Electromagnetic Theory

CREDIT HOURS FIRST LEVEL(PHYSICS /PHYSICS AND COMPUTER SCIENCE PROGRAM)

104 PH

COLLECTED BY DR. FATEMA ALZAHRAA MOHAMMAD

PHYSICS DEPARTMENT-FACULTY OF SCIENCE-DAMIETTA UNIVERSITY-EGYPT)

PHYSICS FOR SCIENTISTS AND ENGINEERS

BY RAYMOND A. SERWAY , JOHN W. JEWETT

.

Chapter 4: Capacitance and Dielectrics

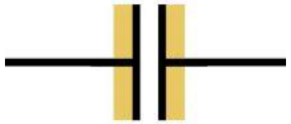
- Definition of Capacitance.
- Calculating Capacitance.
- Combinations of Capacitors.
- Energy Stored in a Charged Capacitors.

Capacitors

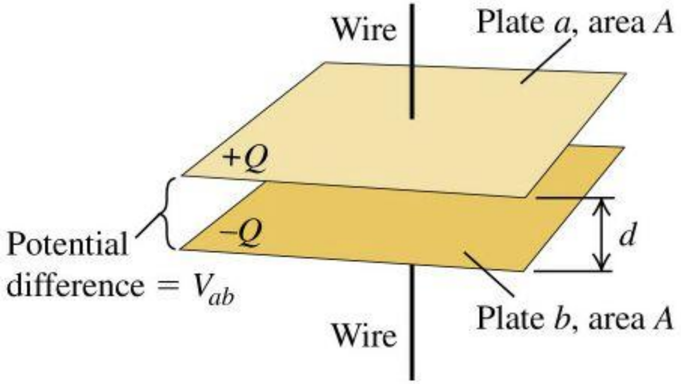
Definition of the capacitors: A *capacitor* is a device used to store electric charge. Capacitors have applications ranging from filtering static out of radio reception to energy storage in heart defibrillators. Typically, commercial capacitors have two conducting parts close to one another, but not touching, such as those in Figure below. (Most of the time an insulator is used between the two plates to provide separation) When battery terminals are connected to an initially uncharged capacitor, equal amounts of positive and negative charge, $+Q$ and $-Q$, are separated into its two plates. The capacitor remains neutral overall, but we refer to it as storing a charge Q in this circumstance.

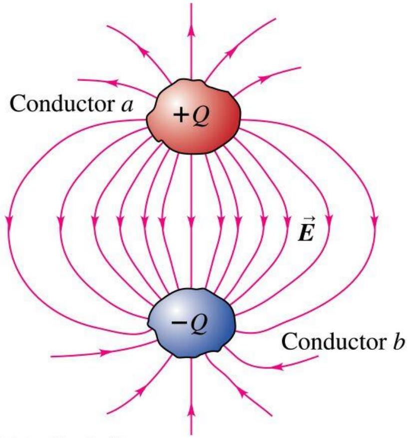
It takes work, which is then stored as potential energy in the electric field that is set up between the two plates, to place charges on the conducting plates of the capacitor.

Symbol in circuits is



We usually talk about capacitors in terms of parallel conducting plates but in fact any charged particles +Q and –Q separated by distance can called it capacitor.





Since there is an electric field between the plates there is also a potential difference between the two plates.

Capacitance

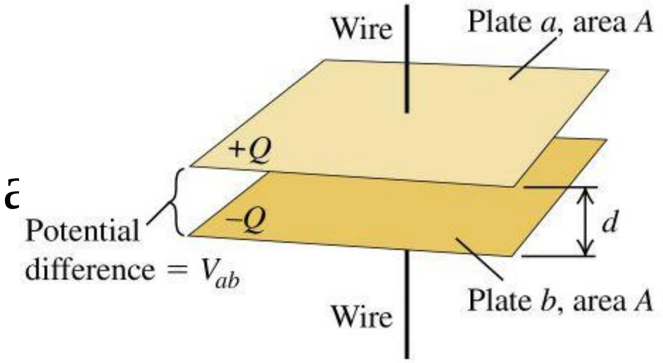
The capacitance is defined to be the ratio of the amount of charge that is on the capacitor to the potential difference between the plates at this point.

$$C = \frac{Q}{V_{ab}}$$

SI Units are

$$1\ farad = \frac{1 Coulomb}{1 Volt}$$

Calculating the Capacitance



- We start with the simplest form – two parallel conducting plates separated by vacuum.
- Let the conducting plates have area A and be separated by a distance d.
- The magnitude of the electric field between the two plates is given **by**

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

- We treat the field as being uniform allowing us to write:

$$V_{ab} = Ed = \frac{Qd}{\varepsilon_0 A}$$

Putting this all together, we have for the capacitance

$$C = \frac{Q}{V_{ab}} = \varepsilon_0 \frac{A}{d}$$

The capacitance is only dependent upon the geometry of the capacitor(A and d).

