GRAPHICS

ContourPlot
Contour plot3D
ListContourPlot

DensityPlot
DensityPlot3D

VectorPlot
VectorPlot3D

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ParametricPlot3D

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SphericalHarmonicY

ContourPlot

ContourPlot creates contours of an expression involving two variables.
The contours are the curves on which the expression is constant.
The contours are drawn on a rectangle.
Ranges for each variable in the expression can be given.

ContourPlot

ContourPlot[f, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}]
generates a contour plot of f as a function of x and y.

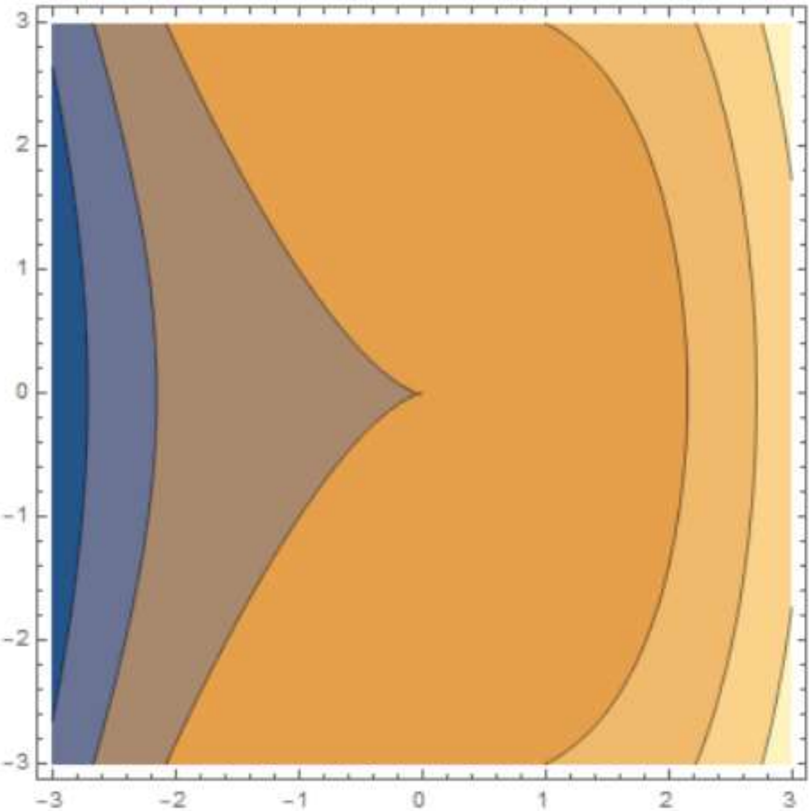
ContourPlot[f == g, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}]
plots contour lines for which $f = g$.

ContourPlot[{f_1 == g_1, f_2 == g_2, ...}, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}]
plots several contour lines.

ContourPlot[..., {x, y} ∈ reg]
takes the variables {x, y} to be in the geometric region reg.

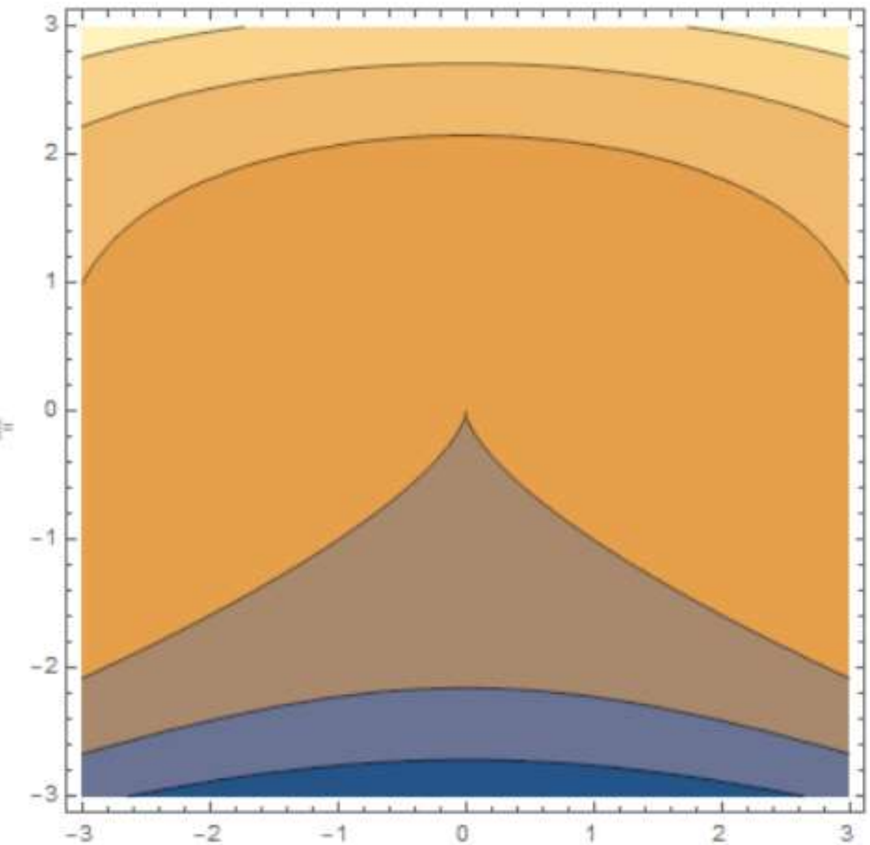
In[2]:= `ContourPlot[x^3 + y^2, {x, -3, 3}, {y, -3, 3}]`

