Electromagnetic Theory

CREDIT HOURS FIRST LEVEL(PHYSICS /PHYSICS AND COMPUTER SCIENCE PROGRAM)

104 PH

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PHYSICS FOR SCIENTISTS AND ENGINEERS

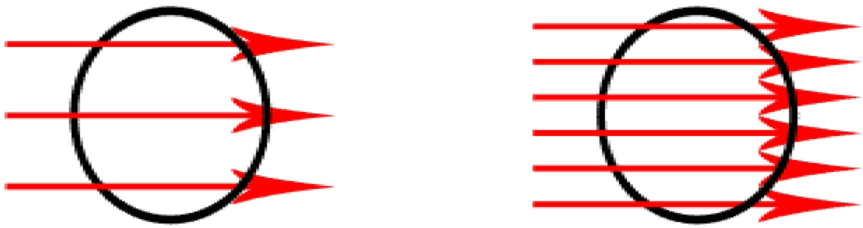
BY RAYMOND A. SERWAY , JOHN W. JEWETT .

Electric flux is the rate of flow of the electric field through a given area (see).

Electric flux is proportional to the number of electric field lines going through a virtual surface.

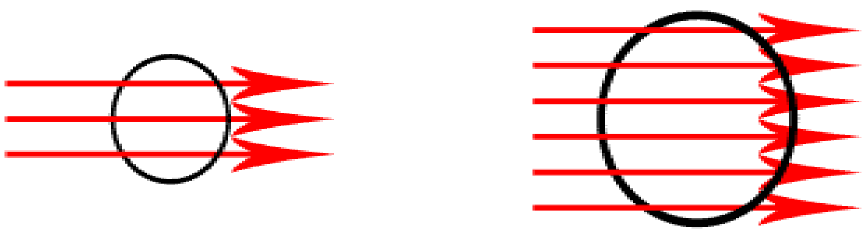
If the **electric field is uniform**, the electric flux passing through a surface of vector area S is Φ(E)=E·S=E S cos(θ) => Φ(E)=E·S=E S cos(θ) where E is the magnitude of the electric field (having units of V/m), S is the area of the surface, and θ is the angle between the electric field lines and the normal (perpendicular) to S.

For a **non-uniform electric field,** the electric flux dΦ(E) through a small surface area dS is given by dΦ(E)=E·dS => dΦ(E)=E·dS (the electric field, E, multiplied by the component of area perpendicular to the field).



Flux is proportional to the density of flow.

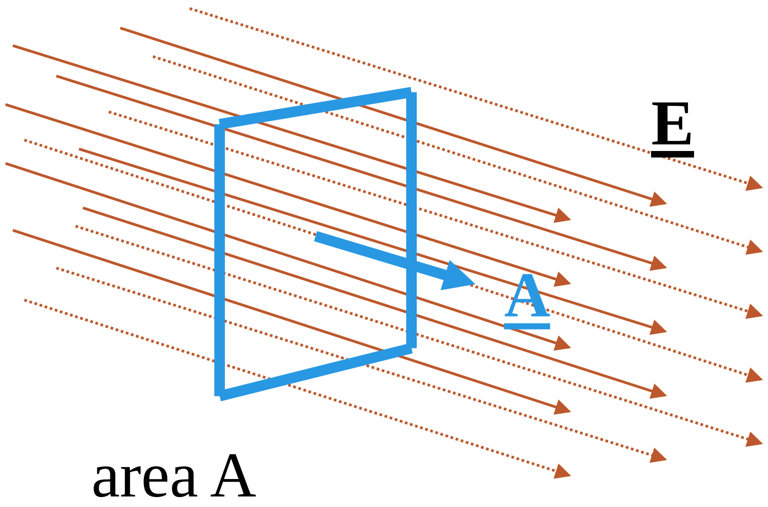
Flux varies by how the boundary faces the direction of flow.



Flux is proportional to the area within the boundary.

Electric Flux

We define the electric flux Φ, of the electric field $\underline{E}$, through the surface A, as:



$$\Phi = \underline{E} \cdot \underline{A}$$

$$\Phi = E\,A\cos(\theta)$$

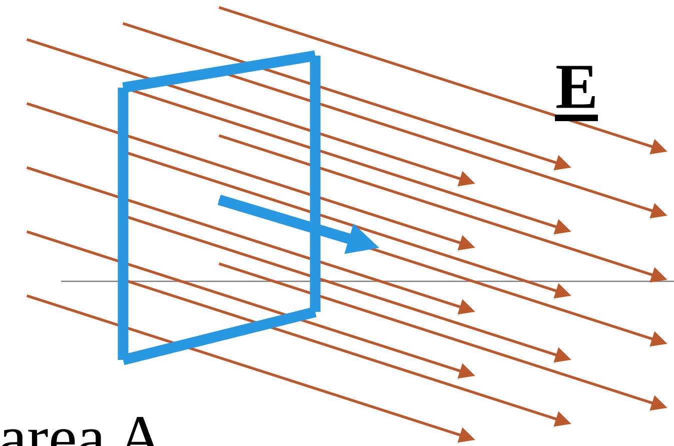
Where:

$\underline{A}$ is a vector normal to the surface (magnitude A, and direction normal to the surface).

θ is the angle between $\underline{E}$ and $\underline{A}$

You can think of the flux through some surface as a measure of the number of field lines which pass through that surface.

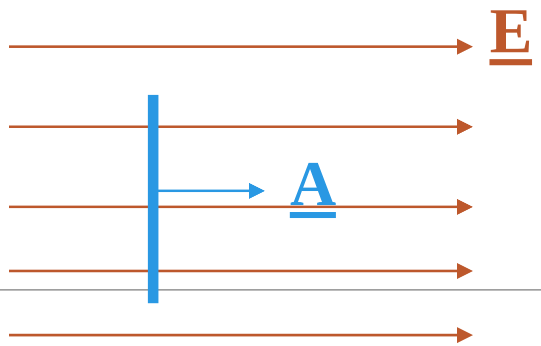
Flux depends on the strength of $\underline{\mathbf{E}}$, on the surface area, and on the relative orientation of the field and surface.



area A

Here the flux is

$$\Phi = \underline{\mathbf{E}} \cdot \underline{\mathbf{A}}$$



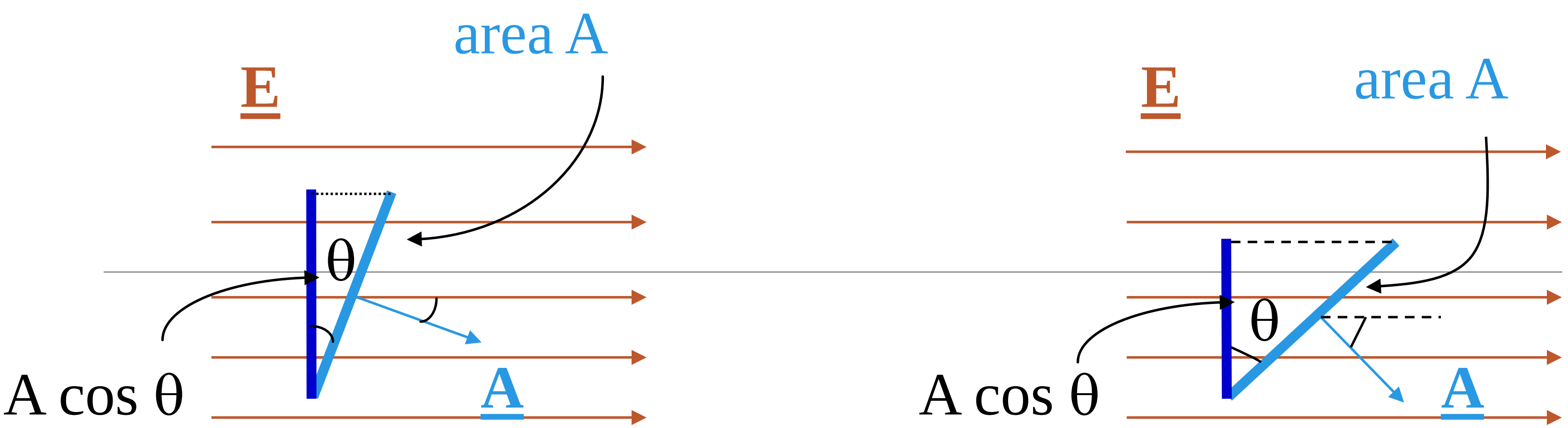
Normal to surface,
magnitude A

$\underline{\mathbf{A}} \equiv \longrightarrow$

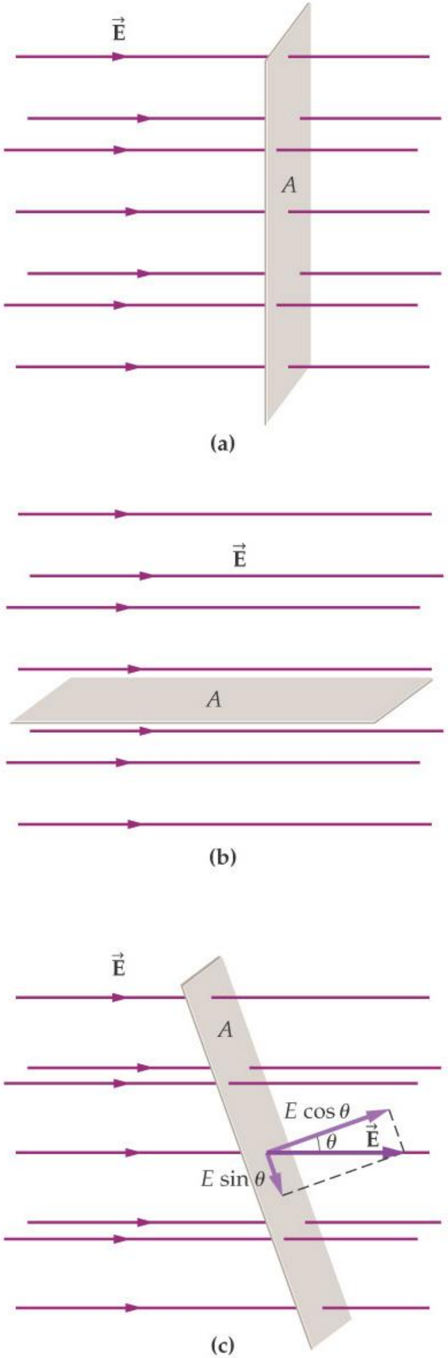
Electric Flux

The flux also depends on orientation

$$\Phi = \underline{\mathbf{E}} \cdot \underline{\mathbf{A}} = E\,A\cos\theta$$



The number of field lines through the tilted surface equals the number through its projection. Hence, the flux through the tilted surface is simply given by the flux through its projection: E (A cosθ).



