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Application

The tension in the cable supporting this person can be found using the concepts in this chapter

Scalar triple product

- $A \times B = (A_x i + A_y i + A_z i) \times (B_x i + B_y i + B_z i)$
- $A \times B = (A_y B_z - A_z B_y)i + (A_z B_x - A_x B_z)j + (A_x B_y - A_y B_x)k$
- $A \times B = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$
- $A \times B.C = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$

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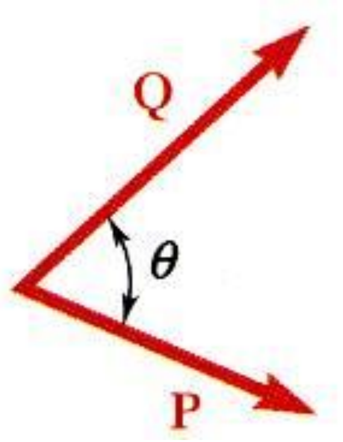
Scalar Product of Two Vectors

- The *scalar product* or *dot product* between two vectors ***P*** and ***Q*** is defined as

  $$\vec{P} \bullet \vec{Q} = PQ\cos\theta \quad (\text{scalar result})$$



- Scalar products:
  - are commutative, $\vec{P} \bullet \vec{Q} = \vec{Q} \bullet \vec{P}$
  - are distributive, $\vec{P} \bullet (\vec{Q}_1 + \vec{Q}_2) = \vec{P} \bullet \vec{Q}_1 + \vec{P} \bullet \vec{Q}_2$
  - are not associative, $(\vec{P} \bullet \vec{Q}) \bullet \vec{S} = \text{undefined}$

- Scalar products with Cartesian unit components,

$$\vec{P} \bullet \vec{Q} = (P_x\vec{i} + P_y\vec{j} + P_z\vec{k}) \bullet (Q_x\vec{i} + Q_y\vec{j} + Q_z\vec{k})$$

$$\vec{i} \bullet \vec{i} = 1 \quad \vec{j} \bullet \vec{j} = 1 \quad \vec{k} \bullet \vec{k} = 1 \quad \vec{i} \bullet \vec{j} = 0 \quad \vec{j} \bullet \vec{k} = 0 \quad \vec{k} \bullet \vec{i} = 0$$

$$\vec{P} \bullet \vec{Q} = P_xQ_x + P_yQ_y + P_zQ_z$$

$$\vec{P} \bullet \vec{P} = P_x^2 + P_y^2 + P_z^2 = P^2$$

Scalar Product of Two Vectors: Applications

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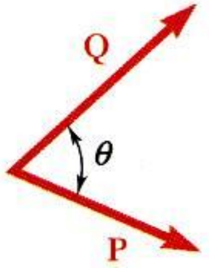
- Angle between two vectors:

$$\vec{P} \bullet \vec{Q} = PQ\cos\theta = P_xQ_x + P_yQ_y + P_zQ_z$$

$$\cos\theta = \frac{P_xQ_x + P_yQ_y + P_zQ_z}{PQ}$$

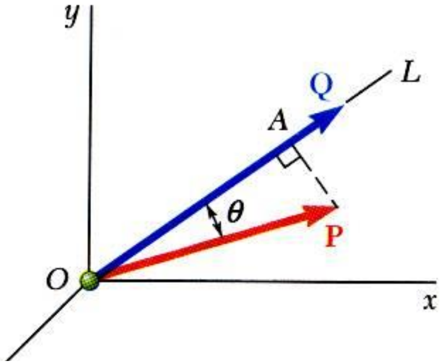


- Projection of a vector on a given axis:

$$P_{OL} = P\cos\theta = \text{projection of } P \text{ along } OL$$

$$\vec{P} \bullet \vec{Q} = PQ\cos\theta$$

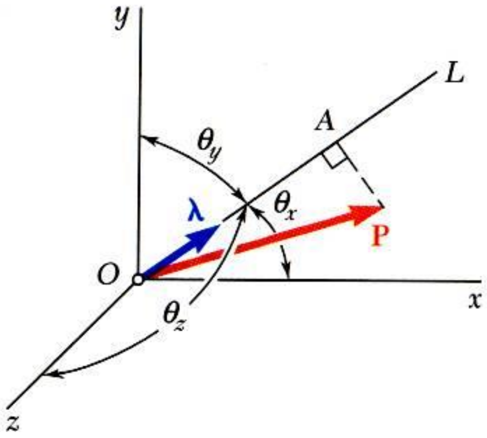
$$\frac{\vec{P} \bullet \vec{Q}}{Q} = P\cos\theta = P_{OL}$$



