

\mathbb{R} . Show that $\sigma(\mathcal{I}) = \mathcal{B}(\mathbb{R})$. (5 Marks)

of following: σ -algebra, an outer measure on X , Lebesgue measurable. (4 Marks)

ity set and $\mathcal{E} = \mathcal{P}(X)$. Define $\mu: \mathcal{E} \rightarrow [0, \infty)$ by

$$\mu(A) = \begin{cases} n & \text{if } A \text{ has } n \text{ element} \\ \infty & \text{otherwise.} \end{cases}$$

ce on $\mathcal{P}(X)$? (5 Marks)

measure space, $A, B \in \mathcal{E}$ and $A \subseteq B$. Prove that $\mu(A) \leq \mu(B)$. (4 Marks)

(17 Marks)

measure space, and (A_n) is a sequence in \mathcal{E} . Prove that

(7 Marks)

$$\mu\left(\bigcup_{n=1}^{\infty} A_n\right) \leq \sum_{n=1}^{\infty} \mu(A_n).$$

an outer measure on X , $\mu^*(E) = 0$ and $E \subset X$. Prove that E is μ^* measurable. (4 Marks)

(4 Marks)

ut outer measure on X and $E_1, E_2 \in \mathcal{M}$, $E_1 \cap E_2 = \emptyset$ where \mathcal{M} is the collection of measurable subsets of X . Show that

$$\mu^*(A \cap (E_1 \cup E_2)) = \mu^*(A \cap E_1) + \mu^*(A \cap E_2)$$

(6 Marks)

(18 Marks)

the Lebesgue outer measure on \mathbb{R} .

(5 Marks)

asure on \mathbb{R} , $A \subset \mathbb{R}$. Prove that there exist an open set V such that $A \subset V$ and $\mu^*(V) < \mu^*(A) + \epsilon$. (4 Marks)

(4 Marks)

erval (a, ∞) , $a \in \mathbb{R}$ is measurable. (5 Marks)

(5 Marks)

ceable space and $E \in \mathcal{E}$, $f: E \rightarrow \mathbb{R}$ and f is measurable. Show that the set $\{x \in E : f(x) \in \mathbb{Q}\}$ is measurable. (4 Marks)

(4 Marks)

(17 Marks)

for a real-valued function is a measurable. (4 Marks)

(4 Marks)

and g are measurable real-valued function defined on a domain $E \in \mathcal{E}$. Show

$|f|$ are measurable functions. (5 Marks)

(5 Marks)

measure space, ϕ non-negative simple functions. Show for any $A, B \in \mathcal{E}$, that

$$\int_A \phi d\mu + \int_B \phi d\mu = \int_{A \cup B} \phi d\mu.$$

(3 Marks)

measure space. For any $A \in \mathcal{E}$ and the function $\nu: \mathcal{E} \rightarrow [0, \infty]$ defined by

show that the function $\nu(A)$ is a measure on X . (5 Marks)

(5 Marks)

مع أحب التمام بالقرآن

بفهمه فليس إلا حكمة

استقلا العقول و الرقاد قلوب

Borel σ -algebra

b- Give the me

c- Let X be a m

Is μ a mea

d- Let (X, \mathcal{E}, μ)

Q2:

a- Let (X, \mathcal{E}, μ)

b- Let X be a

v- Let X be a σ -finite measure space. Let μ^* be the outer measure induced by μ . Show that μ^* is a measure on \mathcal{M} .

Q3:

a- Define and

b- Let μ^* an outer measure on X . Show that $\mu^*(V) < \mu^*(A) + \epsilon$ for any $A \subset X$ and $\epsilon > 0$.

c- Show that if f, g are measurable functions on X , then $af + bg$ is measurable for any $a, b \in \mathbb{R}$.

d- Let (X, \mathcal{E}, μ) be a measure space. Let $f: X \rightarrow \mathbb{R}$ be a measurable function. Show that the set $\{x \in X : f(x) \in \mathbb{Q}\}$ is measurable.

Q4:

a- Give an example of a measurable function $f: \mathbb{R} \rightarrow \mathbb{R}$ such that f is not Lebesgue measurable.

b- Assume that f, g are measurable functions on X . Show that $cf + g$ is measurable for any $c \in \mathbb{R}$.

c- Let (X, \mathcal{E}, μ) be a measure space. Let f, g be measurable functions on X . Show that $\int_A cf + g d\mu = c \int_A f d\mu + \int_A g d\mu$ for any $A \in \mathcal{E}$.

d- Let (X, \mathcal{E}, μ) be a measure space. Let f, g be measurable functions on X . Show that $\int_A cf + g d\mu = c \int_A f d\mu + \int_A g d\mu$ for any $A \in \mathcal{E}$.

e- Let (X, \mathcal{E}, μ) be a measure space. Let f, g be measurable functions on X . Show that $\int_A cf + g d\mu = c \int_A f d\mu + \int_A g d\mu$ for any $A \in \mathcal{E}$.