Exercise: Calculate the flux of the electric fiel through the surface A, in each of the three cases shown:

a) $\Phi =$

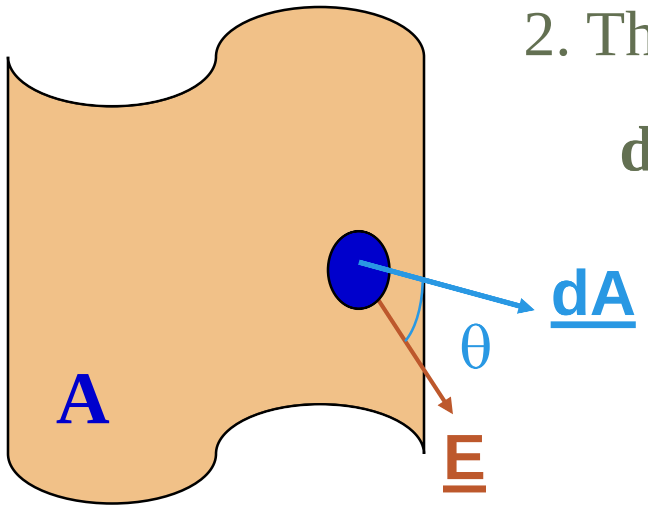
b) $\Phi =$

c) $\Phi =$

What if the surface is curved, or the field varies with position ??

$\Phi = \underline{\mathbf{E}} \cdot \underline{\mathbf{A}}$

1. We divide the surface into small regions with area dA



2. The flux through dA is $d\Phi = E\, dA \cos\theta$

$d\Phi = \underline{\mathbf{E}} \cdot d\underline{\mathbf{A}}$

3. To obtain the total flux we need to integrate over the surface A

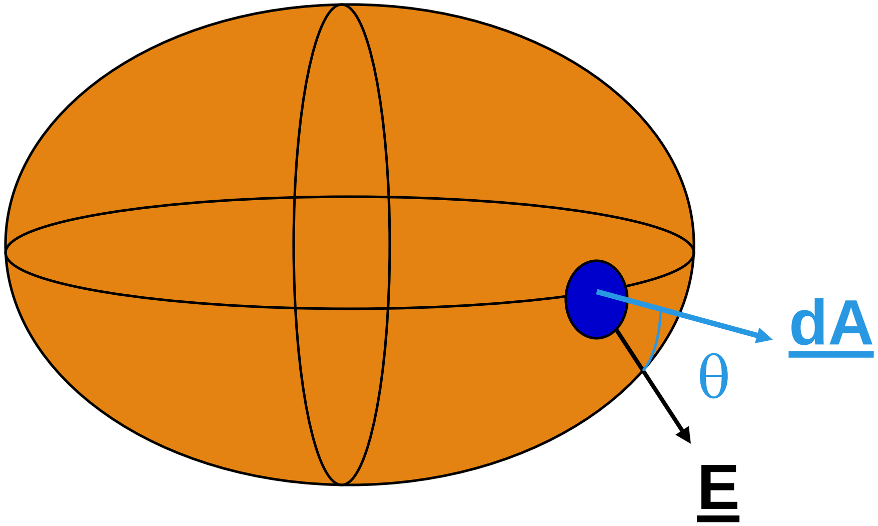
$$\boxed{\Phi = \int d\Phi = \int \underline{\mathbf{E}} \cdot d\underline{\mathbf{A}}}$$

Electric flux has SI units of volt metres (V m), or, equivalently, newton metres squared per coulomb ($N\, m^2\, C^{-1}$). Thus, the SI base units of electric flux are $kg \cdot m^3 \cdot s^{-3} \cdot A^{-1}$.

In the case of a closed surface

$$\Phi = \oint d\Phi = \oint \underline{E} \bullet \underline{dA}$$

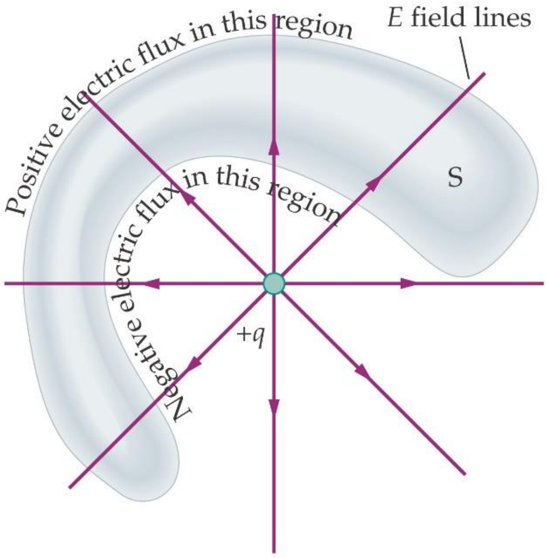
The loop means the integral is over a closed surface.



For a closed surface:

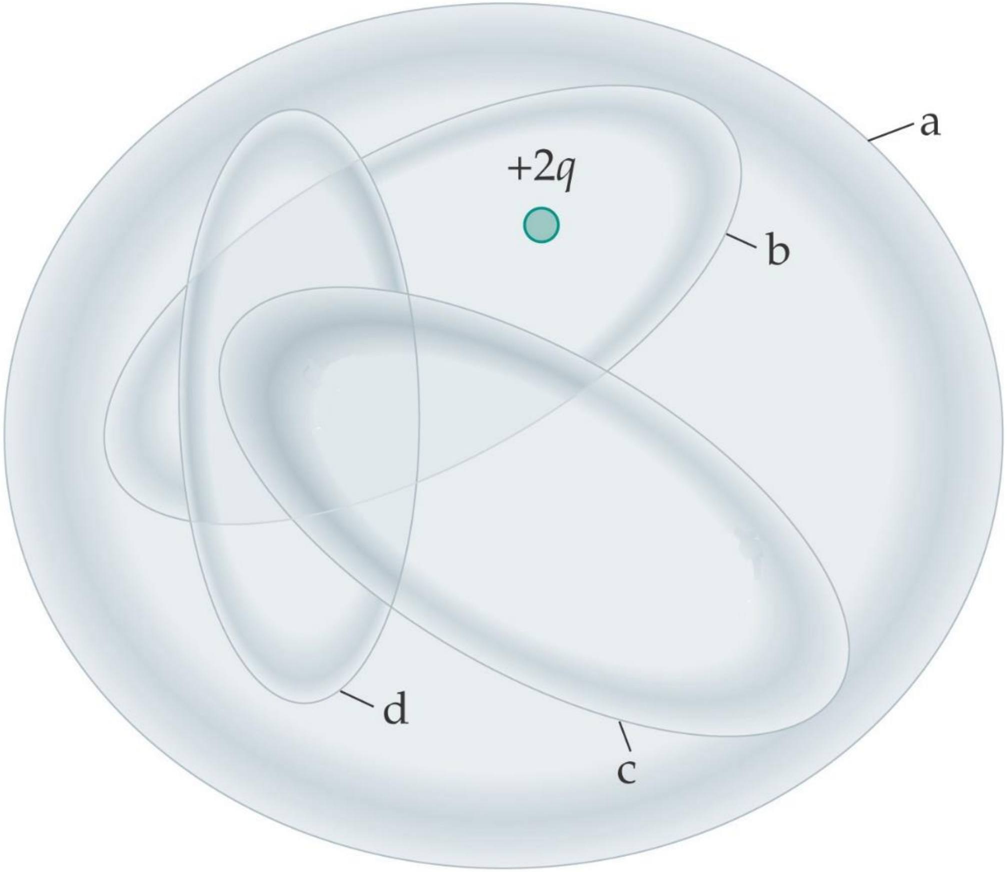
The flux is **positive** for field lines that **leave** the enclosed volume.

The flux is **negative** for field lines that **enter** the enclosed volume.



If a charge is outside a closed surface, **the net flux is zero.**
As many lines leave the surface, as lines enter it.

For which of these closed surfaces (a, b, c, d) the flux of the electric field, produced by the charge +2q, is zero?



Spherical surface with point charge at center

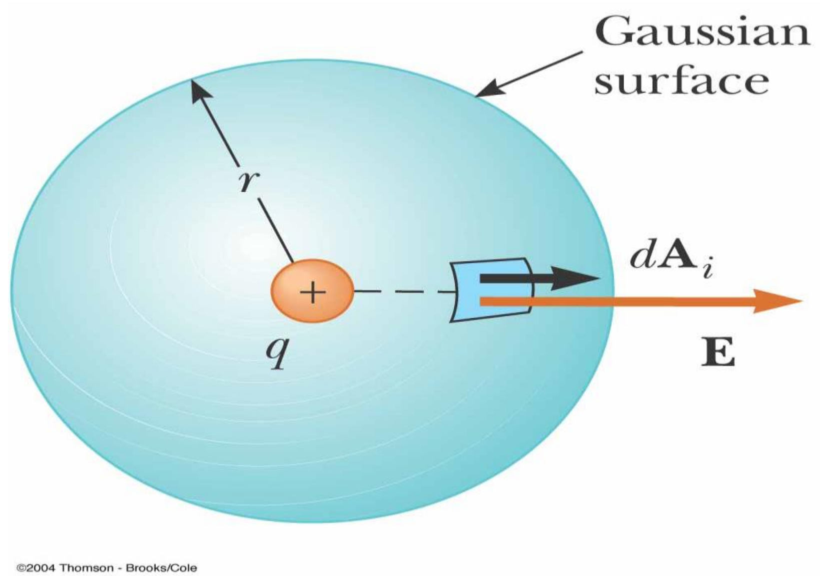
Flux of electric field:

$$\Phi = \oint d\Phi = \oint \vec{E} \cdot \vec{dA}$$

$$\Phi = \oint E\, dA \cos\theta = \oint \frac{1}{4\pi\varepsilon_0} \frac{q\, dA}{r^2}$$

but $\dfrac{dA}{r^2} = d\Omega,$ then:

$$\Phi = \frac{q}{4\pi\varepsilon_0} \oint d\Omega = \frac{q}{4\pi\varepsilon_0} 4\pi = \frac{q}{\varepsilon_0}$$



Gauss's Law

$$\oint \vec{E} \cdot \vec{dA} = \frac{q_{enclosed}}{\varepsilon_0}$$

In geometry, a **solid angle** (symbol: Ω) is a measure of the amount of the field of view from some particular point that a given object covers. That is, it is a measure of how large the object appears to an observer looking from that point.

The point from which the object is viewed is called the *apex* of the solid angle, and the object is said to *subtend* its solid angle from that point.

In the International System of Units (SI), a solid angle is expressed in a dimensionless unit called a *steradian* (symbol: sr).

One steradian corresponds to one unit of area on the unit sphere surrounding the apex, so an object that blocks all rays from the apex would cover a number of steradians equal to the total surface area of the unit sphere.

Solid angles can also be measured in squares of angular measures such as degrees, minutes, and seconds.