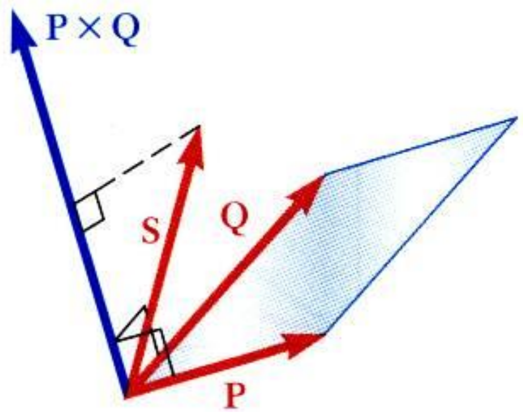
- For an axis defined by a unit vector:

$$P_{OL} = \vec{P} \bullet \vec{\lambda}$$

$$= P_x\cos\theta_x + P_y\cos\theta_y + P_z\cos\theta_z$$



Mixed Triple Product of Three Vectors



- Mixed triple product of three vectors,

$$\vec{S} \bullet \left(\vec{P} \times \vec{Q}\right) = \text{scalar result}$$

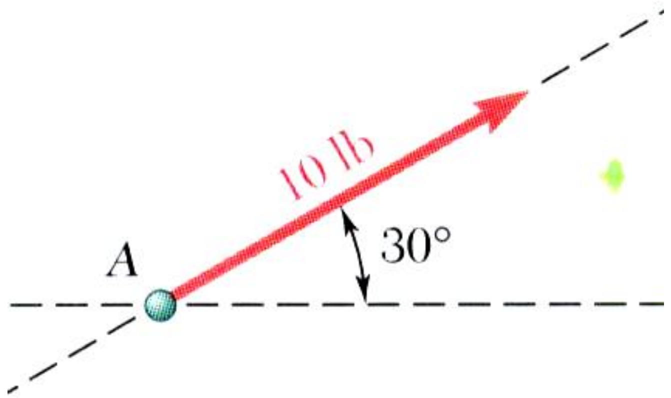
- The six mixed triple products formed from ***S***, ***P***, and ***Q*** have equal magnitudes but not the same sign,

$$\vec{S} \bullet \left(\vec{P} \times \vec{Q}\right) = \vec{P} \bullet \left(\vec{Q} \times \vec{S}\right) = \vec{Q} \bullet \left(\vec{S} \times \vec{P}\right)$$
$$= -\vec{S} \bullet \left(\vec{Q} \times P\right) = -\vec{P} \bullet \left(\vec{S} \times \vec{Q}\right) = -\vec{Q} \bullet \left(\vec{P} \times \vec{S}\right)$$

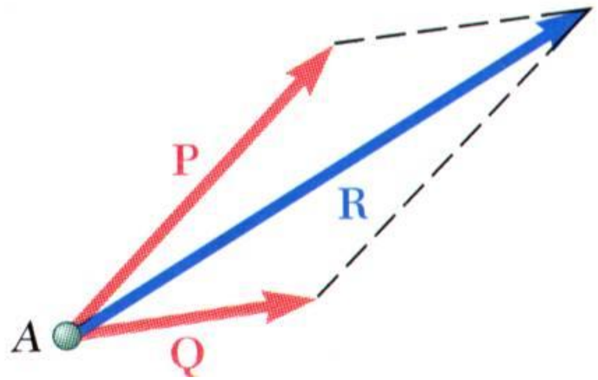
- Evaluating the mixed triple product,

$$\vec{S} \bullet \left(\vec{P} \times \vec{Q}\right) = S_x\left(P_y Q_z - P_z Q_y\right) + S_y\left(P_z Q_x - P_x Q_z\right) + S_z\left(P_x Q_y - P_y Q_x\right)$$
$$= \begin{vmatrix} S_x & S_y & S_z \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

Resultant of Two Forces



- force: action of one body on another; characterized by its *point of application*, *magnitude*, *line of action*, and *sense*.



- Experimental evidence shows that the combined effect of two forces may be represented by a single *resultant* force.
- The resultant is equivalent to the diagonal of a parallelogram which contains the two forces in adjacent legs.
- Force is a *vector* quantity.

