Damietta University
Faculty of science
Mathematical Department

Third Level Mathematics Students
Mid-year Exam 2024/2025
Theory of Elasticity (303 Math)
Time: Three Hours Date: 1-1-2025

Answer all the following questions: **(Total: 105 marks)**

1- a- Defined the Kronecker Delta and Permutation tensors, then expand the following expressions and simplify where possible.

i) $\varepsilon_{3jk}\, a_j\, a_k$ ii) $\varepsilon_{ijk}\, \delta_{kj}$ iii) $\varepsilon_{1jk}\, a_2\, T_{kj}$ **(10 marks)**

b- If A_i is a first-order Cartesian tensor, show that its derivative with respect to x_k, namely $A_{i,k}$ is a second-order Cartesian tensor. **(11 marks)**

2- a- Determine the principal values and principal directions of the second-order tensor **T** whose matrix representation is **(11 marks)**

$$[T_{ij}] = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

b- With respect to axes $Ox_1x_2x_3$ the stress state is given in terms of the coordinates by the matrix

$$[\sigma_{ij}] = \begin{bmatrix} \left(1 - x_1^2\right)x_2 + \frac{2}{3}x_2^3 & -\left(4 - x_2^2\right)x_1 & 0 \\ -\left(4 - x_2^2\right)x_1 & \frac{-1}{3}\left(x_2^3 - 12x_2\right) & 0 \\ 0 & 0 & \left(3 - x_1^2\right)x_2 \end{bmatrix}$$

i) Show that the equilibrium equations are to be satisfied everywhere for zero body forces.**(5 marks)**

ii) Determine the stress vector at point $P(2, -1, 6)$ on the plane whose equation $3x_1 + 6x_2 + 2x_3 = 12$ **(5 marks)**

3- a- Prove the strain compatibility equations. **(6 marks)**

b- Given the deformation expressed by

$$x_1 = X_1 + \alpha X_2, \qquad x_2 = X_2 - \alpha X_1, \qquad x_3 = X_3$$

where α is a constant, determine the finite strain tensor **E** and **e** and show that the circle of particles $X_1^2 + X_2^2 = 1$ deformed into the circle $x_1^2 + x_2^2 = 1 + \alpha^2$. **(15 marks)**

4- a- What is the physical meaning of the diagonal elements of the strain tensor ε_{22}? **(10 marks)**

b- Use Hooke's law for isotropic media to drive the equations of motion in the terms of displacement components (Navier's Equations). **(11 marks)**

5- a- Prove that tensor of elastic coefficients G_{ijkl} is a fourth-order Cartesian tensor, and show that, why it has 36 components only, not 81?. **(10 marks)**

b- Drive the thermoelastic constitutive equations (stress components). **(11 marks)**

With best wishes

Prof. Mohammed Elhagary