Gauss's Law

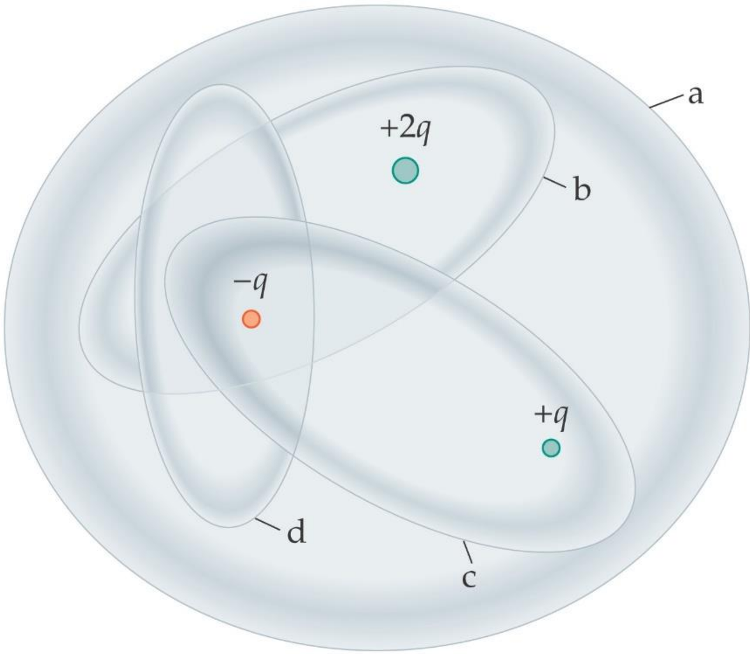
The electric flux through any closed surface equals ☑ enclosed charge / ε_0.

Gauss's law is a law relating the distribution of electric charge to the resulting electric field.

$$\Phi = \oint d\Phi = \oint \underline{E} \bullet \underline{dA} = \frac{\sum_{inside} q}{\varepsilon_0}$$

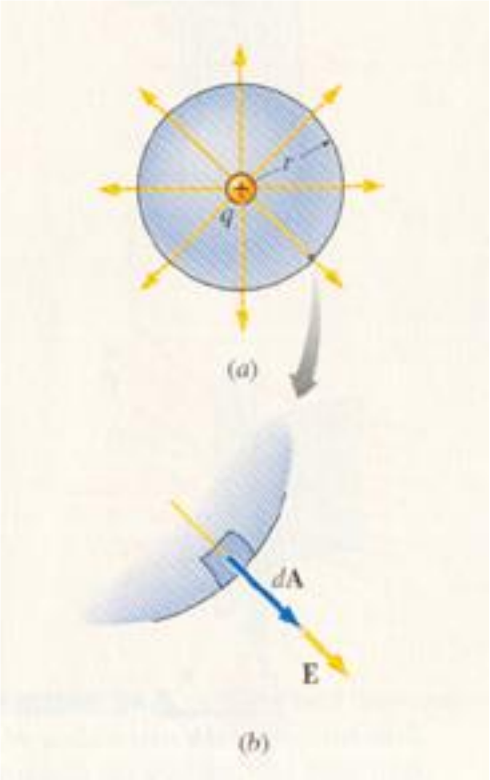
This is always true. Occasionally, it provides a very easy way to find the electric field (for highly symmetric cases).

Calculate the flux of the electric field Φ for each of the closed surfaces a, b, c, and d



Surface a, Φ_a =
Surface b, Φ_b =
Surface c, Φ_c =
Surface d, Φ_d =

Calculate the electric field produced by a point charge using Gauss Law



We choose for the gaussian surface a sphere of radius **r**, centered on the charge **Q**.

Then, the electric field **E**, has the same magnitude everywhere on the surface (radial symmetry)

Furthermore, at each point on the surface, the field **E** and the surface normal **dA** are parallel (both point radially outward).

$$\underline{\mathbf{E}} \cdot \underline{\mathbf{dA}} = \mathbf{E\ dA} \quad [\cos\theta = 1]$$

Is Gauss's Law more fundamental than Coulomb's Law?

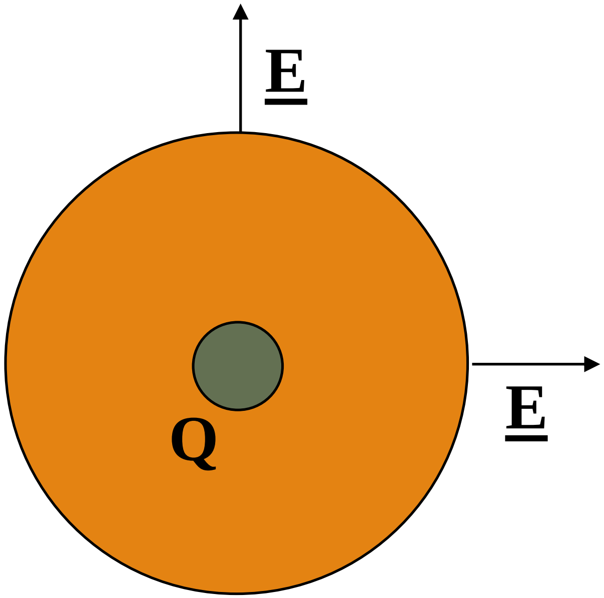
No! Here we derived Coulomb's law for a point charge from Gauss's law.

One can instead derive Gauss's law for a general (even very nasty) charge distribution from Coulomb's law. The two laws are equivalent.

Gauss's law gives us an easy way to solve a few very symmetric problems in electrostatics.

It also gives us great insight into the electric fields in and on conductors and within voids inside metals.

Electric field produced by a point charge



$$\oint \underline{E} \cdot \underline{dA} = Q / \varepsilon_0$$

$$\oint \underline{E} \cdot \underline{dA} = E \oint dA = E\,A$$

$$A = 4\,\pi\,r^2$$

$$E\,A = E\;4\,\pi\,r^2 = Q / \varepsilon_0$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

$k = 1 / 4\,\pi\,\varepsilon_0$

ε_0 = permittivity

$\varepsilon_0 = 8.85\text{x}10^{-12}\ C^2/Nm^2$

→ **Coulomb's Law !**

Gauss's Law

The total flux within a closed surface …

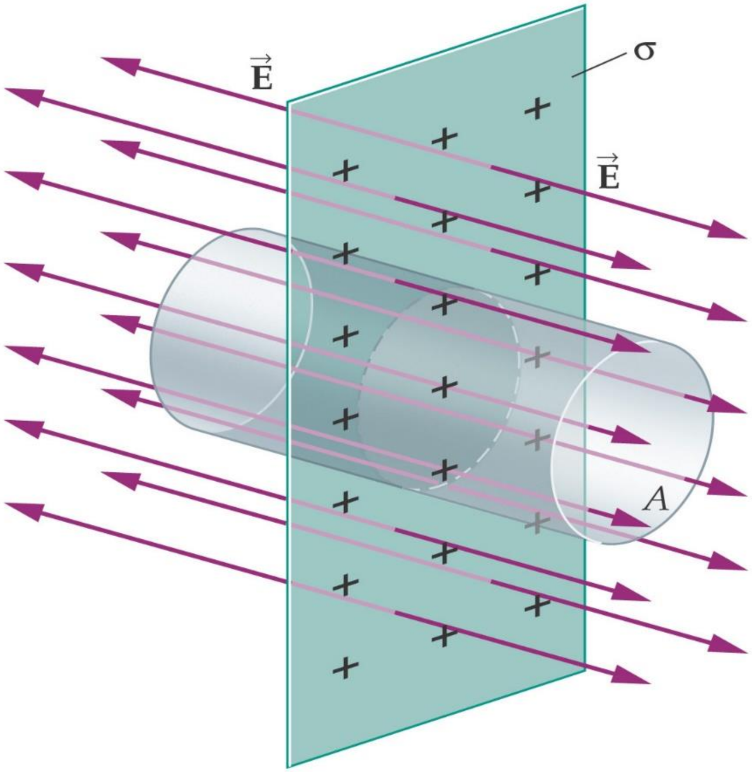
… is proportional to the enclosed charge.

$$\Phi = \oint \vec{\mathbf{E}} \bullet d\vec{\mathbf{A}} = \frac{Q_{enclosed}}{\varepsilon_0}$$

Gauss's Law is always true, but is only useful for certain very simple problems with great symmetry.

Applying Gauss's Law

Gauss's law is useful only when the electric field is constant on a given surface



Infinite sheet of charge

1. Select Gauss surface

In this case a cylindrical pillbox

2. Calculate the flux of the electric field through the Gauss surface

$\Phi = 2\,E\,A$

3. Equate $\Phi = q_{encl}/\varepsilon_0$

$2EA = q_{encl}/\varepsilon_0$

4. Solve for E

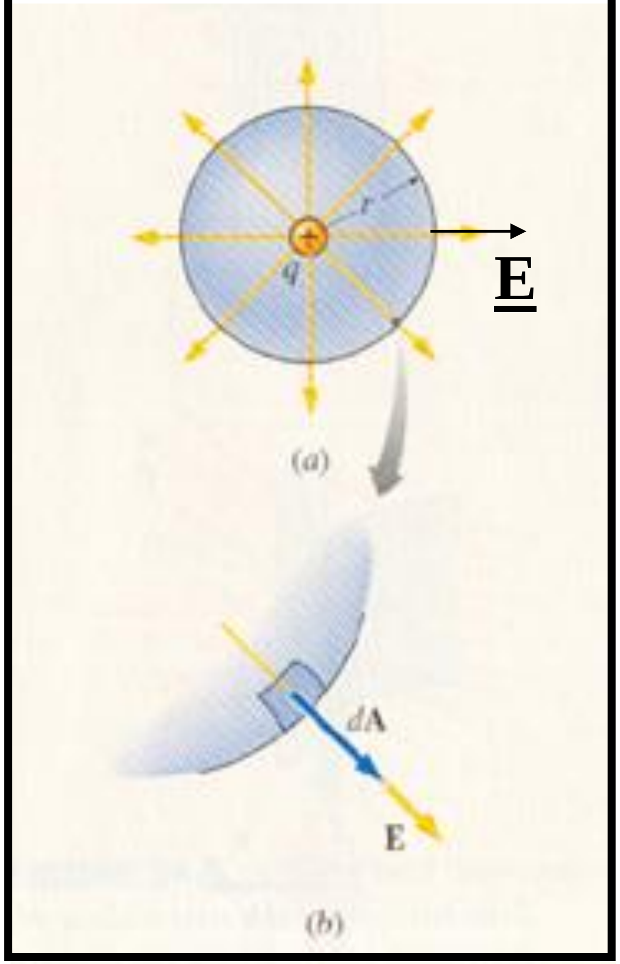
$E = q_{encl} / 2\,A\,\varepsilon_0 = \sigma / 2\,\varepsilon_0$

(with $\sigma = q_{encl} / A$)