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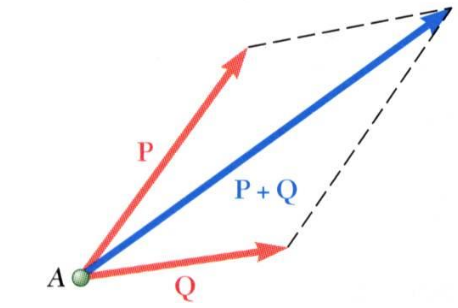
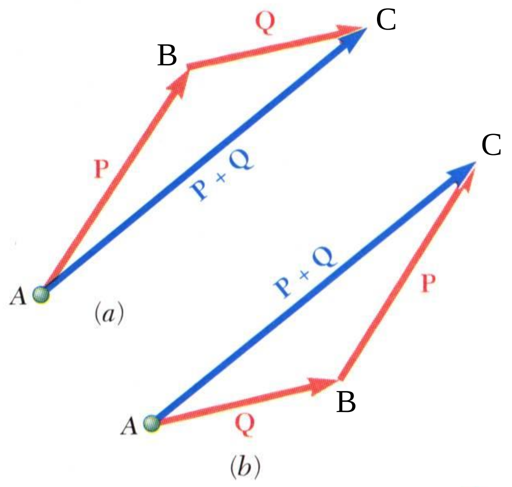
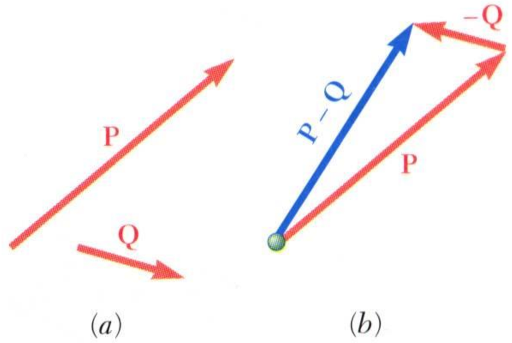
Addition of Vectors







- Trapezoid rule for vector addition
- Triangle rule for vector addition
- Law of cosines,

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$\vec{R} = \vec{P} + \vec{Q}$$

- Law of sines,

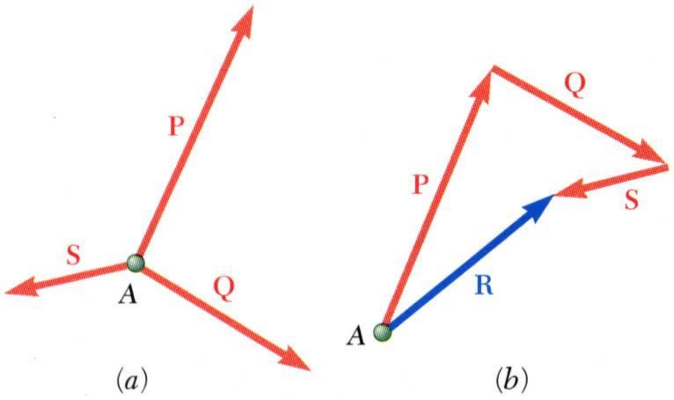
$$\frac{\sin A}{P} = \frac{\sin B}{R} = \frac{\sin C}{Q}$$

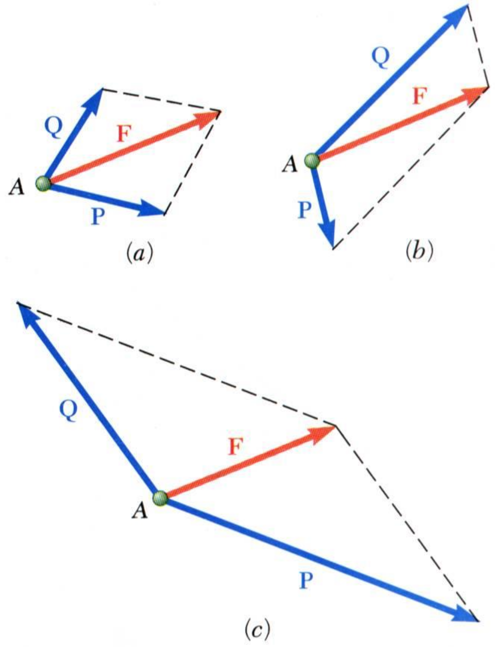
- Vector addition is commutative,

$$\vec{P} + \vec{Q} = \vec{Q} + \vec{P}$$

- Vector subtraction

Resultant of Several Concurrent Forces





- *Concurrent forces*: set of forces which all pass through the same point.