What's about 1 farad Capacitor?

Given a 1 farad parallel plate capacitor having a plate separation of 1mm. What is the area of the plates?

We start with $C = \varepsilon_0 \frac{A}{d}$

And rearrange to solve for *A*, giving

$$A = \frac{C\,d}{\varepsilon_0} = \frac{(1.0F)(1.0\times10^{-3}m)}{8.85\times10^{-12}F/m}$$

$$= 1.1\times10^{8}m^{2}$$

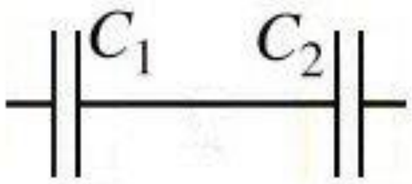
This corresponds to a square about 10km on a side!

The farad is very large unit of capacitance.

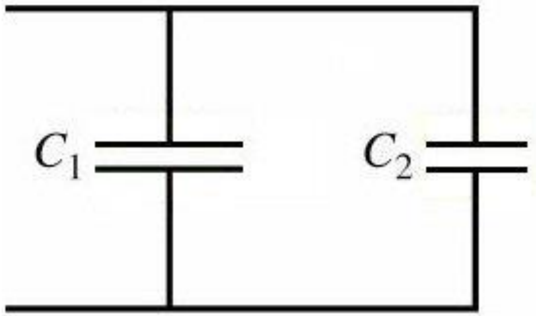
In practice, typical devices have capacitance ranging from Microfarads (μF=10^-6 F) to Picofarads (pF=10^-12 F).,

Series or Parallel Capacitors

Sometimes in order to obtain needed values of capacitance, capacitors are combined in electric circuits.

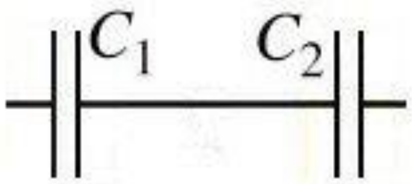
Series 

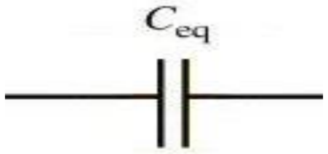
or

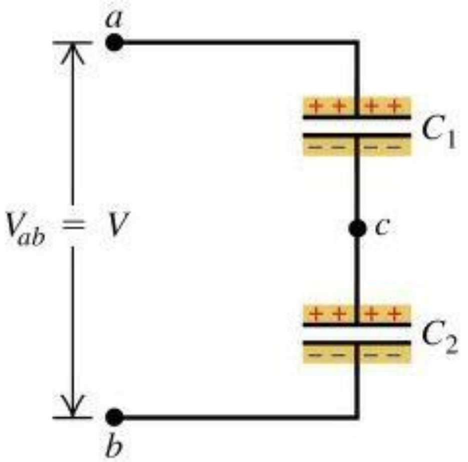
Parallel 

Capacitors in Series

Capacitors are often combined in series and the question then becomes what is the equivalent capacitance?

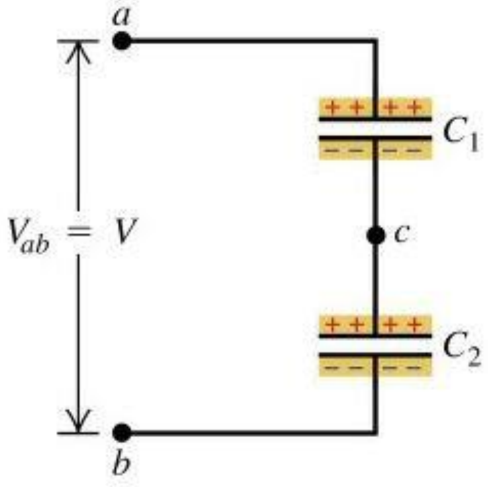
Given 

what is 



We start by putting a voltage, V_{ab}, across the given capacitors

Capacitors in Series



- Capacitors become charged because of V_{ab} .
- If upper plate of C_1 gets a charge of +Q, then the lower plate of C_1 gets a charge of –Q.
- **What happens with C_2?**
- Since there is no source of charge at point c, and we have effectively put a charge of –Q on the lower plate of C_1, the upper plate of C_2 gets a charge of +Q.
- **Charge Conservation**
- This then means that lower plate of C_2 has a charge of –Q.