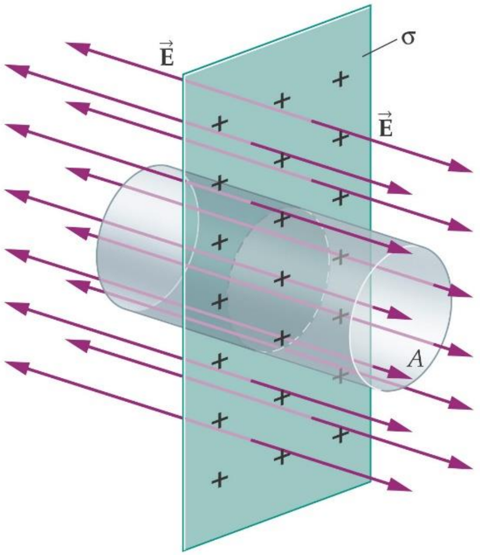
GAUSS LAW – SPECIAL SYMMETRIES

	SPHERICAL (point or sphere)	CYLINDRICAL (line or cylinder)	PLANAR (plane or sheet)
CHARGE DENSITY	Depends only on radial distance from central point	Depends only on perpendicular distance from line	Depends only on perpendicular distance from plane
GAUSSIAN SURFACE	Sphere centered at point of symmetry	Cylinder centered at axis of symmetry	Pillbox or cylinder with axis perpendicular to plane
ELECTRIC FIELD E	E constant at surface $E \parallel A$ - $\cos\theta = 1$	E constant at curved surface and $E \parallel A$ $E \perp A$ at end surface $\cos\theta = 0$	E constant at end surfaces and $E \parallel A$ $E \perp A$ at curved surface $\cos\theta = 0$
FLUX Φ			

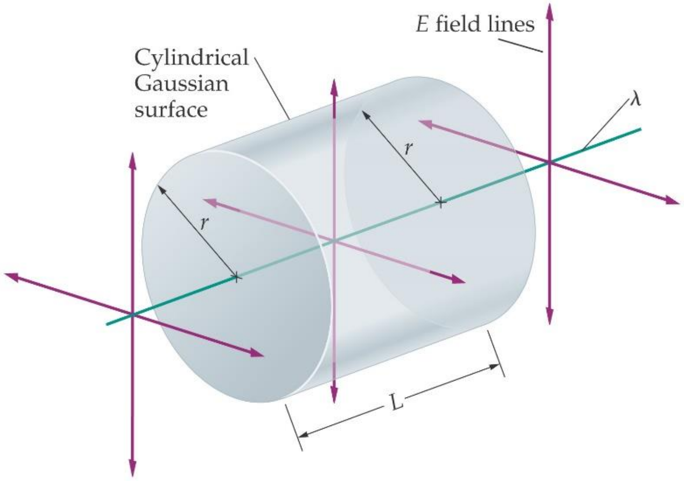
Spherical geometry



Planar geometry

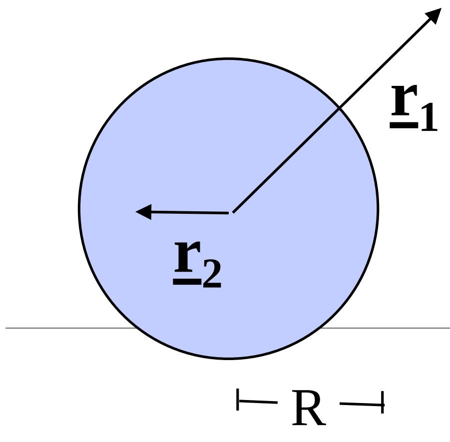


Cylindrical geometry



Problem: Sphere of Charge Q

A charge Q is uniformly distributed through a sphere of radius R. What is the electric field as a function of $\underline{\mathbf{r}}$?. Find $\underline{\mathbf{E}}$ at $\underline{\mathbf{r}}_1$ and $\underline{\mathbf{r}}_2$.



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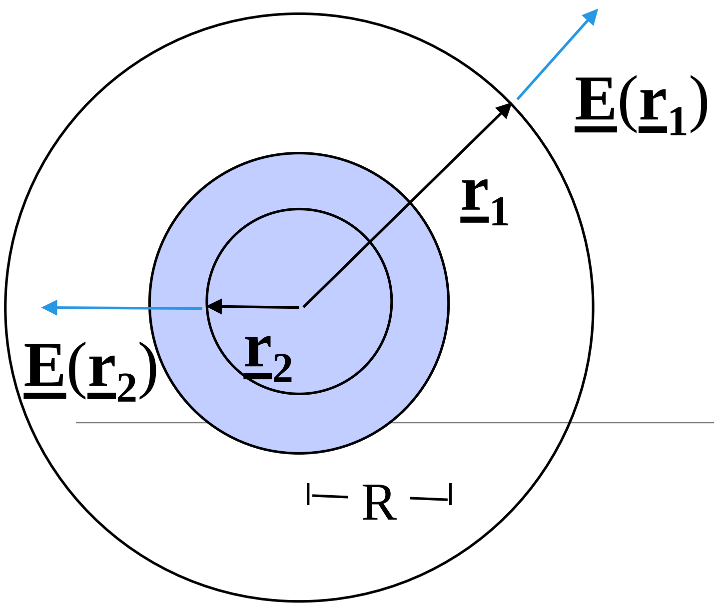
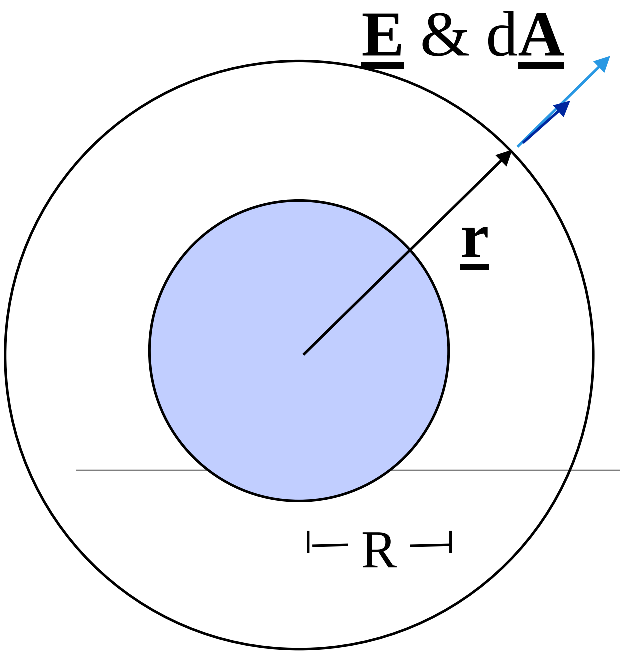


Use symmetry!

This is spherically symmetric. That means that $\underline{\mathbf{E}}(\underline{\mathbf{r}})$ is radially outward, and that all points, at a given radius ($|\underline{\mathbf{r}}|=r$), have the same magnitude of field.

Problem: Sphere of Charge Q

First find **E**(**r**) at a point **outside** the charged sphere. Apply Gauss's law, using as the Gaussian surface the sphere of radius r pictured.



What is the enclosed charge?

Problem: Sphere of Charge Q

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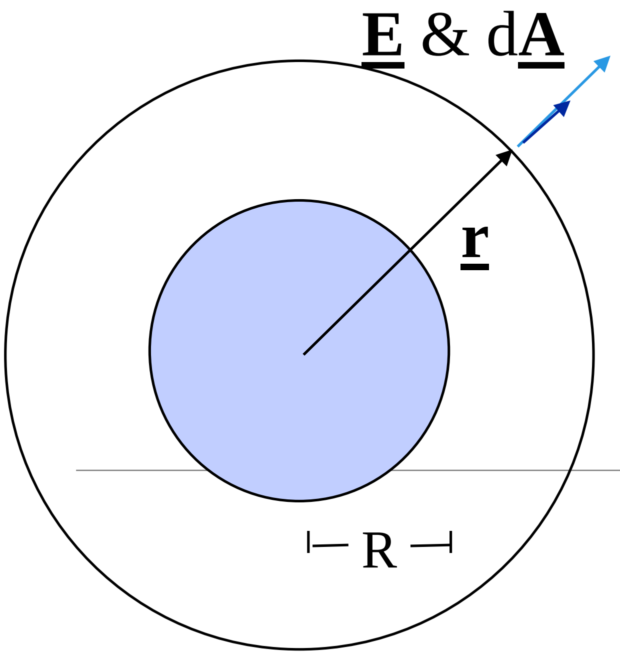


What is the enclosed charge? **Q**

What is the flux through this surface?

$$\Phi = \oint \vec{\mathbf{E}} \bullet d\vec{\mathbf{A}} = \oint E\, dA$$

$$= E \oint dA = EA = E(4\pi r^2)$$

Gauss: $\Phi = Q/\varepsilon_o$

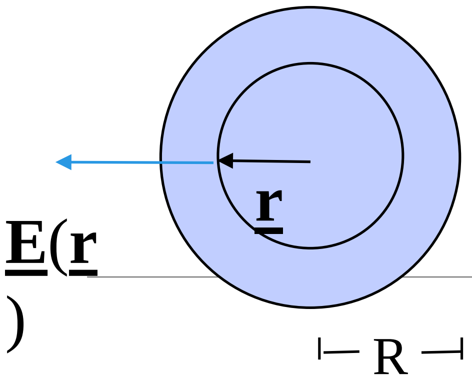
$$Q/\varepsilon_0 = \Phi = E(4\pi r^2)$$

So $\boxed{\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \dfrac{1}{4\pi\varepsilon_o} \dfrac{Q}{r^2}\, \hat{\mathbf{r}}}$

Exactly as though all the charge were at the origin! (for r>R)

Problem: Sphere of Charge Q

Next find **E**(**r**) at a point **inside** the sphere. Apply Gauss's law, using a little sphere of radius r as a Gaussian surface.



What is the enclosed charge?
That takes a little effort. The little sphere has some fraction of the total charge. What fraction?