  A set of concurrent forces applied to a particle may be replaced by a single resultant force which is the vector sum of the applied forces.

- *Vector force components*: two or more force vectors which, together, have the same effect as a single force vector.

Sample Problem

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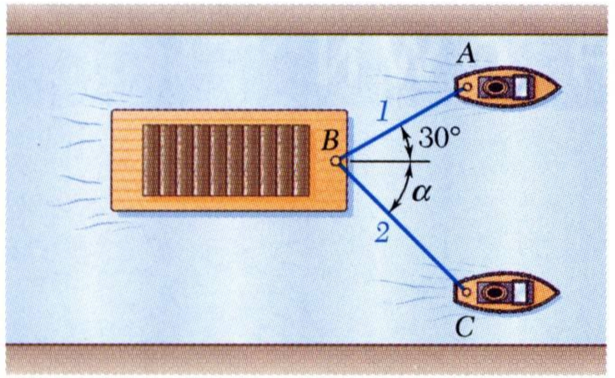


A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is 5000 lbf directed along the axis of the barge, determine the tension in each of the ropes for $\alpha = 45^o$.

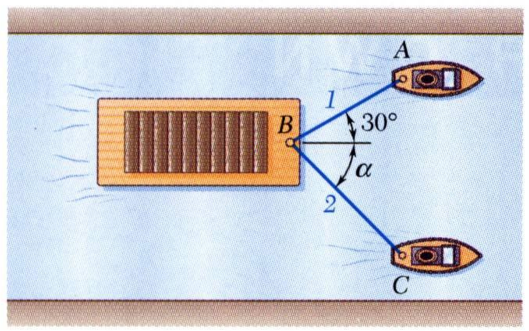
Discuss with a neighbor how you would solve this problem.

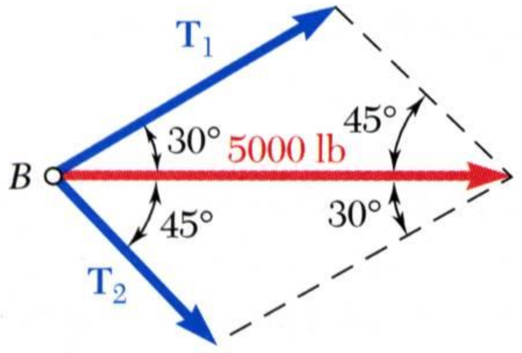
SOLUTION:

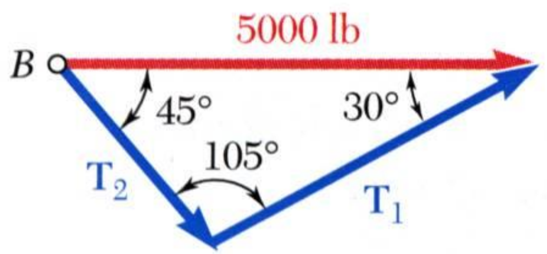
- Find a graphical solution by applying the Parallelogram Rule for vector addition. The parallelogram has sides in the directions of the two ropes and a diagonal in the direction of the barge axis and length proportional to 5000 lbf.

- Find a trigonometric solution by applying the Triangle Rule for vector addition. With the magnitude and direction of the resultant known and the directions of the other two sides parallel to the ropes given, apply the Law of Sines to find the rope tensions.

Sample Problem







- Graphical solution - Parallelogram Rule with known resultant direction and magnitude, known directions for sides.

$$T_1 = 3700\,\text{lbf} \quad T_2 = 2600\,\text{lbf}$$

- Trigonometric solution - Triangle Rule with Law of Sines

$$\frac{T_1}{\sin 45°} = \frac{T_2}{\sin 30°} = \frac{5000\,\text{lbf}}{\sin 105°}$$