We also have to have that the potential across C_1 plus the potential across C_2 should equal the potential drop across the two capacitors.

$$V_{ab} = V_{ac} + V_{cb} = V_1 + V_2$$

We have $V_1 = \frac{Q}{C_1}$ **and** $V_2 = \frac{Q}{C_2}$ **Then** $V_{ab} = \frac{Q}{C_1} + \frac{Q}{C_2}$

Dividing through by *Q*, we have

$$\frac{V_{ab}}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$

The equivalent capacitor will also have the same voltage across it

$$\frac{V_{ab}}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$

The left hand side is the inverse of the definition of capacitance:

$$\frac{1}{C} = \frac{V}{Q}$$

So we then have for the equivalent capacitance:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

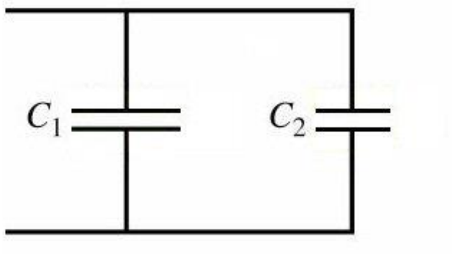
If there are more than two capacitors in series, the resultant capacitance is given by

$$\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i}$$

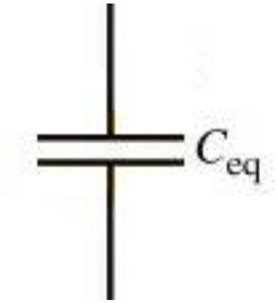
Capacitors in Parallel

Capacitors can also be connected in parallel

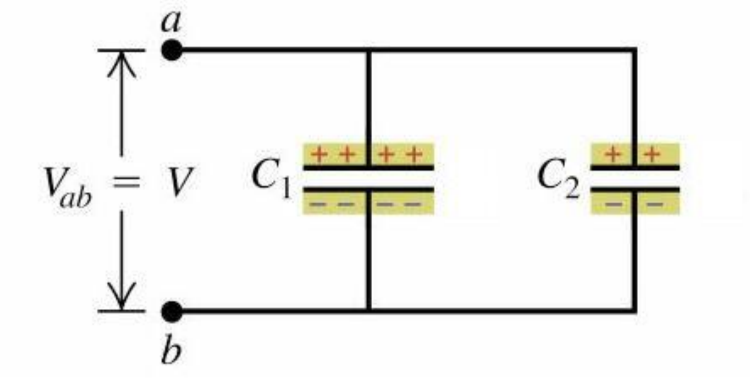
Given



what is



Again we start by putting a voltage across a and b



The upper plates of both capacitors are at the same potential

Likewise for the bottom plates

We have that $V_1 = V_2 = V_{ab}$

Now

$$V_1 = \frac{Q_1}{C_1} \text{ and } V_2 = \frac{Q_2}{C_2}$$

or

$$Q_1 = C_1 V \text{ and } Q_2 = C_2 V$$

- The equivalent capacitor will have the same voltage across it, as do the capacitors in parallel.
- But what about the charge on the equivalent capacitor?
- The equivalent capacitor will have the same total charge $Q = Q_1 + Q_2$