That's given by volume ratio: $Q_{enc} = \dfrac{r^3}{R^3} Q$

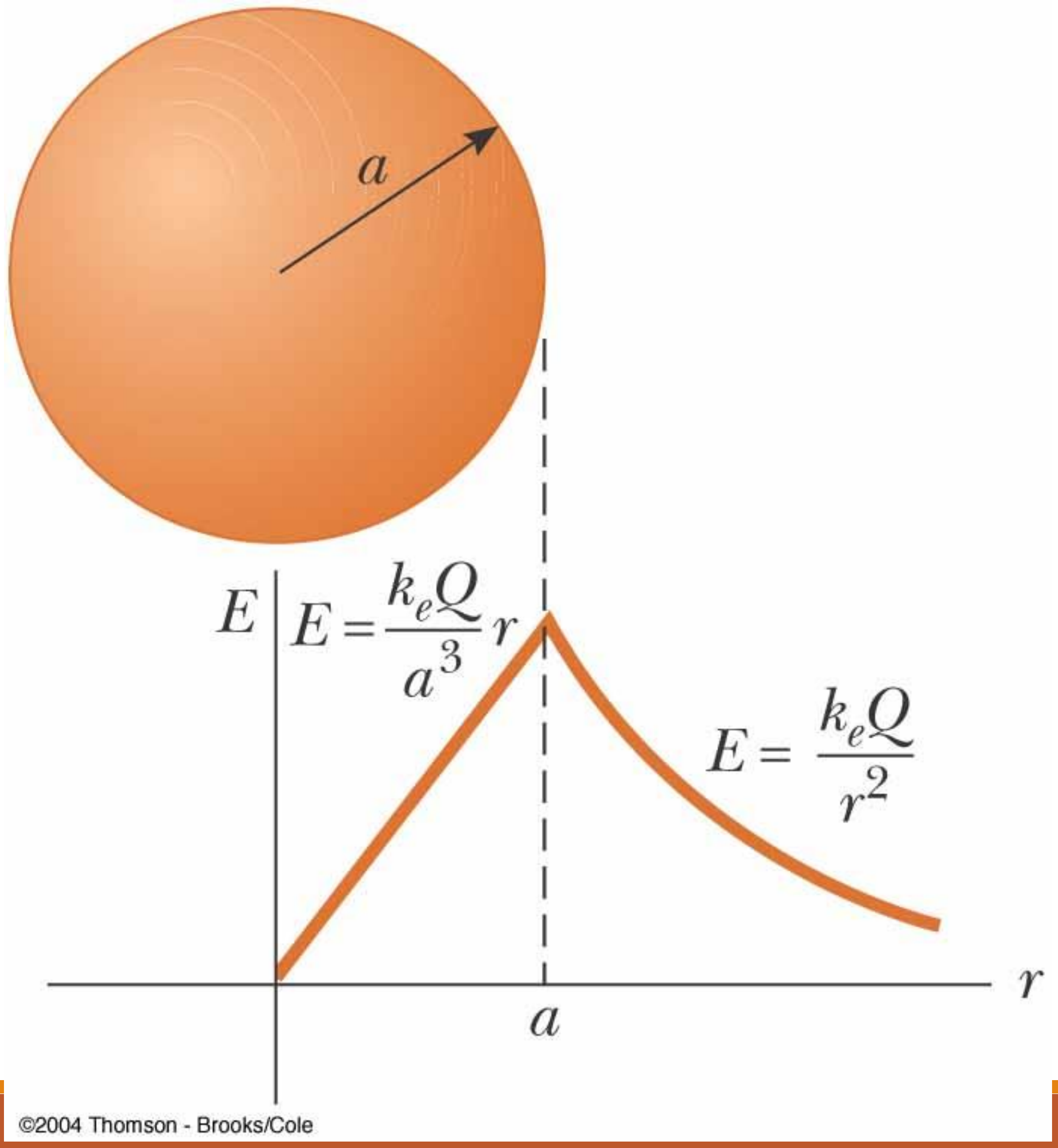
Again the flux is: $\Phi = EA = E(4\pi r^2)$

Setting $\Phi = Q_{enc}/\varepsilon_o$ gives $E = \dfrac{(r^3/R^3)Q}{4\pi\varepsilon_o r^2}$

For r<R

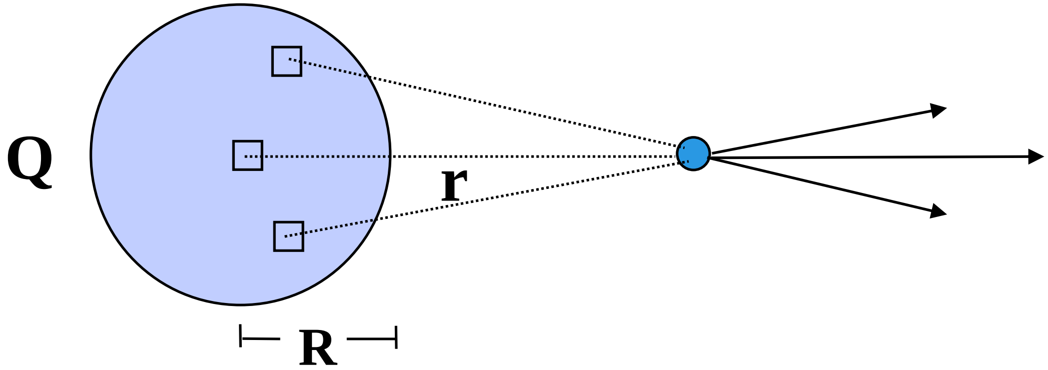
$$\boxed{\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{Q}{4\pi\varepsilon_o R^3}\, r\,\hat{\mathbf{r}}}$$

Problem: Sphere of Charge Q



Problem: Sphere of Charge Q

Look closer at these results. The electric field at comes from a sum over the contributions of all the little bits .



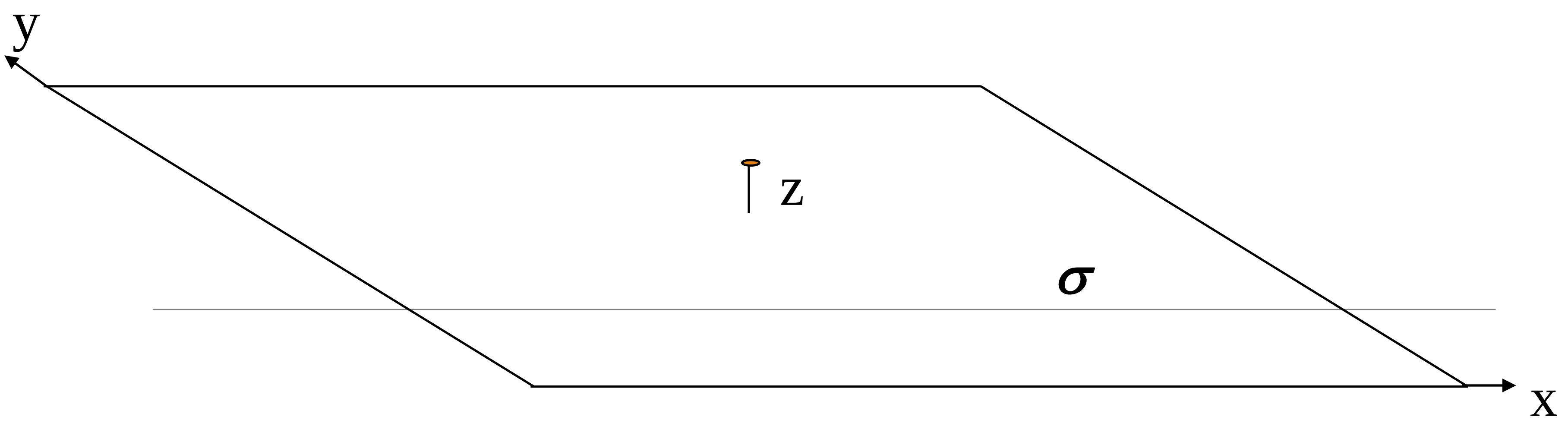
r > R

It's obvious that the net **E** at this point will be horizontal. But the magnitude from each bit is different; and it's completely not obvious that the magnitude E just depends on the distance from the sphere's center to the observation point.

Doing this as a volume integral would be HARD.
Gauss's law is EASY.

Problem: Infinite charged plane

Consider an infinite plane with a constant surface charge density σ (which is some number of Coulombs per square meter).
What is **<u>E</u>** at a point located a distance z above the plane?



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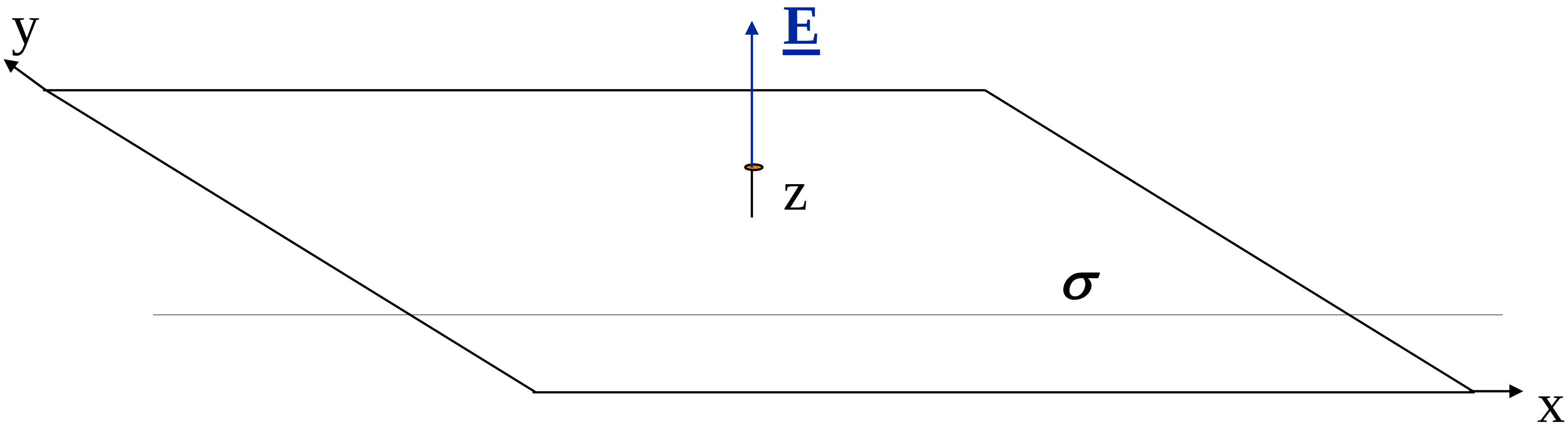
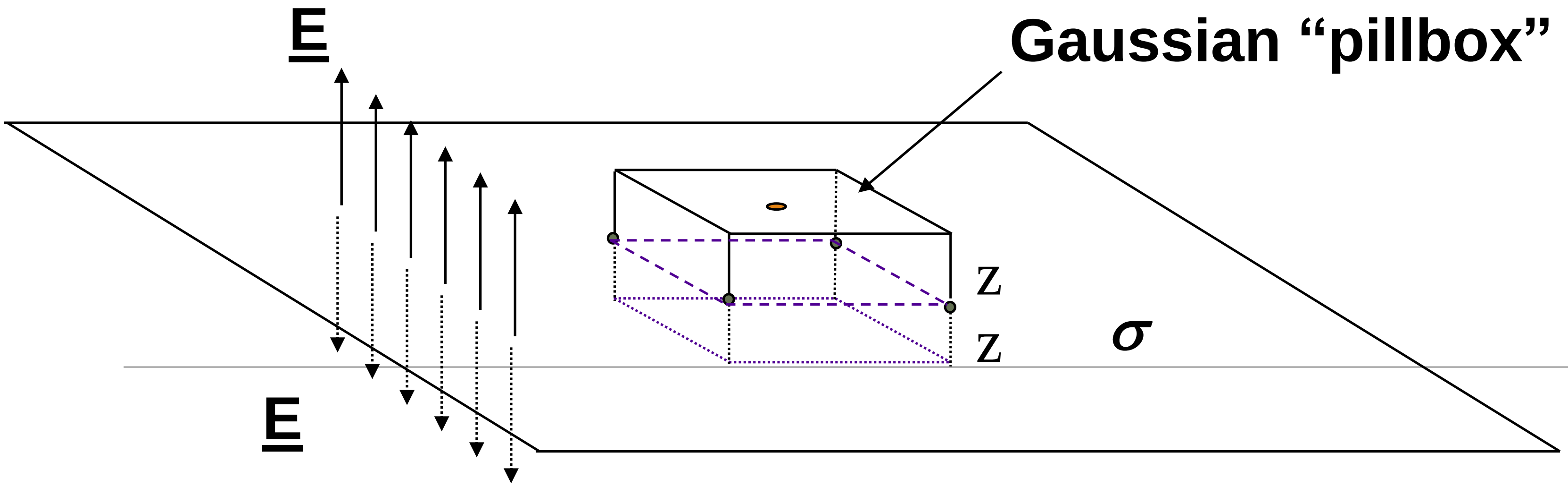


Use symmetry!