$$T_1 = 3660\,\text{lbf} \quad T_2 = 2590\,\text{lbf}$$

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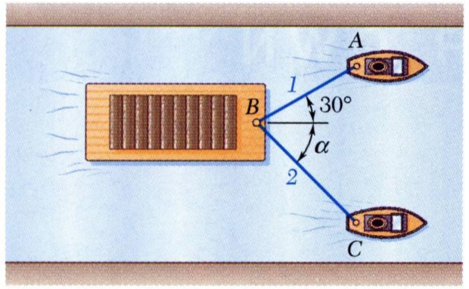
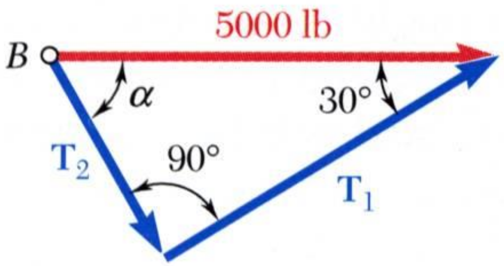
What if…?



- At what value of α would the tension in rope 2 be a minimum?

  Hint: Use the triangle rule and think about how changing α changes the magnitude of T_2. After considering this, discuss your ideas with a neighbor.



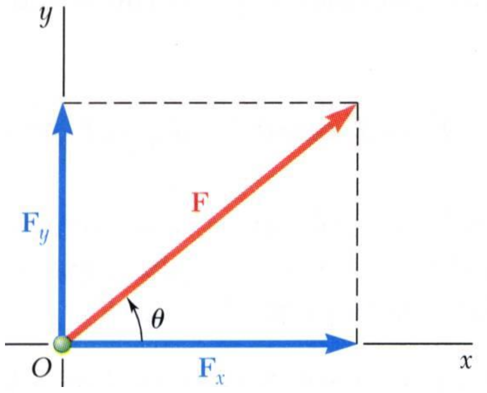
- The minimum tension in rope 2 occurs when $\mathbf{T_1}$ and $\mathbf{T_2}$ are perpendicular.

$$T_2 = (5000\,\text{lbf})\sin 30° \qquad \boxed{T_2 = 2500\,\text{lbf}}$$

$$T_1 = (5000\,\text{lbf})\cos 30° \qquad \boxed{T_1 = 4330\,\text{lbf}}$$

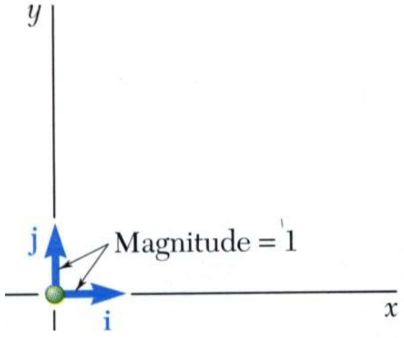
$$\alpha = 90° - 30° \qquad \boxed{\alpha = 60°}$$

Rectangular Components of a Force: Unit Vectors

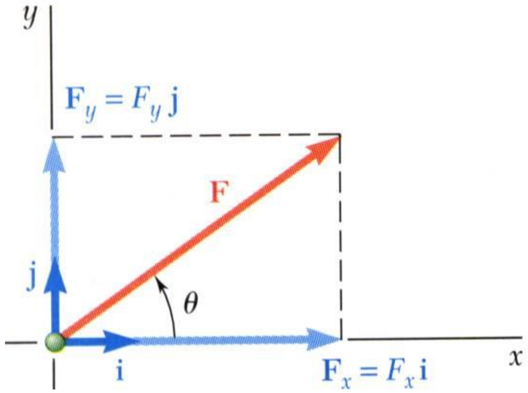


- It's possible to resolve a force vector into perpendicular components so that the resulting parallelogram is a rectangle. $\vec{F}_x$ and $\vec{F}_y$ are referred to as *rectangular vector components* and

$$\vec{F} = \vec{F}_x + \vec{F}_y$$



- Define perpendicular *unit vectors* $\vec{i}$ and $\vec{j}$ which are parallel to the *x* and *y* axes.



