Electromagnetic Theory

CREDIT HOURS FIRST LEVEL(PHYSICS /PHYSICS AND COMPUTER SCIENCE PROGRAM)

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PHYSICS FOR SCIENTISTS AND ENGINEERS

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Chapter 3: Electric Potential

- Electric Potential and Potential Difference.
- Potential Difference in a uniform Electric Field.
- Obtaining the Value of the Electric Field from the Electric Potential.

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Remember that: Energy Considerations

When a force, F, acts on a particle, work is done on the particle in moving from point *a* to point *b*

$$W_{a\to b} = \int_a^b \vec{F} \cdot d\vec{l}$$

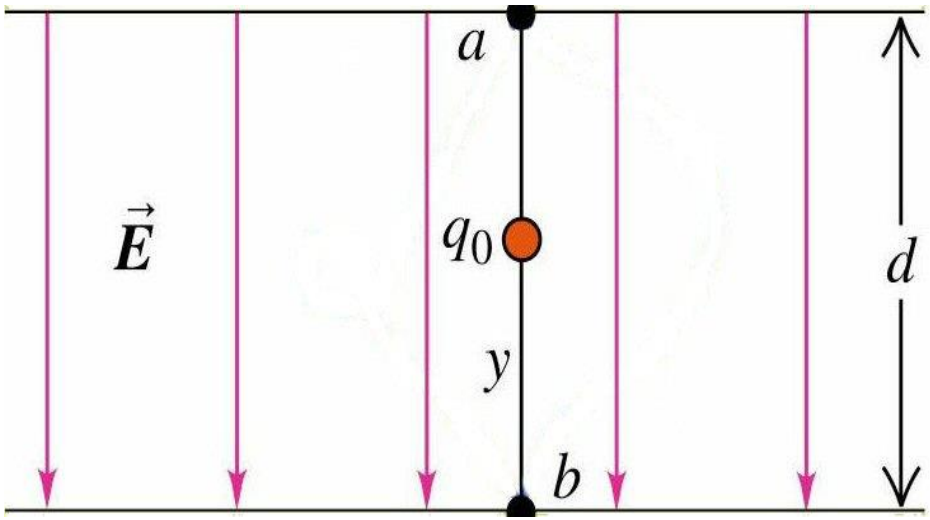
If the force is a conservative, then the work done can be expressed in terms of a change in potential energy

$$W_{a\to b} = -\left(U_b - U_a\right) = -\Delta U$$

Also if the force is conservative, the total energy of the particle remains *constant*

$$KE_a + PE_a = KE_b + PE_b$$

Work Done by Uniform Electric Field



Force on charge is

$$\vec{F} = q_0 \vec{E}$$

Work is done on the charge by field

$$W_{a \to b} = Fd = q_0 E d$$

The work done is *independent* of path taken from point a to point b because

The Electric Force is a *conservative force*

Electric Potential Energy

The work done by the force is the same as the change in the particle's potential energy:

$$W_{a\to b} = -\left(U_b - U_a\right) = -\Delta U$$

$$U_b - U_a = -\int_a^b \vec{F}\cdot d\vec{s} = -qE_{uniform}\left(y_b - y_a\right)$$

The work done only depends upon the *change* in position.

Electric Potential Energy

General Points

1) Potential Energy *increases* if the particle moves in the direction *opposite* to the force on it

Work will have to be done by an external agent for this to occur

and

2) Potential Energy *decreases* if the particle moves in the *same* direction as the force on it