The electric field must point straight away from the plane (if $\sigma > 0$). Maybe the Magnitude of E depends on z, but the direction is fixed. And **E** is independent of x and y.

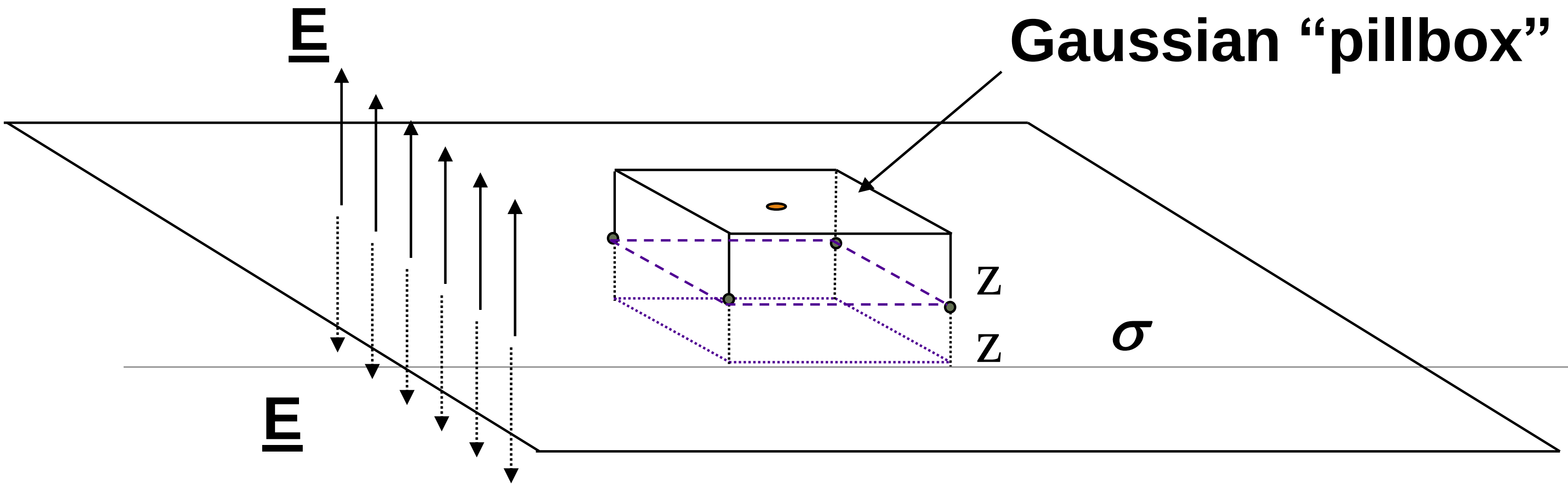
Problem: Infinite charged plane

So choose a Gaussian surface that is a “pillbox”, which has its top above the plane, and its bottom below the plane, each a distance z from the plane. That way the observation point lies in the top.



Problem: Infinite charged plane

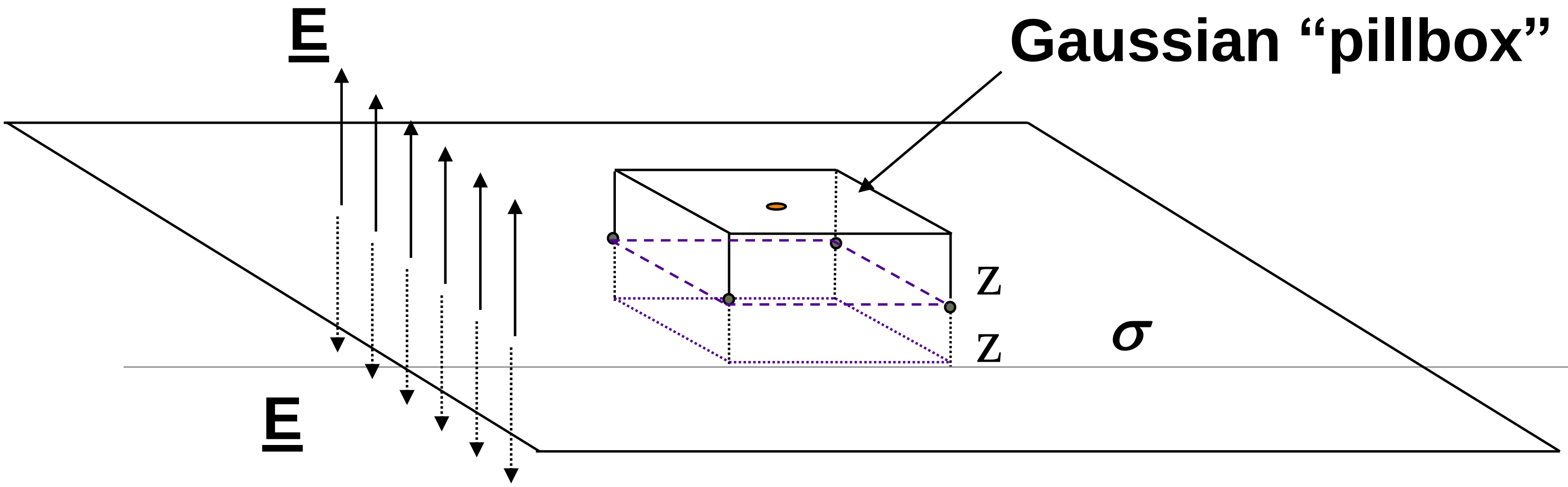
Let the area of the top and bottom be A.



Total charge enclosed by box = Aσ

Problem: Infinite charged plane

Let the area of the top and bottom be A.



Outward flux through the top: EA

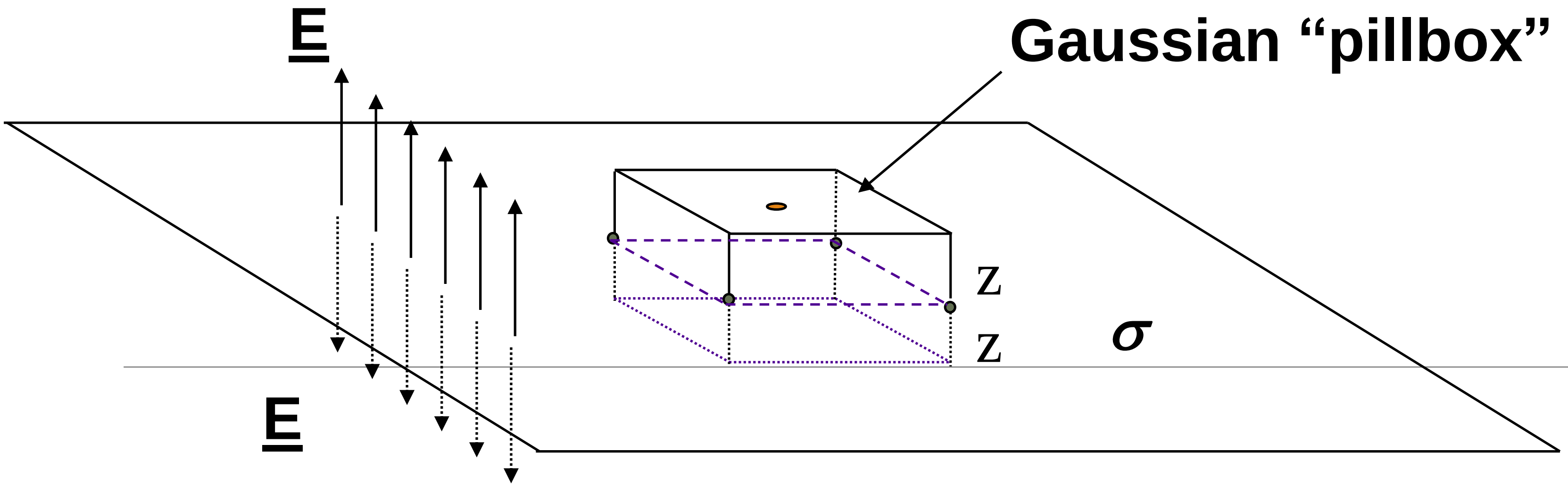
Outward flux through the bottom: EA

Outward flux through the sides: E x (some area) x $\cos(90^0) = 0$

So the total flux is: 2EA

Problem: Infinite charged plane

Let the area of the top and bottom be A.