- Vector components may be expressed as products of the unit vectors with the scalar magnitudes of the vector components.

$$\vec{F} = F_x \vec{i} + F_y \vec{j}$$

F_x and F_y are referred to as the *scalar components* of $\vec{F}$

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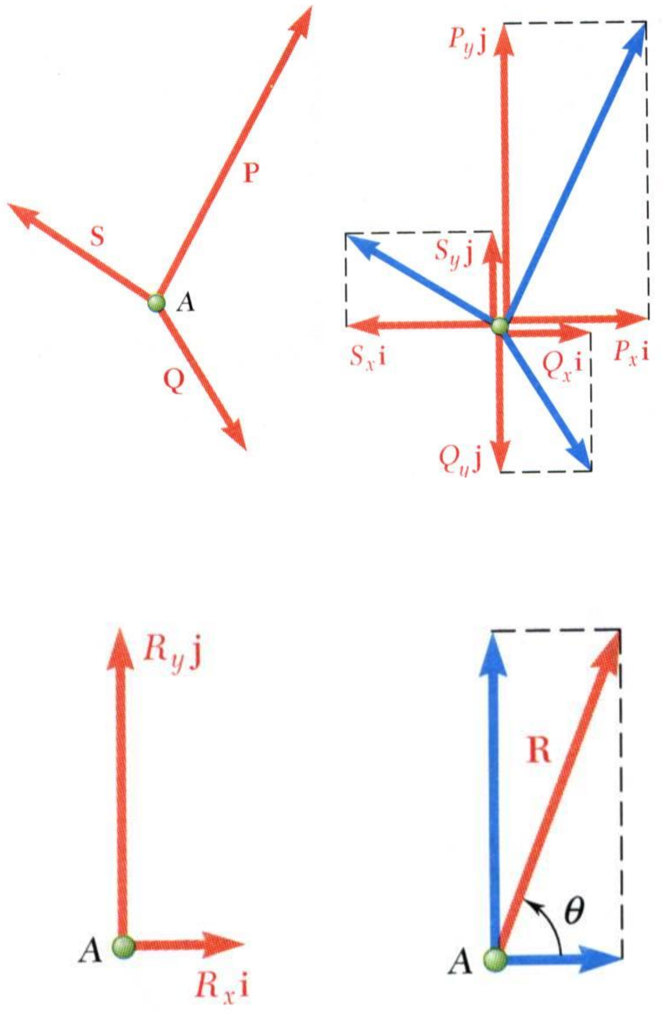
Addition of Forces by Summing Components



- To find the resultant of 3 (or more) concurrent forces,

$$\vec{R} = \vec{P} + \vec{Q} + \vec{S}$$

- Resolve each force into rectangular components, then add the components in each direction:

$$R_x\vec{i} + R_y\vec{j} = P_x\vec{i} + P_y\vec{j} + Q_x\vec{i} + Q_y\vec{j} + S_x\vec{i} + S_y\vec{j}$$
$$= (P_x + Q_x + S_x)\vec{i} + (P_y + Q_y + S_y)\vec{j}$$

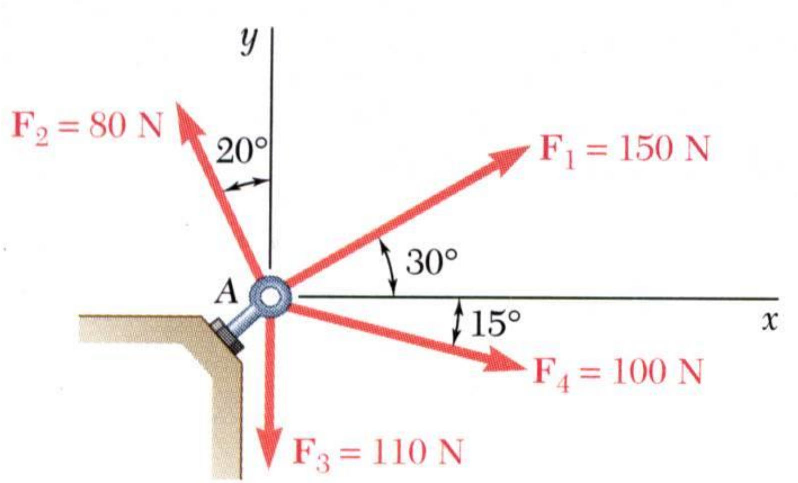
- The scalar components of the resultant vector are equal to the sum of the corresponding scalar components of the given forces.

$$R_x = P_x + Q_x + S_x = \sum F_x \qquad R_y = P_y + Q_y + S_y = \sum F_y$$

- To find the resultant magnitude and direction,

$$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \tan^{-1}\frac{R_y}{R_x}$$