Potential Energy

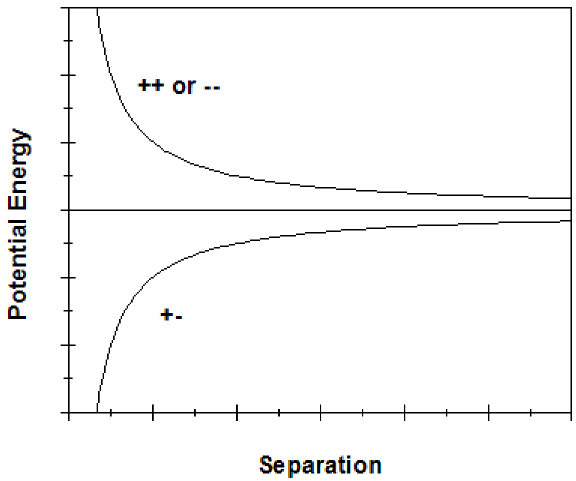
Looking at the work done we notice that there is the same *functional* at points a and b and that we are taking the difference

$$W_{a\to b} = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

We define this functional to be the potential energy

$$U = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r}$$

The signs of the charges are included in the calculation

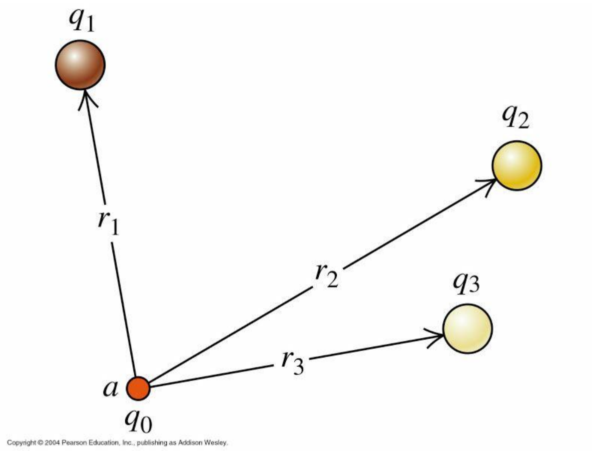


The potential energy is taken to be zero when the two charges are infinitely separated

Potential Energy of one charge with respect to others

Given several charges, $q_1 \ldots q_n$, in place

Now a test charge, q_0, is brought into position



Work must be done against the electric fields of the original charges

This work goes into the potential energy of q_0

We calculate the potential energy of q_0 with respect to each of the other charges and then

Just sum the individual potential energies

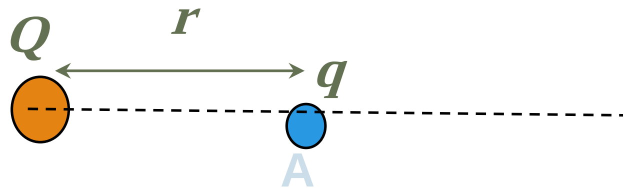
$$PE_{q_0} = \sum_i \frac{1}{4\pi\varepsilon_0} \frac{q_0 q_i}{r_i}$$

Remember - Potential Energy is a Scalar

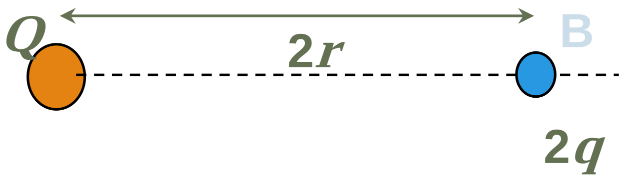
Example 1

Two test charges are brought separately to the vicinity of a positive charge Q

Charge $+q$ is brought to pt A, a distance r from Q



Charge $+2q$ is brought to pt B, a distance $2r$ from Q



I) Compare the potential energy of q (U_A) to that of $2q$ (U_B)

(a) $U_A < U_B$ **(b)** $U_A = U_B$ **(c)** $U_A > U_B$

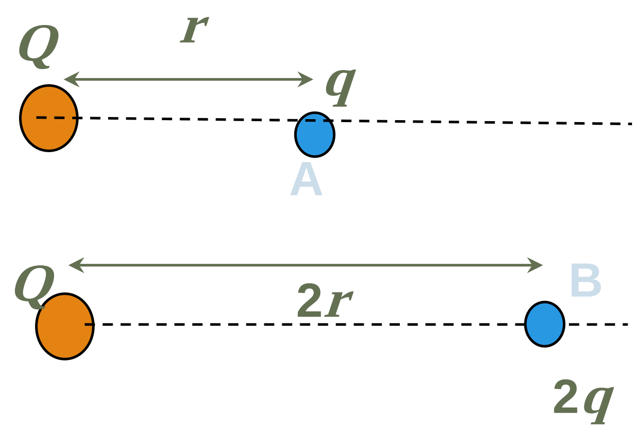
II) Suppose charge $2q$ has mass m and is released from rest from the above position (a distance $2r$ from Q). What is its velocity v_f as it approaches $r = \infty$?