Out[2]=



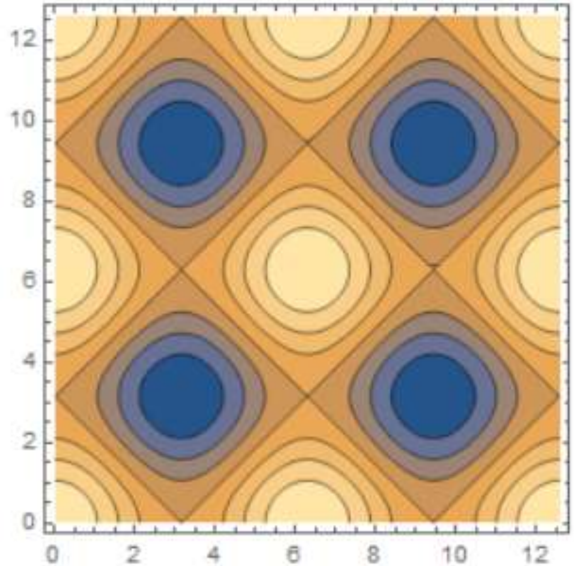
In[17]:= `ContourPlot[x^2 + y^3, {x, -3, 3}, {y, -3, 3}]`

Out[17]=



```
In[1]:= ContourPlot[Cos[x] + Cos[y], {x, 0, 4 Pi}, {y, 0, 4 Pi}]
```

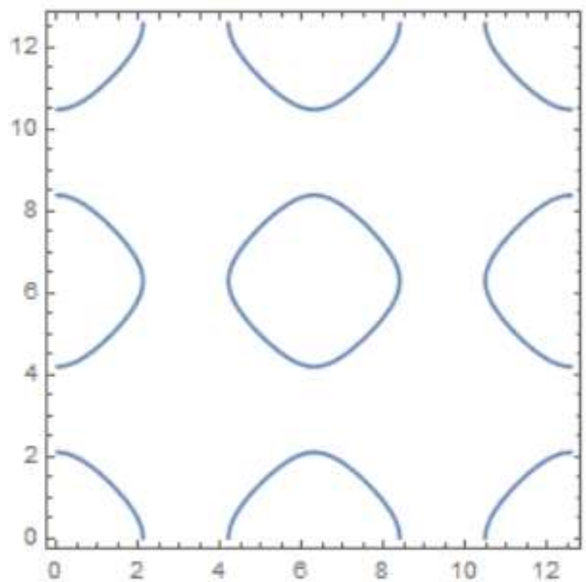
Out[1]=



Plot an equation:

```
In[1]:= ContourPlot[Cos[x] + Cos[y] == 1 / 2, {x, 0, 4 Pi}, {y, 0, 4 Pi}]
```

