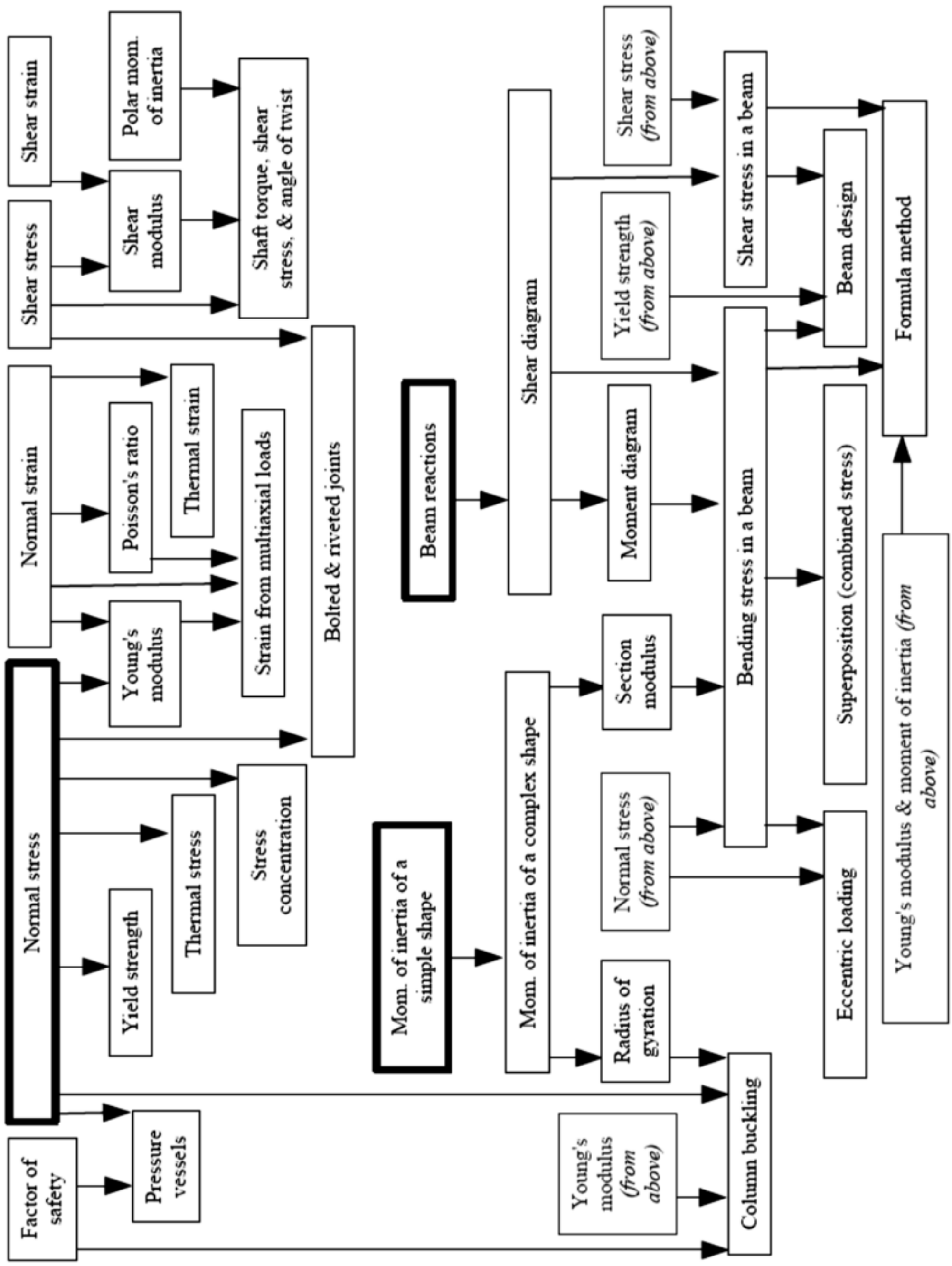


Figure 1-1

This diagram shows how the major topics in Strength are linked to each other and to three topics in Statics.