(a) $v_f = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{Qq}{mr}}$ **(b)** $v_f = \sqrt{\frac{1}{2\pi\varepsilon_0}\frac{Qq}{mr}}$ **(c)** $v_f = 0$

Two test charges are brought separately to the vicinity of a positive charge Q

Charge $+q$ is brought to pt A, a distance r from Q

Charge $+2q$ is brought to pt B, a distance $2r$ from Q



I) Compare the potential energy of q (U_A) to that of $2q$ (U_B)

(a) $U_A < U_B$ $\quad$ (b) $\boxed{U_A = U_B}$ $\quad$ (c) $U_A > U_B$

The potential energy of q is proportional to Qq/r

The potential energy of $2q$ is proportional to $Q(2q)/(2r) = Qq/r$

Therefore, the potential energies U_A and U_B are EQUAL!!!

II) Suppose charge $2q$ has mass m and is released from rest from the above position (a distance $2r$ from Q). What is its velocity v_f as it approaches $r = \infty$?

(a) $v_f = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{Qq}{mr}}$ **(b)** $\boxed{v_f = \sqrt{\frac{1}{2\pi\varepsilon_0}\frac{Qq}{mr}}}$ **(c)** $v_f = 0$

The principle at work here is *CONSERVATION OF ENERGY*.

Initially:

The charge has no kinetic energy since it is at rest.

The charge does have potential energy (electric) = U_B.

Finally:

The charge has no potential energy ($U \propto 1/R$)

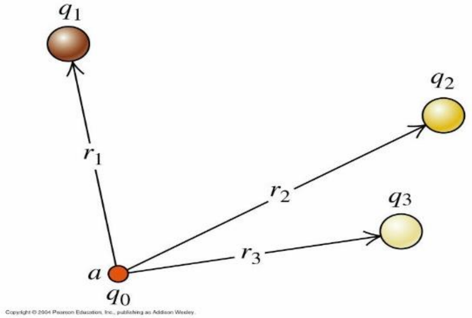
The charge does have kinetic energy = *KE*

$$U_B = KE \quad \Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{Q(2q)}{2r} = \frac{1}{2}mv_f^2 \quad \Rightarrow \quad v_f^2 = \frac{1}{2\pi\varepsilon_0}\frac{Qq}{mr}$$

Electric Potential

Recall Case 1 from before

The potential energy of the test charge, q_0, was given by



$$PE_{q_0} = \sum_i \frac{1}{4\pi\varepsilon_0} \frac{q_0 q_i}{r_i}$$

Notice that there is a part of this equation that would remain the same regardless of the test charge, *q_0*, placed at point *a*

The value of the test charge can be pulled out from the summation

$$PE_{q_0} = q_0 \sum_i \frac{1}{4\pi\varepsilon_0} \frac{q_i}{r_i}$$

Electric Potential

We define the term to the right of the summation as the electric potential at point *a*

$$Potential_a = \sum_i \frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i}$$

Like energy, potential is a *scalar*

We define the potential of a given point charge as being

$$Potential = V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

This equation has the convention that the potential is zero at infinite distance

The potential at a given point

Represents the potential energy that a positive unit charge would have, if it were placed at that point

It has units of

$$\text{Volts} = \frac{\text{Energy}}{\text{charge}} = \frac{joules}{coulomb}$$

General Points for either positive or negative charges

The Potential *increases* if you move in the direction *opposite* to the electric field

and

The Potential *decreases* if you move in the *same* direction as the electric field

Example 2

Points A, B, and C lie in a uniform electric field.