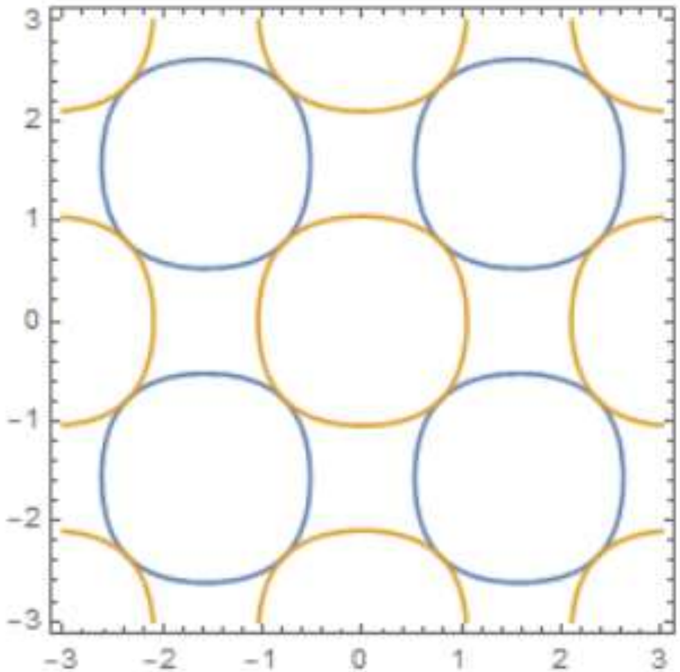
Out[1]=



Plot several equations:

```
In[1]:= ContourPlot[{Abs[Sin[x] Sin[y]] == 0.5, Abs[Cos[x] Cos[y]] == 0.5}, {x, -3, 3}, {y, -3, 3}]
```

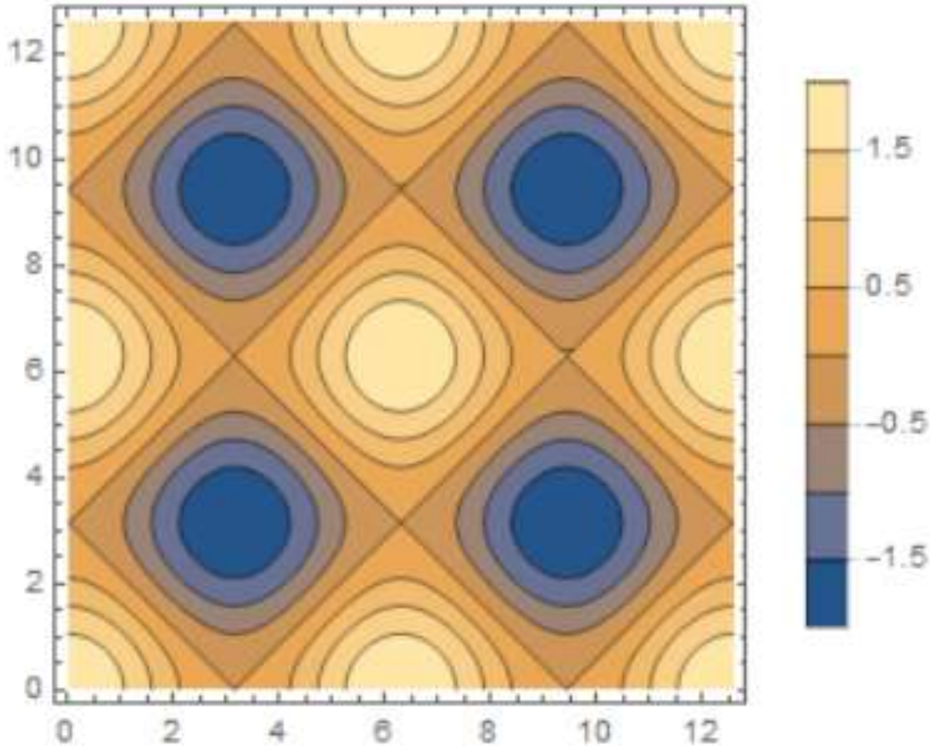
Out[1]=



Show a legend for the contours:

In[1]:= `ContourPlot[Cos[x] + Cos[y], {x, 0, 4 Pi}, {y, 0, 4 Pi}, PlotLegends → Automatic]`

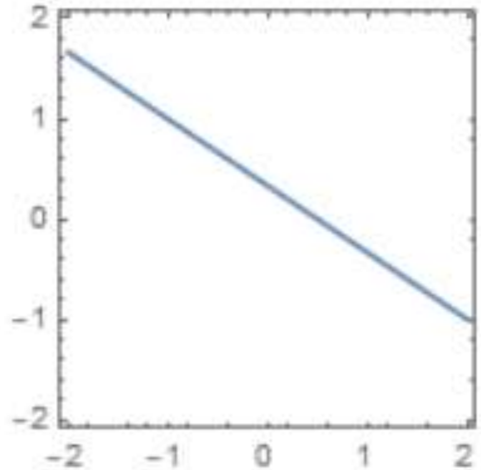
Out[1]=



Simple shapes, including a line:

```
In[1]:= ContourPlot[2 x + 3 y == 1, {x, -2, 2}, {y, -2, 2}]
```