Using this we then have

$$Q = Q_1 + Q_2$$

$$C_{eq}V = C_1V + C_2V$$

or

$$C_{eq} = C_1 + C_2$$

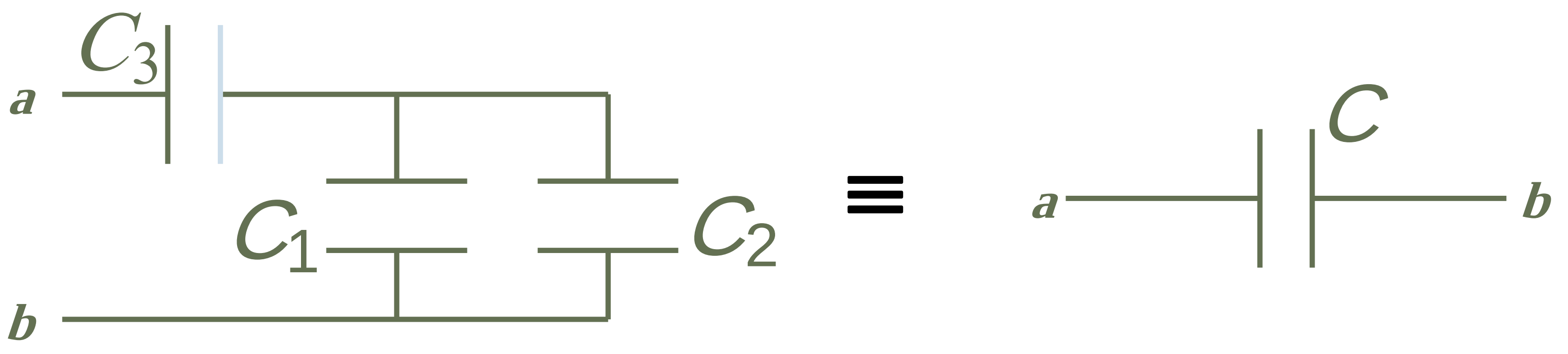
The equivalent capacitance is just the sum of the two capacitors

If we have more than two, the resultant capacitance is just the sum of the individual capacitances

$$C_{eq} = \sum_i C_i$$

Example 1

in the given circuit calculate the equivalent capacitance?



Solution

Recognize that C_1 and C_2 are parallel with each other and combine these to get C_{12}

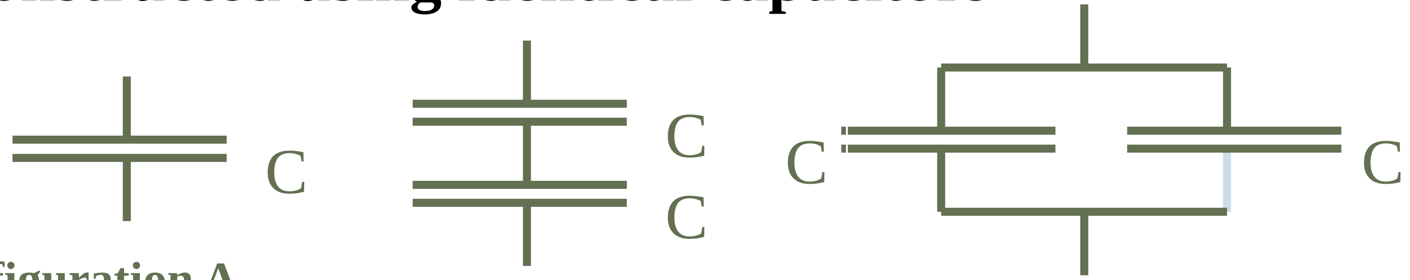
This C_{12} is then in series with with C_3

The resultant capacitance is then given by

$$\frac{1}{C} = \frac{1}{C_3} + \frac{1}{C_1 + C_2} \quad \Rightarrow \quad C = \frac{C_3(C_1 + C_2)}{C_1 + C_2 + C_3}$$

Example 2

Three configurations are constructed using identical capacitors



Configuration A

Configuration B

Configuration C

Which of these configurations has the lowest overall capacitance?

a) Configuration A

b) Configuration B

c) Configuration C

1. The net capacitance for A is just C.
2. In B, the caps are in series and the resultant is given by

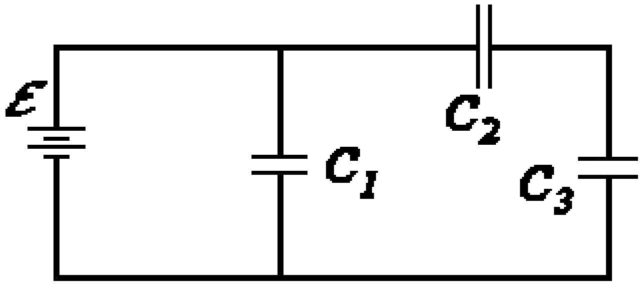
$$\frac{1}{C_{net}} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C} \Rightarrow C_{net} = \frac{C}{2}$$

3. In C, the caps are in parallel and the resultant is given by $C_{net} = C + C = 2C$

Problem 1

A circuit consists of three unequal capacitors C_1, C_2, and C_3 which are connected to a battery of emf E. The capacitors obtain charges Q_1 Q_2, and Q_3, have voltages across their plates V_1, V_2, and V_3. C_{eq} is the equivalent capacitance of the circuit.