1- 2 Forces and moments

A force is a measure of its tendency to cause a body to move or translate in the direction of the force. A complete description of a force includes its magnitude and direction. The magnitude of a force acting on a structure is usually measured by Newton (N), or kilo newton (KN). In stress analysis, a force can be categorized as either external or internal. External forces include, for example, applied surface loads, force of gravity and support reactions, and the internal forces are the resisting forces generated within loaded structural elements. Typical examples of applied external forces include the following:

(a) Point load, where force is applied through a point of a structure (Figure 1.2(a))

(b) Distributed load, where force is applied over an area of a structure (Figure 1.2(b))

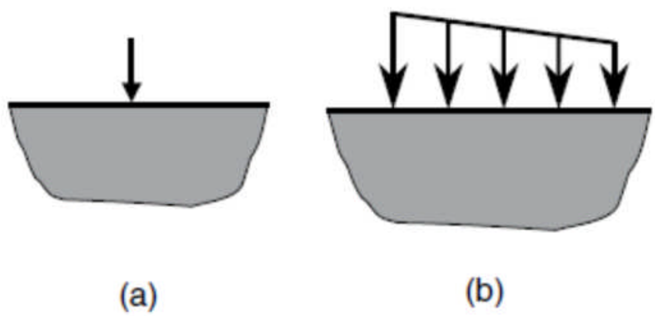


Figure 1.2

The moment of a force is a measure of its tendency to cause a body to rotate about a specific point or axis. In order to develop a moment about, for example, a specific axis, a force must act such that the body would begin to twist or bend about the axis. The magnitude of the moment of a force acting about a point or axis is directly proportional to the distance of the force from the point or axis. It is defined as the product of the force and the lever arm. The lever arm is the perpendicular distance between the line of action of the force and the point about which the force causes rotation. A moment is usually measured by Newton-meters (N m), or kilo newton meters (KN m). Figure 1.3 shows how a moment about the beam–column connection is caused by the applied point load F.

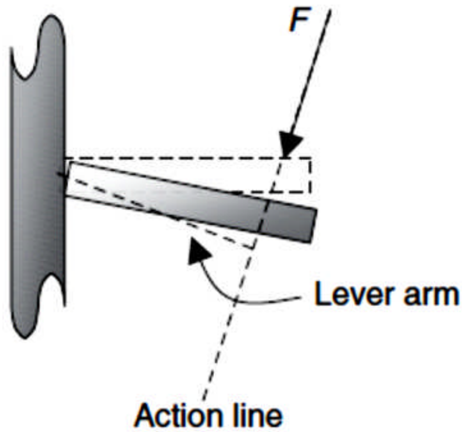


Figure 1.3

1-3 Types of force and deformation

1-3-1 Force

On a cross-section of a material subject to external loads, there exist four different types of internal force (Figure 1.4):

1. Normal force, F, which is perpendicular to the cross-section;
2. Shear force, V, which is parallel to the cross-section;
3. Bending moment, M, which bends the material; and
4. Twisting moment (torque), T, which twists the material about its central axis.