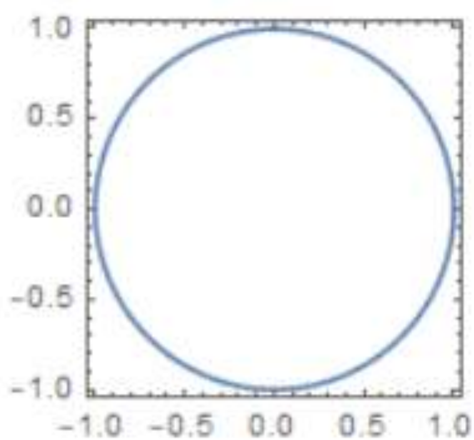
Out[1]=



Circle:

```
In[2]:= ContourPlot[x^2 + y^2 == 1, {x, -1, 1}, {y, -1, 1}]
```

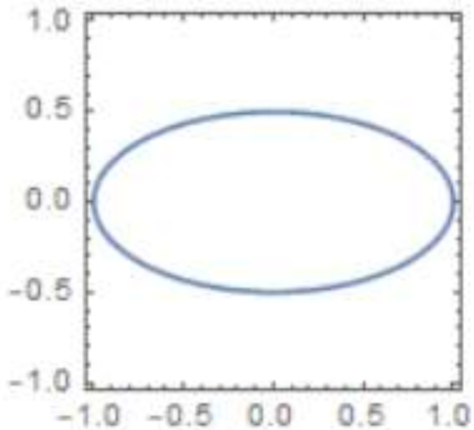
Out[2]=



Ellipse:

```
In[3]:= ContourPlot[x^2 + (2 y)^2 == 1, {x, -1, 1}, {y, -1, 1}]
```

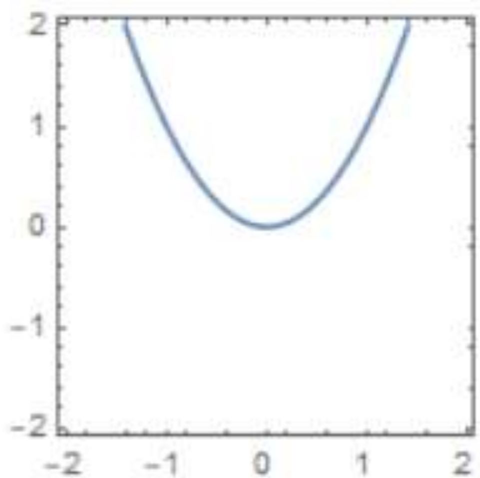
Out[3]=



Parabola:

```
In[4]:= ContourPlot[x^2 == y, {x, -2, 2}, {y, -2, 2}]
```

Out[4]=



ContourPlot3D

ContourPlot3D[f, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}, {z, z_{min}, z_{max}}]
produces a three–dimensional contour plot of f as a function of x, y, and z.

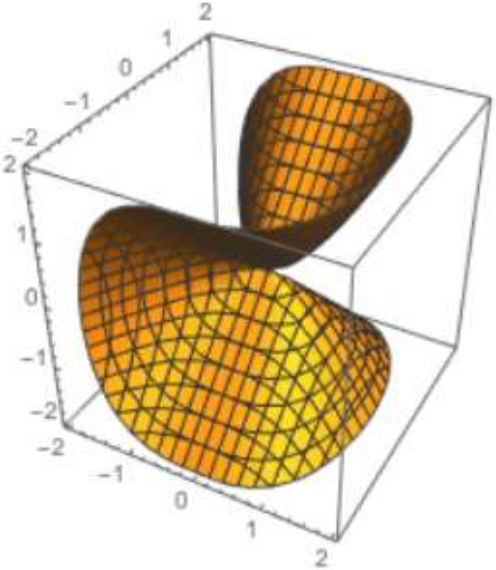
ContourPlot3D[f == g, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}, {z, z_{min}, z_{max}}]
plots the contour surface for which $f = g$.

- The contour surfaces plotted by ContourPlot3D can contain disconnected parts.
- By default, ContourPlot3D shows each contour level as an opaque white surface, with normals pointing outward.