Check all of the following that apply:

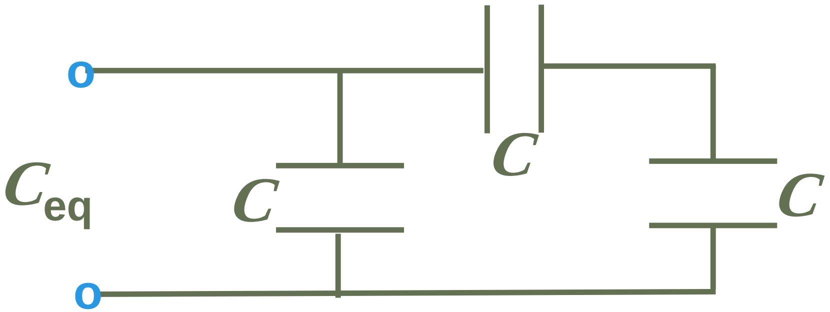


a) $Q_1 = Q_2$ b) $Q_2 = Q_3$ c) $V_2 = V_3$ d) $E = V_1$

e) $V_1 < V_2$ f) $C_{eq} > C_1$

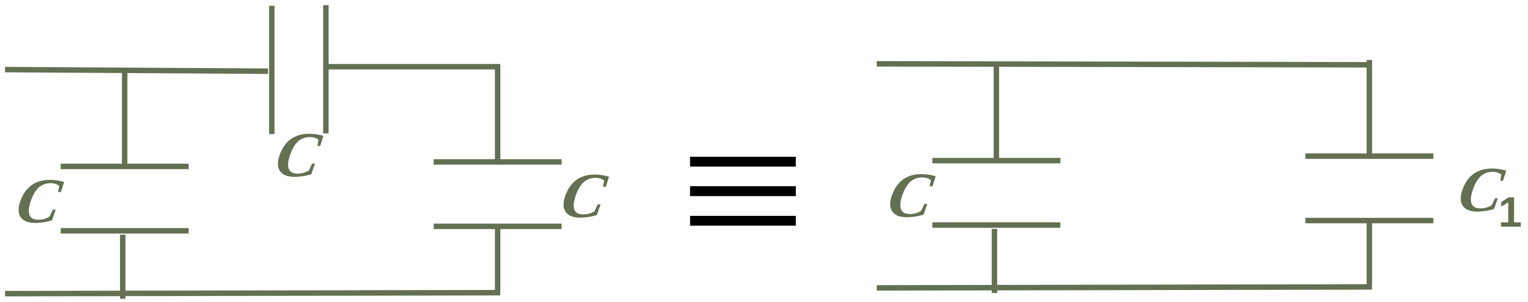
Example 3

What is the equivalent capacitance, C_{eq}, of the combination shown?



(a) $C_{eq} = (3/2)C$ **(b)** $C_{eq} = (2/3)C$ **(c)** $C_{eq} = 3C$

Solution



$$\frac{1}{C_1} = \frac{1}{C} + \frac{1}{C} \Rightarrow C_1 = \frac{C}{2} \Rightarrow C_{eq} = C + \frac{C}{2} = \frac{3}{2}C$$

Energy Stored in a Capacitor

- Electrical Potential energy is stored in a capacitor.
- The energy comes from the work that is done in charging the capacitor.
- **Let q and v be the intermediate charge and potential on the capacitor**
- The incremental done in bringing an incremental charge, dq, to the capacitor is then given by

$$dW = v\,dq = \frac{q\,dq}{C}$$

The total work done is just the integral of this equation from 0 to Q

$$W = \frac{1}{C}\int_0^Q q\,dq = \frac{Q^2}{2C}$$

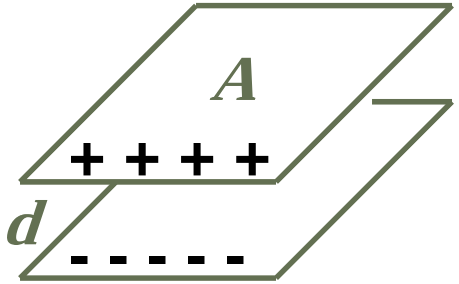
Using the relationship between capacitance, voltage and charge we also obtain

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

where U is the stored potential energy

Example 4

Suppose the capacitor as shown is charged to Q and then the battery is *disconnected*



Now suppose you pull the plates(from d) further apart so that the final separation is d_1

Which of the quantities *Q, C, V, U, E* change?