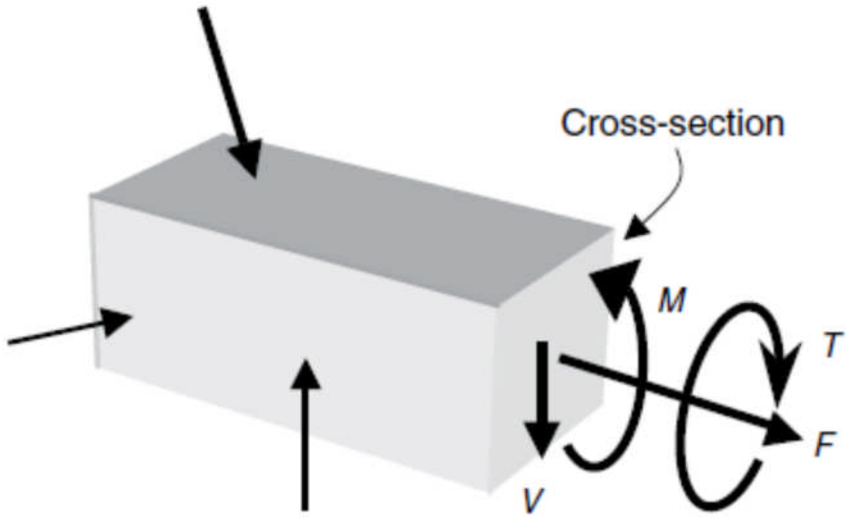


Figure 1.4

1-3-2 Deformation

Table 1.1 shows the most common types of force and their associated deformations. In a practical design, the deformation of a member can be a combination of the basic deformations shown in Table 1.1.

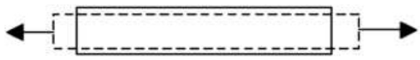
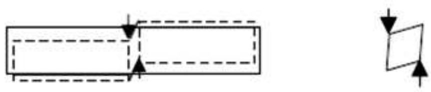
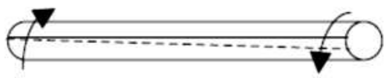
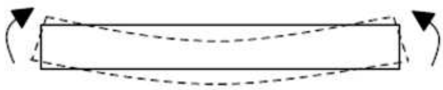
1- 4 Equilibrium system

In static structural and stress analysis, a system in equilibrium implies that:

- The resultant of all applied forces, including support reactions, must be zero;
- The resultant of all applied moments, including bending and twisting moments, must be zero

The two equilibrium conditions are commonly used to determine support reactions and internal forces on cross-sections of structural members.

Table 1.1 Basic types of deformation

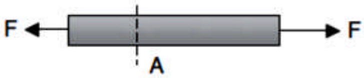
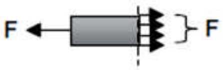
Force	*Deformation*	*Description*
Normal force, Axial force, Thrust		The member is being stretched by the axial force and is in *tension*. The deformation is characterized by axial *elongation*.
Normal force, Axial force, Thrust		The member is being compressed by the axial force and is in *compression*. The deformation is characterized by axial *shortening*.
Shear force		The member is being sheared. The deformation is characterized by *distorting* a rectangle into a parallelogram.
Torque, Twist moment		The member is being twisted and is in torsion. The deformation is characterized by *angle of twist*.
Bending moment		The member is being bent and the deformation is characterized by a *bent shape*.

1-4-1 The method of section

One of the most basic analyses is the investigation of the internal resistance of a structural member, that is, the development of internal forces within the member to balance the effect of the externally applied forces. The method of section is normally used for this purpose. Table 1.2 shows how the method of section works.

In summary, if a member as a whole is in equilibrium, any part of it must also be in equilibrium. Thus, the externally applied forces acting on one side of an arbitrary section must be balanced by the internal forces developed on the section.

Table 1.2 The method of section

	Consider a single bar under tension and the internal force on section A.
	Cut the bar into two parts at A and separate.
	Take one of the parts and consider equilibrium. The resultant force developed on section A must be equal to F.
	The force is also acting on the face of the right-hand-side part but in an opposite direction.

1-4 -2 The method of joint

The analysis or design of a truss requires the calculation of the forces in each of its members. Taking the entire truss as a free body, the forces in the members are internal forces. In order to determine the internal forces in the members jointed at a particular joint, for example, joint A in Figure 1.5, the joint can be separated from the truss system by cutting all the members around it. On the sections of the cuts there exist axial forces that can be further determined by considering the equilibrium of the joint under the action of the internal forces and the externally applied loads at the joint, that is, by resolving the forces in the x and y directions, respectively, and letting the resultants be zero.