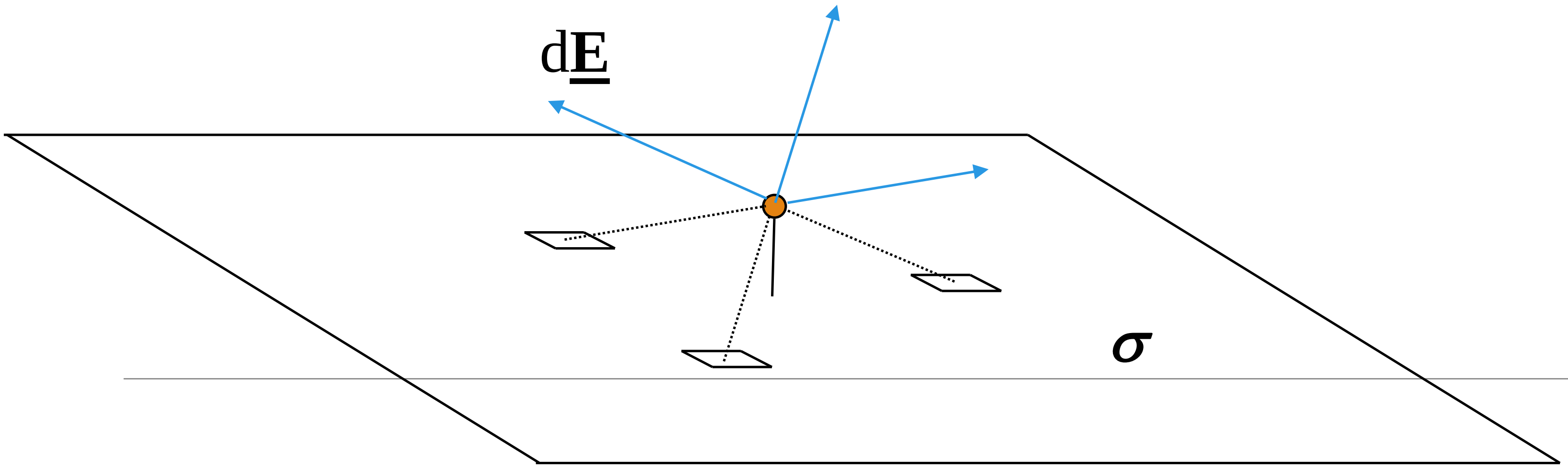
Gauss's law then says that $A\sigma/\varepsilon_0 = 2EA$ so that $\boxed{\underline{\mathbf{E}} = \sigma/2\varepsilon_0 \text{, outward.}}$

This is constant everywhere in each half-space!

Notice that the area A canceled: this is typical!

Problem: Infinite charged plane

Imagine doing this with an integral over the charge distribution: break the surface into little bits dA …



Doing this as a surface integral would be HARD.
Gauss's law is EASY.

Consider a long cylindrical charge distribution of radius R, with charge density $\rho = a - b\,r$ (with a and b positive). Calculate the electric field for:

a) $r < R$
b) $r = R$
c) $r > R$

Conductors

A conductor is a material in which charges can move relatively freely.

Usually these are metals (Au, Cu, Ag, Al).

Excess charges (of the same sign) placed on a conductor will move as far from each other as possible, since they repel each other.

For a charged conductor, in a static situation, all the charge resides at the surface of a conductor.

For a charged conductor, in a static situation, the electric field is zero everywhere inside a conductor, and perpendicular to the surface just outside

Conductors

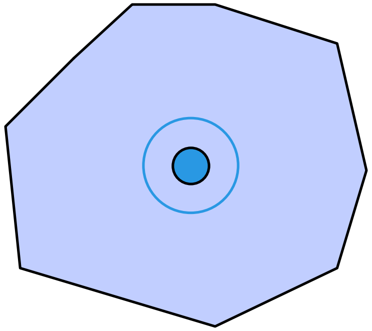
Why is E = 0 inside a conductor?

Conductors are full of free electrons, roughly one per cubic Angstrom. These are free to move. If **E** is nonzero in some region, then the electrons there feel a force -e**E** and start to move.

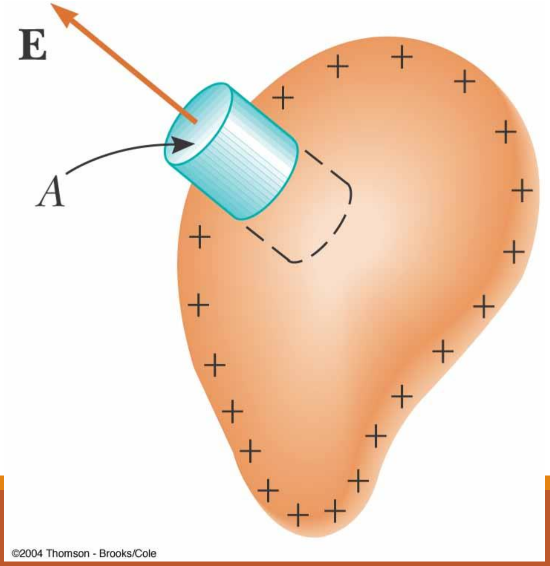
In an electrostatics problem, the electrons adjust their positions until the force on every electron is zero (or else it would move!). That means when equilibrium is reached, **E**=0 everywhere inside a conductor.

Conductors

Because E = 0 inside, the inside of a conductor is neutral.

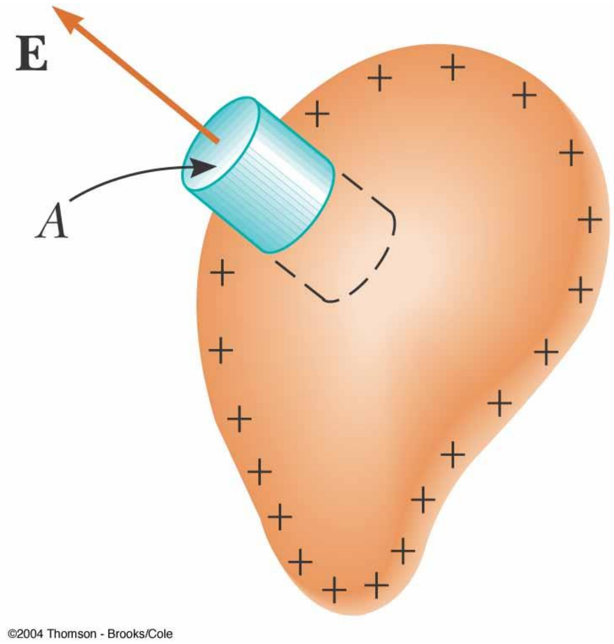


Suppose there is an extra charge inside.
Gauss's law for the little spherical surface
says there would be a nonzero E nearby.
But there can't be, within a metal!



Consequently the interior of a metal is neutral.
Any excess charge ends up on the surface.

Electric field just outside a charged conductor



$$\Phi = \oint \vec{E} \cdot \vec{dA} = EA$$

$$EA = \frac{q_{enclosed}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

The electric field just outside a charged conductor is perpendicular to the surface and has magnitude $E = \sigma / \varepsilon_0$

Properties of Conductors

In a conductor there are large number of electrons free to move. This fact has several interesting consequences

Excess charge placed on a conductor moves to the exterior surface of the conductor.

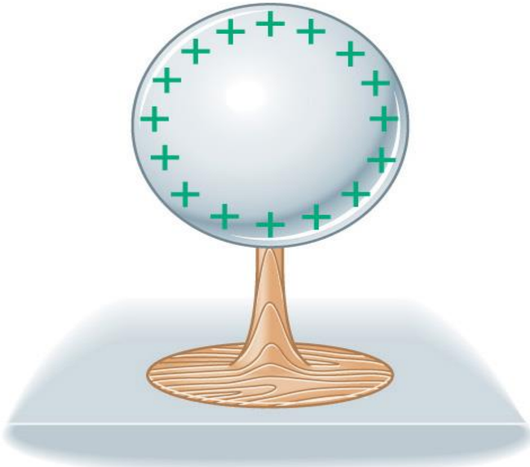
The electric field inside a conductor is zero when charges are at rest.

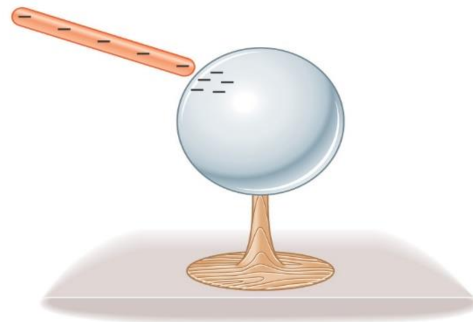
A conductor shields a cavity within it from external electric fields.

Electric field lines contact conductor surfaces at right angles

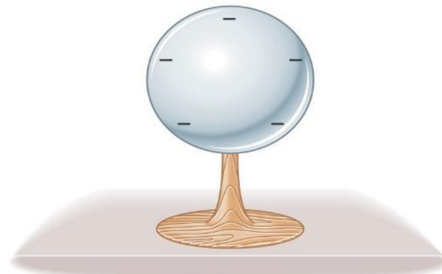
A conductor can be charged by contact or induction

Connecting a conductor to ground is referred to as grounding
The ground can accept of give up an unlimited number of electrons

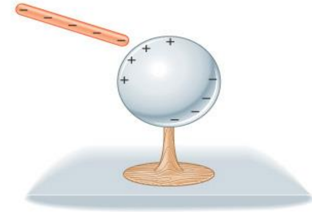




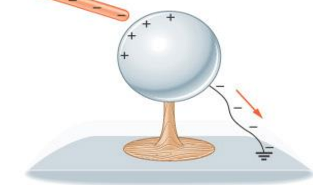
(a)



(b)



(a)



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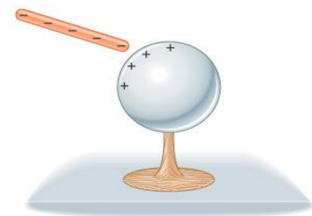

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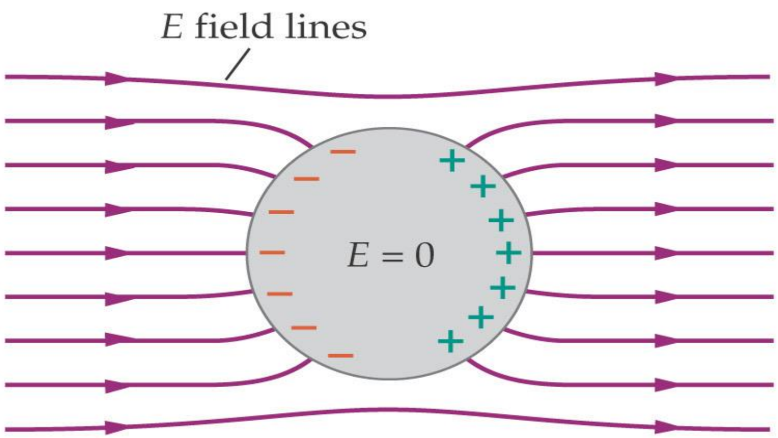
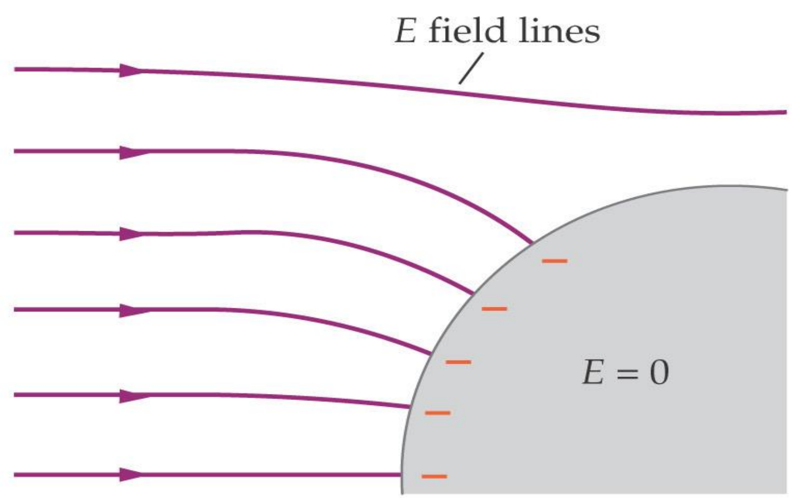


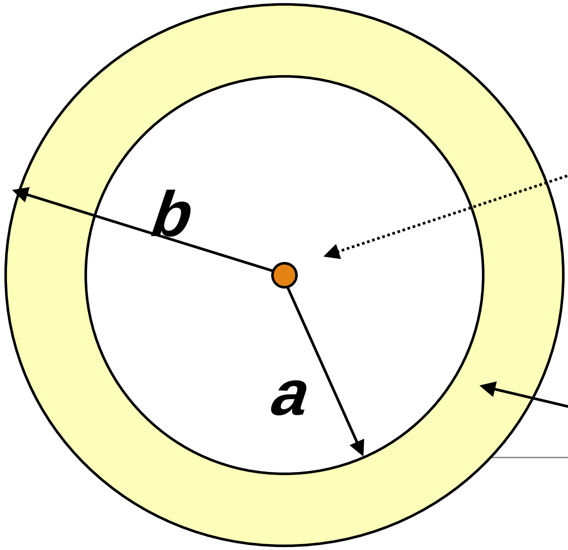
(a)



(b)

Problem: Charged coaxial cable

This picture is a cross section of an infinitely long line of charge, surrounded by an infinitely long cylindrical conductor. Find **E**.



