جامعة دمياط
Damietta University
جامعة دمياط
كلية العلوم
قسم الرياضيات

نموذج امتحان نهائي
الفصل الدراسي الثانى لعام
٢٠٢٣-٢٠٢٤

المستوى: الرابع (ساعات معتمدة)
البرنامج: الرياضيات
المادة: نظرية القياس
الكود: (٤١٦ ر)
التاريخ: ٢ / ٦ / ٢٠٢٤
الزمن: ٢ ساعات
الدرجة الكلية: ٧٠ درجة

Answer the following questions:

Q1: **(18 Marks)**

a- Let $I = \{[a,b): a,b \in \mathbb{R}, a \leq b\}$, $\sigma(I)$ be the smallest σ- algebra containing I and $\beta(\mathbb{R})$ be Borel σ-algebra of $\mathbb{R}$. Show that $\sigma(I) = \beta(\mathbb{R})$. **(5 Marks)**

b- Give the meaning of following: **σ −algebra, an outer measure on X, Lebesgue measurable.** **(4 Marks)**

c- Let X be a nonempty set and $\Sigma = P(X)$. Define $\mu: \Sigma \to [0,\infty)$ by

$$\mu(A) = \begin{cases} n & \text{if } A \text{ has n element} \\ \infty & \text{otherwise.} \end{cases}$$

Is μ a measure space on $P(X)$? **(5 Marks)**

d- Let (X, Σ, μ)be a measure space, $A, B \in \Sigma$ and $A \subseteq B$. Prove that $\mu(A) \leq \mu(B)$. **(4 Marks)**

Q2: **(17 Marks)**

a- Let (X, Σ, μ)be a measure space, and (A_n) is a sequence in Σ. Prove that **(7 Marks)**

$$\mu\left(\bigcup_{n=1}^{\infty} A_n\right) \leq \sum_{n=1}^{\infty} \mu(A_n).$$

b- Let X be a set, μ^*an outer measure on X, $\mu^*(E) = 0$ and $E \subset X$. Prove that E is μ^* measurable. **(4 Marks)**

c- Let X be a set, μ^*an outer measure on X and $E_1, E_2 \in M$, $E_1 \cap E_2 = \emptyset$ where M is the collection of all μ^*-measurable subsets of X. Show that

$$\mu^*(A \cap (E_1 \cup E_2)) = \mu^*(A \cap E_1) + \mu^*(A \cap E_2)$$ **(6 Marks)**

Q3: **(18 Marks)**

a- Define and prove the Lebesgue outer measure on $\mathbb{R}$. **(5 Marks)**

b- Let μ^*an outer measure on R, $A \subset R$. Prove that there exist an open set V such that $A \subset V$ and $\mu^*(V) < \mu^*(A) + \epsilon$. **(4 Marks)**

c- Show that the interval (a, ∞), $a \in \mathbb{R}$ is measurable. **(5 Marks)**

d- Let (X, Σ) be a measurable space and $E \in \Sigma$, $f: E \to \mathbb{R}$ and f is measurable. Show that the set $\{x \in E: f(x) < \alpha, \alpha \in \mathbb{R}\}$ is measurable. **(4 Marks)**

Q4: **(17 Marks)**

a- Give an example for a real-valued function is a measurable. **(4 Marks)**

b- Assume that f and g are measurable real-valued function defined on a domain $E \in \Sigma$. Show that $cf, f \wedge g$ and $|f|$ are measurable functions. **(5 Marks)**

c- Let (X, Σ, μ) be a measure space, φ non-negative simple functions. Show for any $A, B \subseteq X$, that $\int_{A \cup B} \varphi d\mu = \int_A \varphi d\mu + \int_B \varphi d\mu$. **(3 Marks)**

d- Let (X, Σ, μ) be a measure space. For any $A \in \Sigma$ and the function $\nu: \Sigma \to [0,\infty]$ defined by $\nu(A) = \int_A \varphi d\mu$, Show that the function $\nu(A)$ is a measure on X. **(5 Marks)**

مع أطيب التمنيات بالتوفيق

أستاذ المقرر: د / وفاء قوطه　　　　رئيس قسم الرياضيات: أ.د/حسن المرشدي