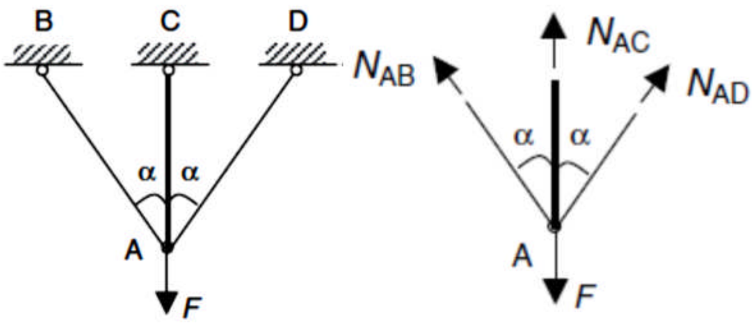


Figure 1.4

1-5 Stresses

Stress can be defined as the intensity of internal force that represents internal force per unit area at a point on a cross-section. Stresses are usually different from point to point. There are two types of stresses, namely normal and shear stresses.

1-5-1 Normal stress

Normal stress is a stress perpendicular to a cross-section or cut. For example, for the simple structural element shown in Figure 1.6(a), the normal stress on section m–m can be calculated as

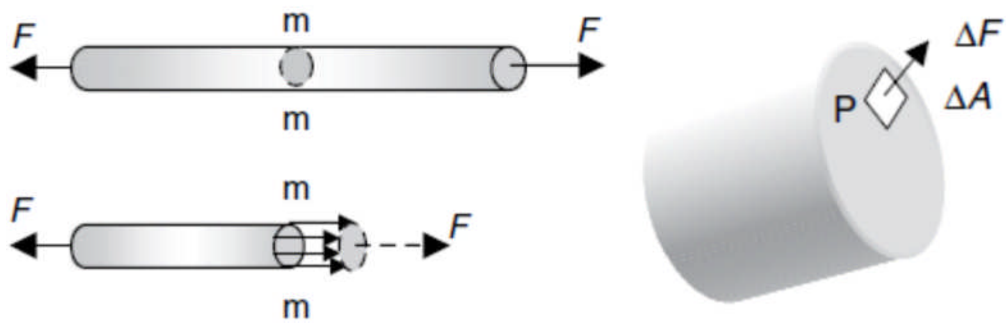
$$\text{Normal stress } (\sigma) = \frac{\text{force (on section m–m)}}{\text{area (of section m–m)}} \tag{1.1a}$$

The basic unit of stress is N/m2, which is also called a Pascal.

In general a stress varies from point to point (Figure 1.6(b)). A general stress can be calculated by

$$\textit{stress at point } \text{P} = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \tag{1.1b}$$

where ΔF is the force acting on the infinitesimal area, ΔA, surrounding P.



(a) (b)

Figure 1.6

1- 5- 2 Shear stress

Shear stress is a stress parallel to a cross-section or cut. For example, for the plates connected by a bolt, shown in Figure 1.7(a), the forces are transmitted from one part of structure to the other part by causing stresses in the plane parallel to the applied forces. These stresses are shear stresses. To compute the shear stresses, a cut is taken through the parallel plane and uniform distribution of the stresses over the cut is assumed. Thus:

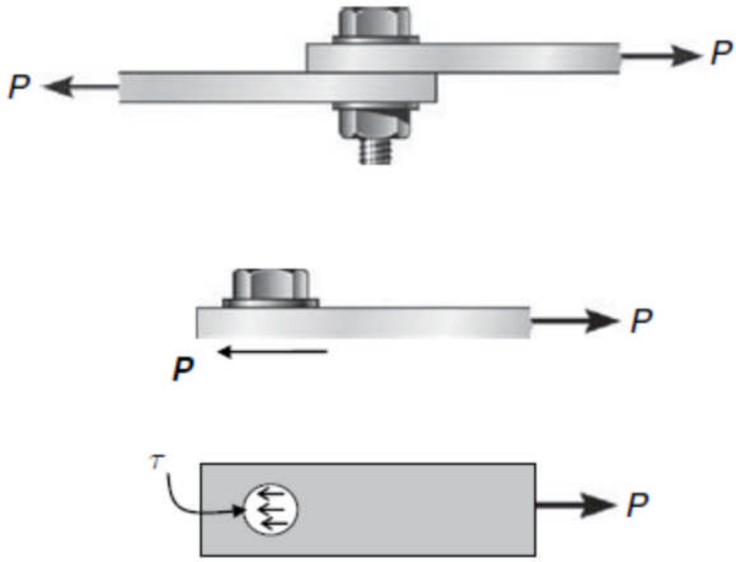
$$\tau = \frac{\text{force}}{\text{area}} = \frac{P}{A} \qquad (1.2)$$

Where A is the cross-sectional area of the bolt.