What is the potential difference between points A and B?

$\Delta V_{AB} = V_B - V_A$

a) $\Delta V_{AB} > 0$ b) $\Delta V_{AB} = 0$ c) $\Delta V_{AB} < 0$

The electric field, *E*, points in the direction of decreasing potential

Since points A and B are in the same relative horizontal location in the electric field there is on potential difference between them

Example 3

Points A, B, and C lie in a uniform electric field.

Point C is at a higher potential than point A.

True False

As stated previously the electric field points in the direction of *decreasing* potential

Since point C is further to the right in the electric field and the electric field is pointing to the right, point C is at a lower potential

The statement is therefore false

Example 4

Points A, B, and C lie in a uniform electric field.

If a negative charge is moved from point A to point B, its electric potential energy

a) Increases.

b) decreases.

c) doesn’t change.

The potential energy of a charge at a location in an electric field is given by the product of the charge and the potential at the location

As shown in Example 2, the potential at points A and B are the same

Therefore the electric potential energy also doesn’t change

Example 5

Points A, B, and C lie in a uniform electric field.

Compare the potential differences between points A and C and points B and C.

a) $V_{AC} > V_{BC}$ b) $V_{AC} = V_{BC}$ c) $V_{AC} < V_{BC}$

In Example 4 we showed that the the potential at points A and B were the same

Therefore the potential difference between A and C and the potential difference between points B and C are the same

Also remember that potential and potential energy are scalars and directions do not come into play

Work and Potential

The work done by the electric force in moving a test charge from point *a* to point *b* is given by

$$W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l} = \int_a^b q_0\vec{E}\cdot d\vec{l}$$

Dividing through by the test charge q_0 we have

$$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$$

Rearranging so the order of the subscripts is the same on both sides

$$V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{l}$$

Potential

From this last result

$$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}$$

We get

$$dV = -\vec{E} \cdot d\vec{l} \text{ or } \frac{dV}{dx} = -E$$

We see that the electric field points in the direction of *decreasing* potential

We are often more interested in potential differences as this relates directly to the work done in moving a charge from one point to another

Example 6

If you want to move in a region of electric field without changing your electric potential energy. You would move

a) **Parallel to the electric field**

b) **Perpendicular to the electric field**

The work done by the electric field when a charge moves from one point to another is given by

$$W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l} = \int_a^b q_0\vec{E}\cdot d\vec{l}$$

The way no work is done by the electric field is if the integration path is perpendicular to the electric field giving a zero for the dot product

Example 7

A positive charge is released from rest in a region of electric field. The charge moves:

$\boxed{\text{a) towards a region of smaller electric potential}}$

b) along a path of constant electric potential

c) towards a region of greater electric potential

A positive charge placed in an electric field will experience a force given by $F = qE$

But *E* is also given by $E = -\dfrac{dV}{dx}$

Therefore $F = qE = -q\dfrac{dV}{dx}$

Since q is positive, the force F points in the direction opposite to increasing potential or in the direction of decreasing potential

Units for Energy

There is an additional unit that is used for energy in addition to that of joules

A particle having the charge of *e* (1.6 x 10^{-19} C) that is moved through a potential difference of 1 Volt has an increase in energy that is given by

$$W = qV = 1.6 \times 10^{-19}\ joules$$

$$= 1\, eV$$

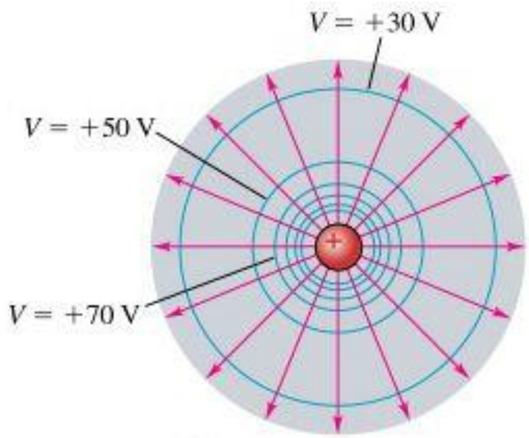
Equipotential Surfaces

It is possible to move a test charge from one point to another without having any net work done on the charge