Sample Problem



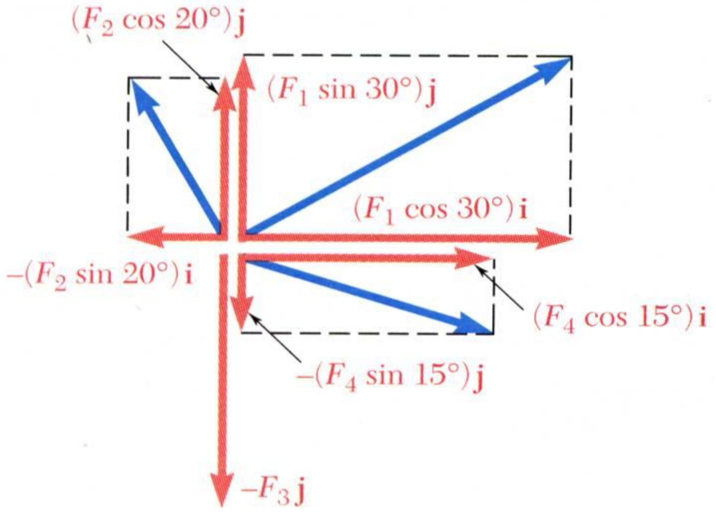
Four forces act on bolt *A* as shown. Determine the resultant of the force on the bolt.

SOLUTION:

- Resolve each force into rectangular components.
- Determine the components of the resultant by adding the corresponding force components in the x and y directions.
- Calculate the magnitude and direction of the resultant.

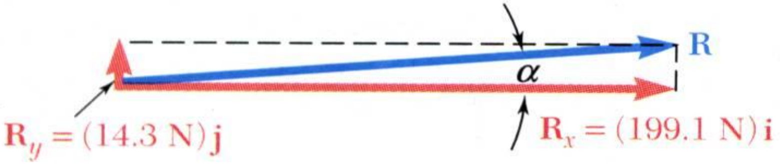
Sample Problem

SOLUTION: Resolve each force into rectangular components.



force	mag	x − comp	y − comp
$\vec{F}_1$	150	+129.9	+75.0
$\vec{F}_2$	80	−27.4	+75.2
$\vec{F}_3$	110	0	−110.0
$\vec{F}_4$	100	+96.6	−25.9
		$R_x = +199.1$	$R_y = +14.3$

- Determine the components of the resultant by adding the corresponding force components.



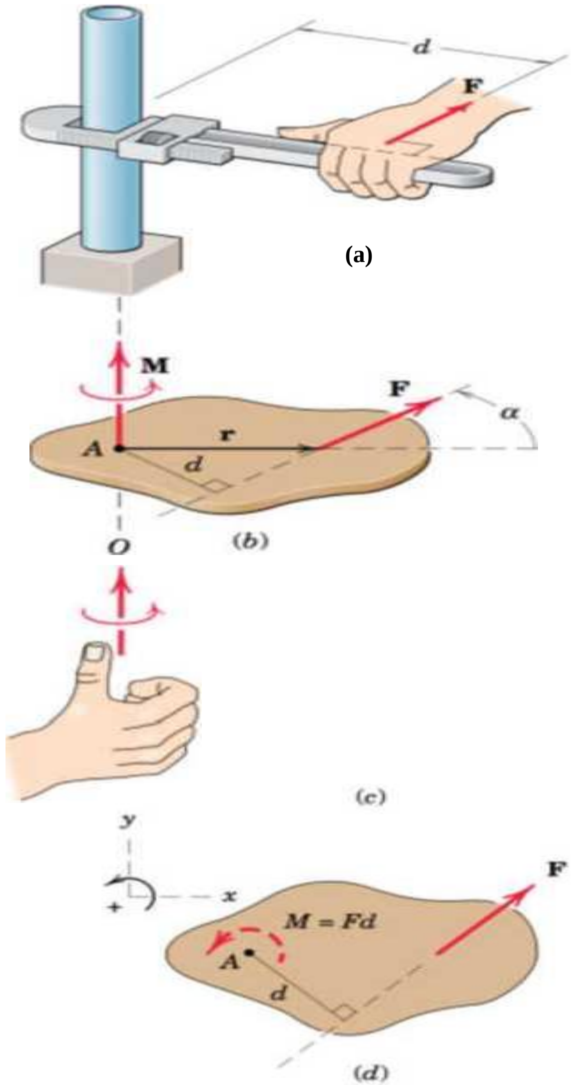
- Calculate the magnitude and direction.

$$R = \sqrt{199.1^2 + 14.3^2} \qquad R = 199.6\,\mathrm{N}$$

$$\tan\alpha = \frac{14.3\,\mathrm{N}}{199.1\,\mathrm{N}} \qquad \alpha = 4.1^\circ$$

MOMENT

- $M = Fd$
- $\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$
- $M_O = \mathbf{r} \times \mathbf{R} = \mathbf{r} \times \mathbf{P} + \mathbf{r} \times \mathbf{Q}$



Moment of a Force About a Point

- A force vector is defined by its magnitude and direction. Its effect on the rigid body also depends on it point of application.