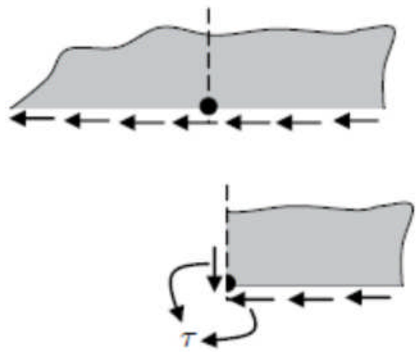
At a point in a material, shear stresses always appear in pair acting on two mutually perpendicular planes. They are equal in magnitude, but in an opposite sense, that is, either towards or away from the point (Figure 1.7(b)).

From the definition of normal and shear stresses, the following three characteristics must be specified in order to define a stress:

1. The magnitude of the stress;
2. The direction of the stress; and
3. The plane (cross-section) on which the stress is acting.



(a)



(b)

Figure 1.7

1- 6 Strains

Strain is a measure of relative deformation. Strains can be categorized as normal and shear strains.

Normal strain is a measure of the change in length per unit length under stress (Figure 1.8). It is measured by the following formula:

$$\text{Normal strain}(\varepsilon) = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta L}{L} \quad (1.3)$$

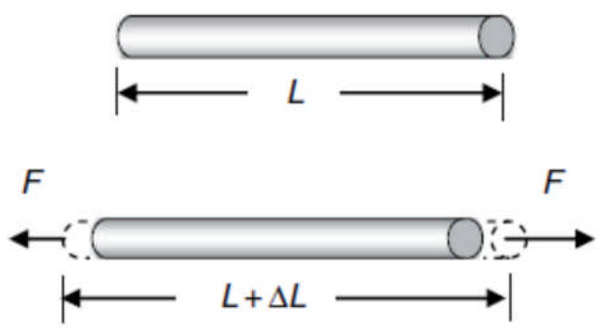


Figure 1.8

Shear strain is a measure of the change caused by shear stresses in the right angle between two fibers within a plane (Figure 1.9).

$$\text{Shear strain} \qquad \gamma = \alpha_1 + \alpha_2 \quad (1.4)$$

Shear strains are dimensionless.

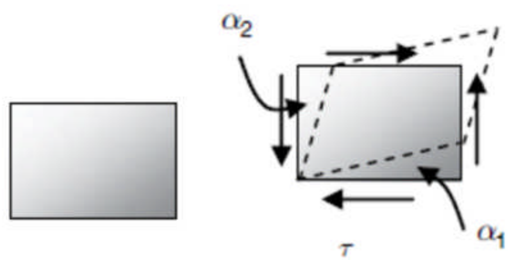
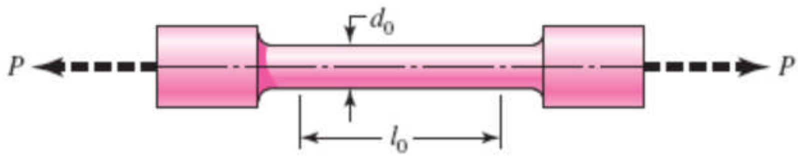