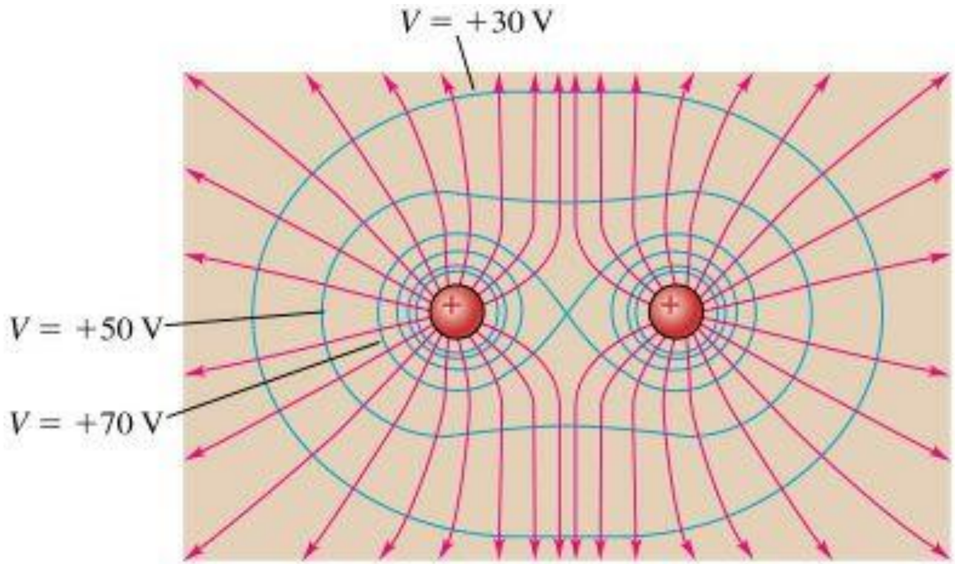
This occurs when the beginning and end points have the same potential

It is possible to map out such points and a given set of points at the same potential form an *equipotential surface*

Examples of equipotential surfaces



Point Charge



Two Positive Charges

The electric field does no work as a charge is moved along an equipotential surface

Since no work is done, there is no force, qE, along the direction of motion

The electric field is *perpendicular* to the equipotential surface

Potential Gradient

The equation that relates the derivative of the potential to the electric field that we had before

$$\frac{dV}{dx} = -E$$

can be expanded into three dimensions

$$\vec{E} = -\vec{\nabla} V$$

$$\vec{E} = -\left(\hat{i}\frac{dV}{dx} + \hat{j}\frac{dV}{dy} + \hat{k}\frac{dV}{dz} \right)$$

Example 8

The electric potential in a region of space is given by

$$V(x) = 3x^2 - x^3$$

The *x*-component of the electric field E_x at $x = 2$ is

$\boxed{\textbf{(a)} \quad E_x = 0}$ **(b)** $E_x > 0$ **(c)** $E_x < 0$

We know *V(x)* "everywhere"

To obtain E_x "everywhere", use

$$\vec{E} = -\vec{\nabla} V \Rightarrow E_x = -\frac{dV}{dx} \Rightarrow E_x = -6x + 3x^2$$

$$E_x(2) = -6(2) + 3(2)^2 = 0$$

Example 9

You have a 12.0-V motorcycle battery that can move 5000 C of charge, and a 12.0-V car battery that can move 60,000 C of charge. How much energy does each deliver?

Solution

For the motorcycle battery, $q=5000C$ and $\Delta V=12.0V$. The total energy delivered by the motorcycle battery is

$$\Delta U_{cycle}=(5000C)(12.0V)=(5000C)(12.0J/C)=6.00\times10^{4}J.$$

Similarly, for the car battery, $q=60,000C$ and

$$\Delta U_{car}=(60,000C)(12.0V)=7.20\times10^{5}J.$$