Q: **Charge on the capacitor does not change**

C: **Capacitance Decreases**

V: **Voltage Increases**

U: **Potential Energy Increases**

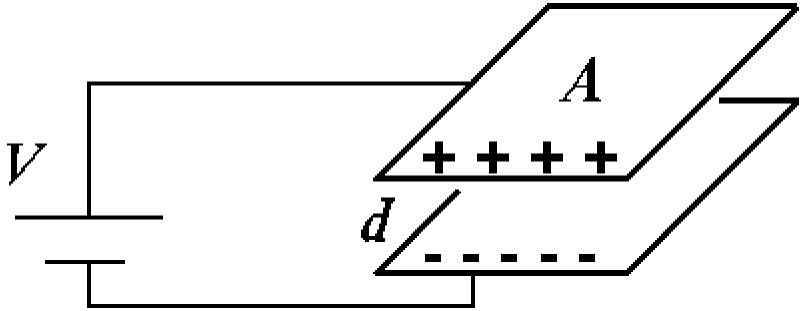
E: **Electric Field does not change**

How do these quantities change?

Answers: $C_1 = \frac{d}{d_1} C$ $V_1 = \frac{d_1}{d} V$ $U_1 = \frac{d_1}{d} U$

Example 5

Suppose the battery (V) is kept attached to the capacitor



Again pull the plates apart from d to d_1

Now which quantities, if any, change?

Q: Charge Decreases

C: Capacitance Decreases

V: Voltage on capacitor does not change

U: Potential Energy Decreases

E: Electric Field Decreases

How much do these quantities change?

Answers: $Q_1 = \frac{d}{d_1} Q$ $C_1 = \frac{d}{d_1} C$ $U_1 = \frac{d}{d_1} U$ $E_1 = \frac{d}{d_1} E$

Electric Field Energy Density

The potential energy that is stored in the capacitor can be thought of as being stored in the electric field that is in the region between the two plates of the capacitor.

The quantity that is of interest is in fact the *energy density*

Using $C = \varepsilon_0 \frac{A}{d}$ **and** $V = E\,d$ **we then have**

$$\text{Energy Density} = u = \frac{\frac{1}{2}CV^2}{Ad}$$

where *A* and d are the area of the capacitor plates and their separation, respectively

$$u = \frac{1}{2}\varepsilon_0 E^2$$

Even though we used the relationship for a parallel capacitor, this result holds for *all* capacitors regardless of configuration

This represents the energy density of the electric field in general

Dielectrics

Most capacitors have a nonconducting material between their plates, this nonconducting material is called *a dielectric*, accomplishes three things:

1. Solves mechanical problem of keeping the plates separated.
2. Increases the maximum potential difference allowed between the plates.
3. Increases the capacitance of a given capacitor over what it would be without the dielectric.

Suppose we have a capacitor of value C_0 that is charged to a potential difference of V_0 and then removed from the charging source

We would then find that it has a charge of $Q = C_0 V_0$

We now insert the dielectric material into the capacitor

We find that the potential difference *decreases* by a factor K

$$V = \frac{V_0}{K}$$

And equivalently the capacitance has *increased* by a factor of K

$$C = K C_0$$

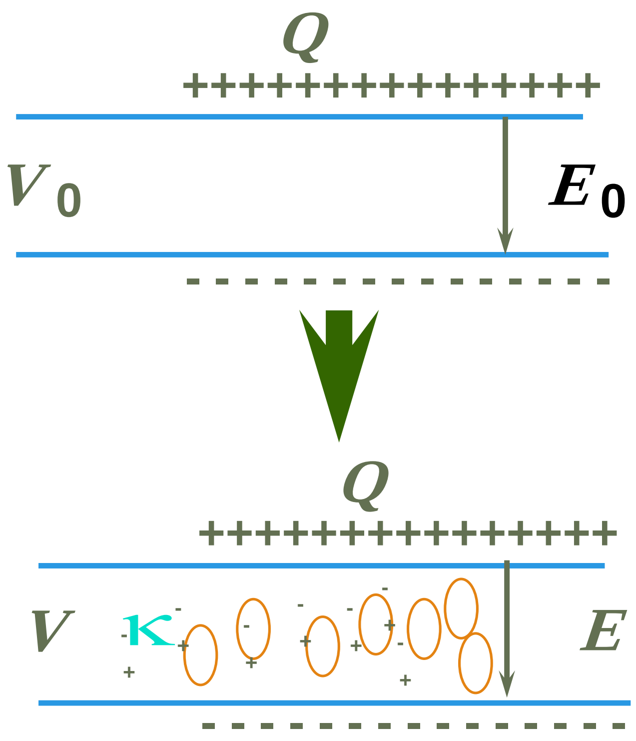
This constant K is known as the dielectric constant and is dependent upon the material used and is a number greater than 1.