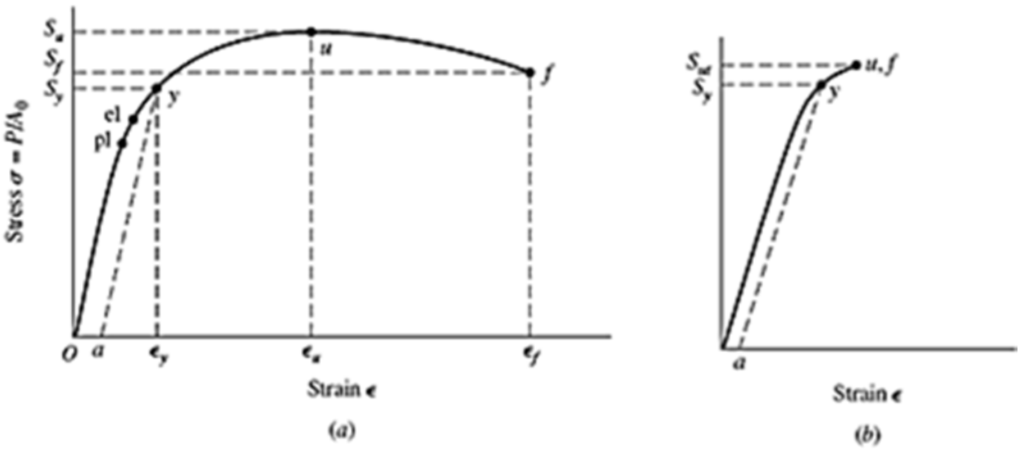
Figure 1.9

1-7 Strain–stress relation

Strains and stresses always appear in pair and their relationship depends on the properties of materials. Many mechanical properties of materials are determined from tests, some of which give relationships between stresses and strains as shown by the curves in the accompanying figures.



A typical tension-test specimen.



Stress- strain curves obtained from the standard tensile test (*a*) Ductile material; (*b*) brittle material. pl marks the proportional limit; el, the elastic limit; *y*, the offset-yield strength as defined by offset strain *a*; *u*, the maximum or ultimate strength; and *f*, the fracture strength.

Proportional limit is the point on a stress-strain curve at which it begins to deviate from the straight-line relationship between stress and strain.

Elastic limit is the maximum stress to which a test specimen may be subjected and still return to its original length upon release of the load. A material is said to be stressed within the elastic region when the working stress does not exceed the elastic limit, and to be stressed in the plastic region when the working stress does exceed the elastic limit. The elastic limit for steel is for all practical purposes the same as its proportional limit.

Yield point is a point on the stress-strain curve at which there is a sudden increase in strain without a corresponding increase in stress. Not all materials have a yield point.

Yield strength, S_y, is the maximum stress that can be applied without permanent deformation of the test specimen. This is the value of the stress at the elastic limit for materials for which there is an elastic limit. Because of the difficulty in determining the elastic limit, and because many materials do not have an elastic region, yield strength is often determined by the offset method as illustrated by the accompanying figures. Yield strength in such a case is the stress value on the stress-strain curve corresponding to a definite amount of permanent set or strain, usually 0.1 or 0.2 per cent of the original dimension.

Ultimate strength, S_u, (also called tensile strength) is the maximum stress value obtained on a stress-strain curve.

Strain–stress relation is also termed as Hooke's law, which determines how much strain occurs under a given stress. For materials undergoing linear elastic deformation, stresses are proportional to strains. Thus, for the simple load cases shown in Figures 1.8 and 1.9, the strain–stress relations are:

$$\sigma = E\varepsilon$$

$$\tau = G\gamma \qquad (1.5)$$

Where E is called modulus of elasticity or Young's modulus. G is termed as shear modulus. They are all material-dependent constants and measure the unit of stress, for example, N/mm^2, since strains are dimensionless. For isotropic materials, for example, most metals, E and G have the following relationship:

$$G = \frac{E}{2(1+\nu)} \tag{1.6}$$

In Equation (1.6), v is called Poisson's ratio, which is also an important material constant. Figure 1.10 shows how a Poisson's ratio is defined by comparing axial elongation and lateral contraction of a prismatic bar in tension. Poisson's ratio is defined as:

$$\text{Poisson's ratio } (\nu) = \left|\frac{\text{lateral strain}}{\text{axial strain}}\right| = -\frac{\text{lateral strain (contraction)}}{\text{axial strain (tension)}} \tag{1.7}$$

A negative sign is usually assigned to a contraction. Poisson's ratio is a dimensionless quality that is constant in the elastic range for most materials and has a value between 0 and 0.5.