- The *moment* of $\boldsymbol{F}$ about O is defined as

$$\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$$

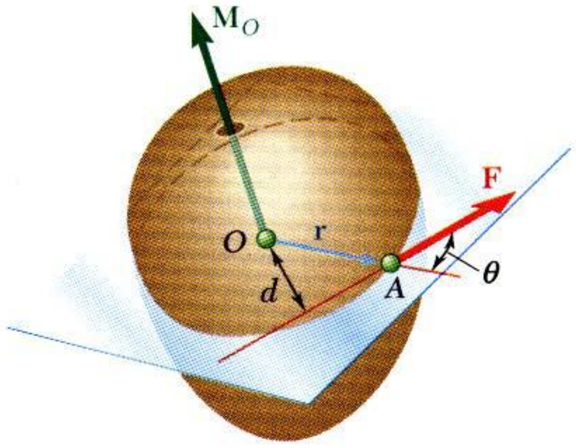
- The moment vector $\boldsymbol{M_O}$ is perpendicular to the plane containing O and the force $\boldsymbol{F}$.

- Magnitude of $\boldsymbol{M_O}$ measures the tendency of the force to cause rotation of the body about an axis along $\boldsymbol{M_O}$.

$$M_O = rF\sin\theta = Fd$$

  The sense of the moment may be determined by the right-hand rule.

- Any force $\boldsymbol{F'}$ that has the same magnitude and direction as $\boldsymbol{F}$, is *equivalent* if it also has the same line of action and therefore, produces the same moment.



(a)

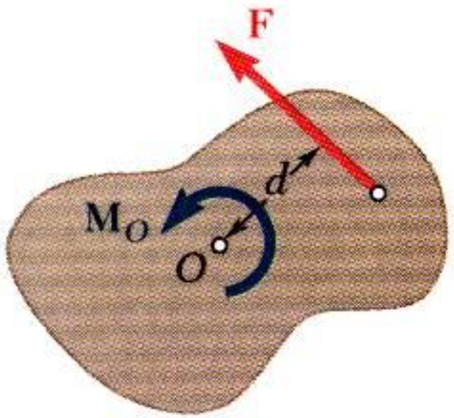
(b)

Moment of a Force About a Point

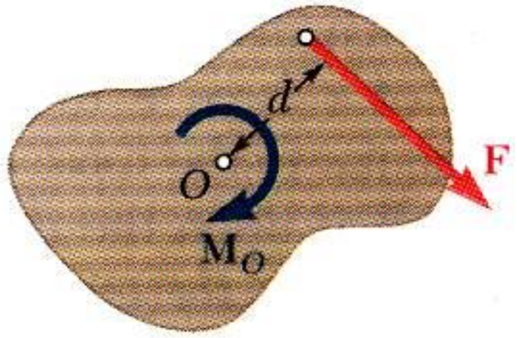
- *Two-dimensional structures* have length and breadth but negligible depth and are subjected to forces contained in the plane of the structure.

- The plane of the structure contains the point *O* and the force ***F***. $\boldsymbol{M_O}$, the moment of the force about *O* is perpendicular to the plane.

- If the force tends to rotate the structure counterclockwise, the sense of the moment vector is out of the plane of the structure and the magnitude of the moment is positive.
- If the force tends to rotate the structure clockwise, the sense of the moment vector is into the plane of the structure and the magnitude of the moment is negative.



(a) $\boldsymbol{M}_O = + Fd$



(b) $\boldsymbol{M}_O = - Fd$

Varignon's Theorem

- The moment about a given point O of the resultant of several concurrent forces is equal to the sum of the moments of the various moments about the same point O.

$$\vec{r} \times \left(\vec{F}_1 + \vec{F}_2 + \cdots\right) = \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2 + \cdots$$

- Varigon's Theorem makes it possible to replace the direct determination of the moment of a force ***F*** by the moments of two or more component forces of ***F***.