Polarization

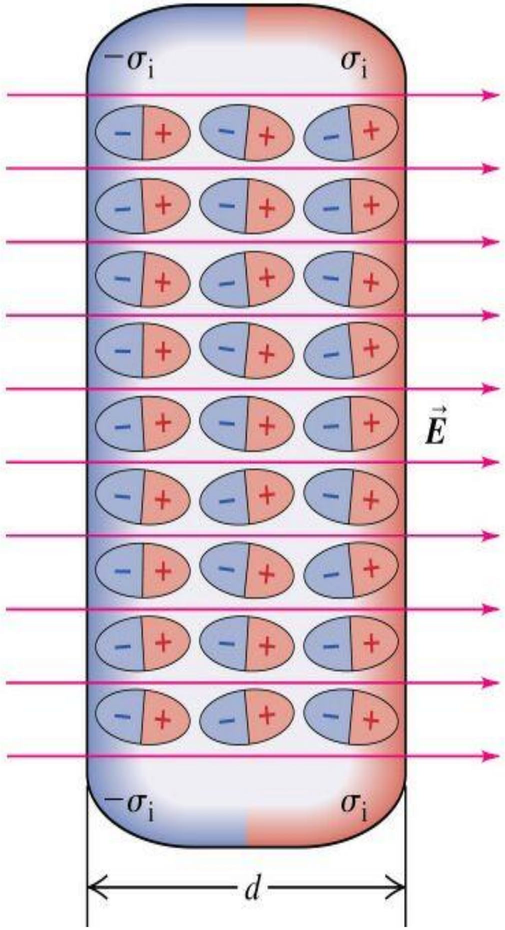
Without the dielectric in the capacitor, we have

The electric field points undiminished from the positive to the negative plate

With the dielectric in place we have



The electric field between the plates of the capacitor is reduced because some of the material within the dielectric rearranges so that their negative charges are oriented towards the positive plate.



These rearranged charges set up an internal electric field that opposes the electric field due to the charges on the plates

The net electric field is given by

$$\boldsymbol{E} = \frac{\boldsymbol{E}_0}{\boldsymbol{K}}$$

We now redefine several quantities using the dielectric constant.

We define the permittivity of the dielectric as being

$$\varepsilon = K\,\varepsilon_0$$

Capacitance:

$$C = KC_0 = K\varepsilon_0 \frac{A}{d} = \varepsilon \frac{A}{d}$$

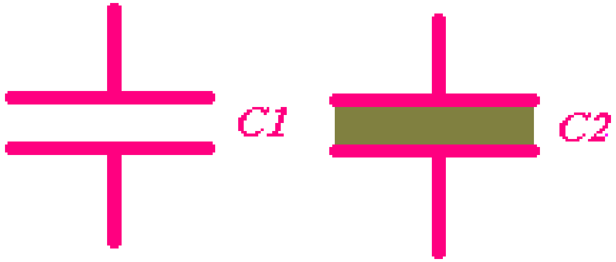
with the last two relationships holding for a parallel plate capacitor

Energy Density

$$u = \frac{1}{2} K\varepsilon_0 E^2 = \frac{1}{2} \varepsilon E^2$$

Example 6

Two identical parallel plate capacitors are given the same charge *Q*, after which they are disconnected from the battery. After C_2 has been charged and disconnected it is filled with a dielectric.



Compare the voltages of the two capacitors.

a) $V_1 > V_2$ **b)** $V_1 = V_2$ **c)** $V_1 < V_2$

We have that $Q_1 = Q_2$ and that $C_2 = KC_1$

We also have that C = Q/V or V= Q/C

Then $V_1 = \frac{Q_1}{C_1}$ **and** $V_2 = \frac{Q_2}{C_2} = \frac{Q_1}{KC_1} = \frac{1}{K}V_1$