ℝ. Show that $\sigma(J) = \mathcal{B}(\mathbb{R})$ (5 Marks)

of following: σ −algebra, an outer measure on X, Lebesgue measurable. (4 Marks)

ty set and $\Sigma = P(X)$. Define $\mu: \Sigma \to [0, \infty)$ by

$$\mu(A) = \begin{cases} n & \text{if } A \text{ has n element} \\ \infty & \text{otherwise.} \end{cases}$$

ce on $P(X)$? (5 Marks)

measure space, $A, B \in \Sigma$ and $A \subseteq B$. Prove that $\mu(A) \leq \mu(B)$. (4 Marks)

(17 Marks)

measure space, and (A_n) is a sequence in Σ. Prove that (7 Marks)

$$\mu\left(\bigcup_{n=1}^{\infty} A_n\right) \leq \sum_{n=1}^{\infty} \mu(A_n).$$

an outer measure on X, $\mu^*(E) = 0$ and $E \subset X$. Prove that E is μ^* measurable. (4 Marks)

an outer measure on X and $E_1, E_2 \in M$, $E_1 \cap E_2 = \emptyset$ where M is the collection ble subsets of X. Show that

$A \cap (E_1 \cup E_2)) = \mu^*(A \cap E_1) + \mu^*(A \cap E_2)$ (6 Marks)

(18 Marks)

the Lebesgue outer measure on ℝ. (5 Marks)

asure on ℝ, $A \subset \mathbb{R}$. Prove that there exist an open set V such that $A \subset V$ and

ϵ. (4 Marks)

rval (a, ∞), $a \in \mathbb{R}$ is measurable. (5 Marks)

easurable space and $E \in \Sigma$, $f: E \to \mathbb{R}$ and f is measurable. Show that the set

$\alpha \in \mathbb{R}\}$ is measurable. (4 Marks)

(17 Marks)

for a real-valued function is a measurable. (4 Marks)

d g are measurable real-valued function defined on a domain $E \in \Sigma$. Show

$|f|$ are measurable functions. (5 Marks)

measure space, φ non-negative simple functions. Show for any $A, B \subseteq X$, that

$\int_A \varphi d\mu + \int_B \varphi d\mu$. (3 Marks)

measure space. For any $A \in \Sigma$ and the function $v: \Sigma \to [0, \infty]$ defined by

Show that the function $v(A)$ is a measure on X. (5 Marks)

مع أطيب التمنيات بالتوفيق

رئيس قسم الرياضيات / د.

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Borel σ-alge

b- Give the me

c- Let X be a no

Is μ a measur

d- Let (X, Σ, μ)

Q2:

a- Let (X, Σ, μ)

b- Let X be a s

c- Let X be a s

of all μ^*-me

Q3:

a- Define and p

b- Let μ^* an ou

$\mu^*(V) < \mu^*$

c- Show that th

d- Let (X, Σ) b

$\{x \in E: f(x)$

Q4:

a- Give an exa

b- Assume that

that $cf, f \wedge$

c- Let (X, Σ, μ)

$\int_{A \cup B} \varphi d$

d- Let (X, Σ, μ)

$v(A) = \int$