This represents the line of charge. Say it has a linear charge density of λ (some number of C/m).

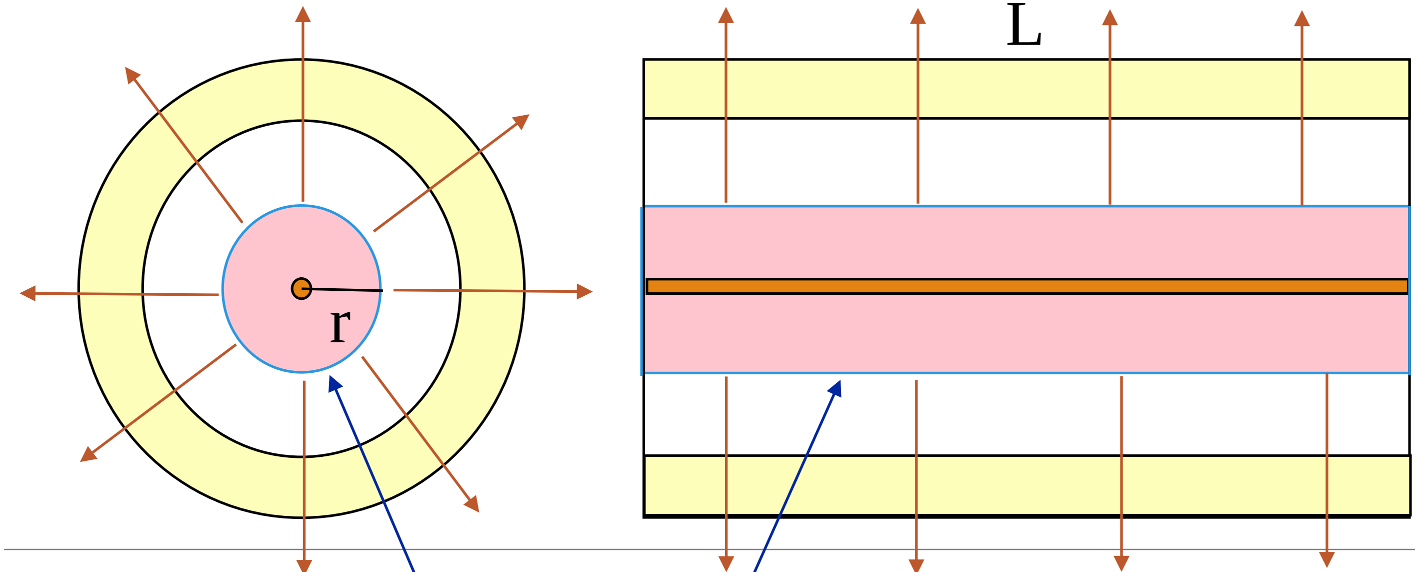
This is the cylindrical conductor. It has inner radius a, and outer radius b.

Use symmetry!

Clearly **E** points straight out, and its amplitude depends only on r.

Problem: Charged coaxial cable

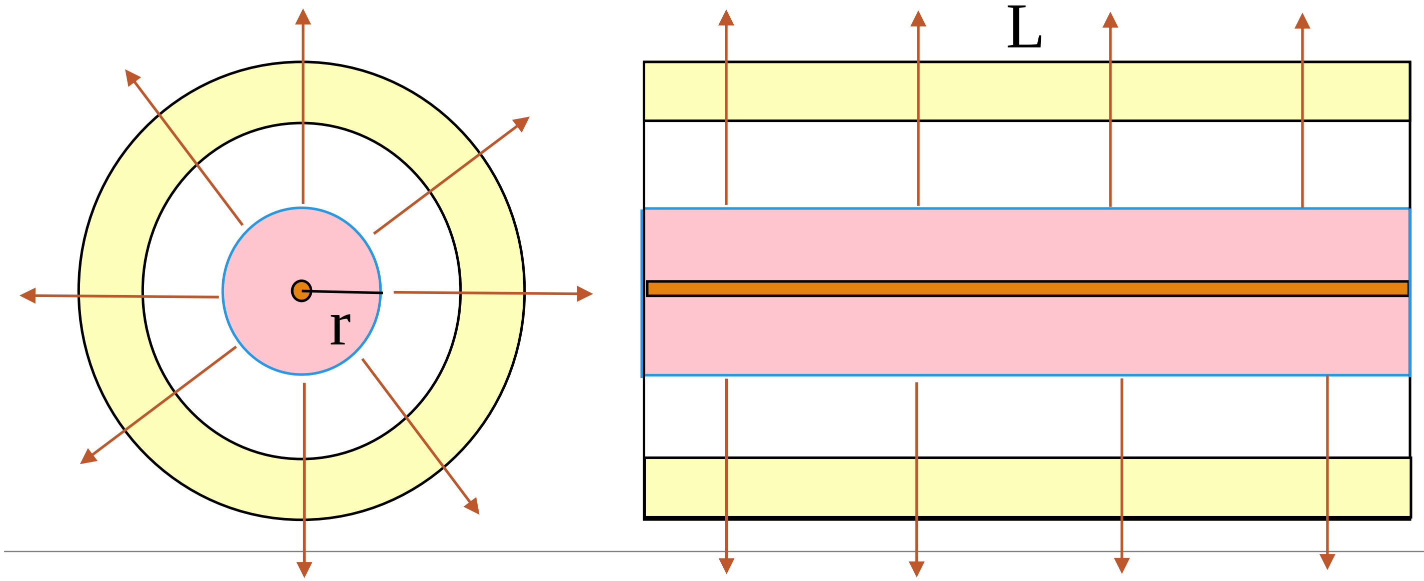
First find E at positions in the space inside the cylinder (r<a).



Choose as a Gaussian surface:
a cylinder of radius r, and length L.

Problem: Charged coaxial cable

First find E at positions in the space inside the cylinder (r<a).



What is the charge enclosed? $\Rightarrow$ λL

What is the flux through the end caps? $\Rightarrow$ zero ($\cos 90^0$)

What is the flux through the curved face? $\Rightarrow$ E x (area) = $E(2\pi rL)$

Total flux = $E(2\pi rL)$

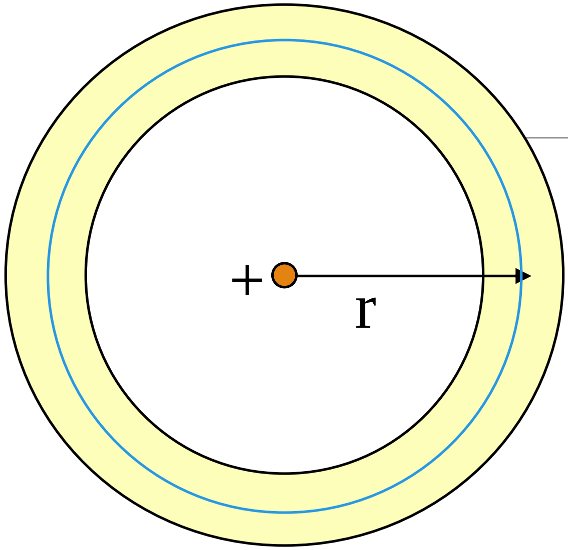
Gauss's law $\Rightarrow$ $E(2\pi rL) = \lambda L/\varepsilon_0$ so $\boxed{\mathbf{E(r) = \lambda / 2\pi r\varepsilon_0}}$

Problem: Charged coaxial cable

Now find E at positions within the cylinder (a<r<b).

There's no work to do: within a conductor **E**=0.

Still, we can learn something from Gauss's law.

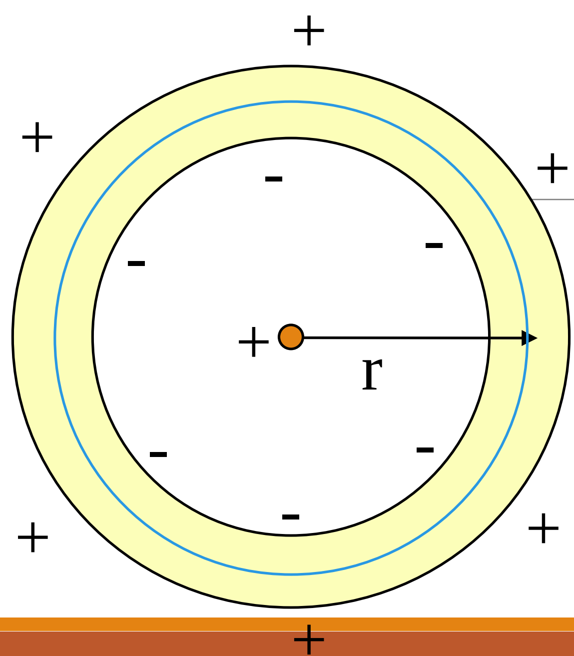


Make the same kind of cylindrical Gaussian surface. Now the curved side is entirely within the conductor, where **E**=0; hence the flux is zero.

Thus the total charge enclosed by this surface must be zero.

Problem: Charged coaxial cable

There must be a net charge per unit length $-\lambda$ attracted to the inner surface of the metal so that the total charge enclosed by this Gaussian surface is zero.



And since the cylinder is neutral, these negative charges must have come from the outer surface. So the outer surface has a charge density per unit length of $+\lambda$ spread around the outer perimeter.

So what is the field for r>b?