Plot a 3D contour surface:

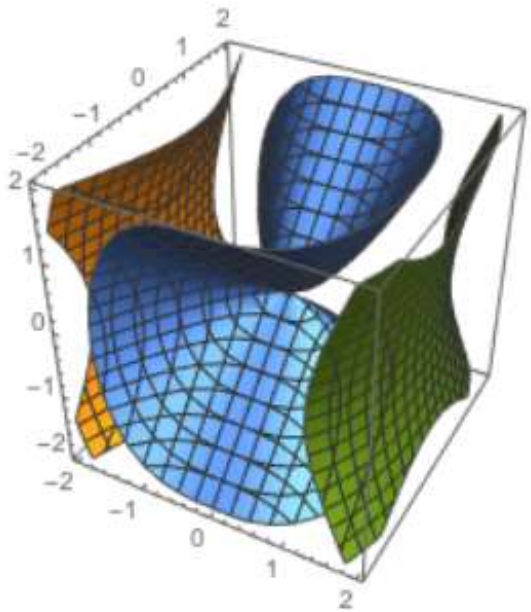
In[1]:= `ContourPlot3D[x^3 + y^2 - z^2 == 0, {x, -2, 2}, {y, -2, 2}, {z, -2, 2}]`

Out[1]=



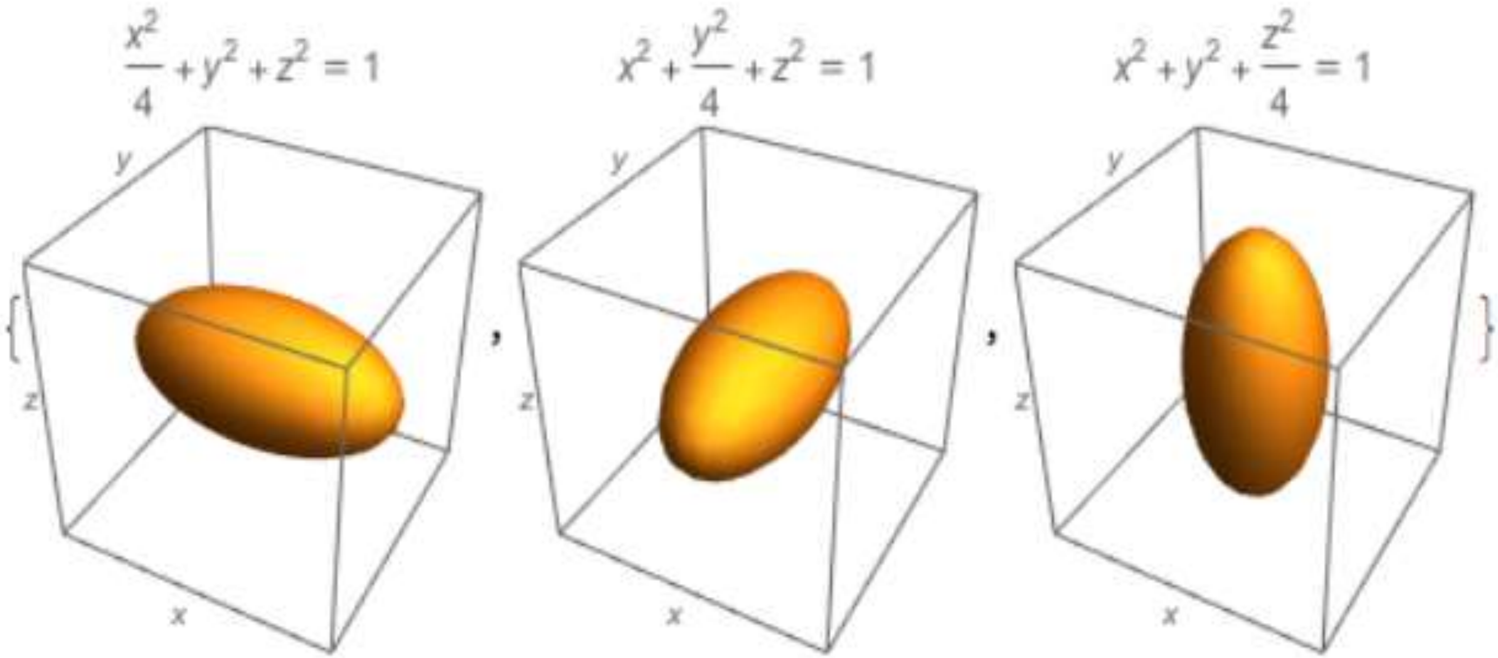
In[1]:= `ContourPlot3D[x^3 + y^2 - z^2, {x, -2, 2}, {y, -2, 2}, {z, -2, 2}]`

Out[1]=



```
In[1]:= Table[ContourPlot3D[Evaluate[e], {x, -2, 2}, {y, -2, 2},
          {z, -2, 2}, Mesh → None, AxesLabel → Automatic, Ticks → None, PlotLabel → e],
         {e, {(x / 2) ^2 + y^2 + z^2 == 1, x^2 + (y / 2) ^2 + z^2 == 1, x^2 + y^2 + (z / 2) ^2 == 1}}]
```

Out[1]=



ListContourPlot

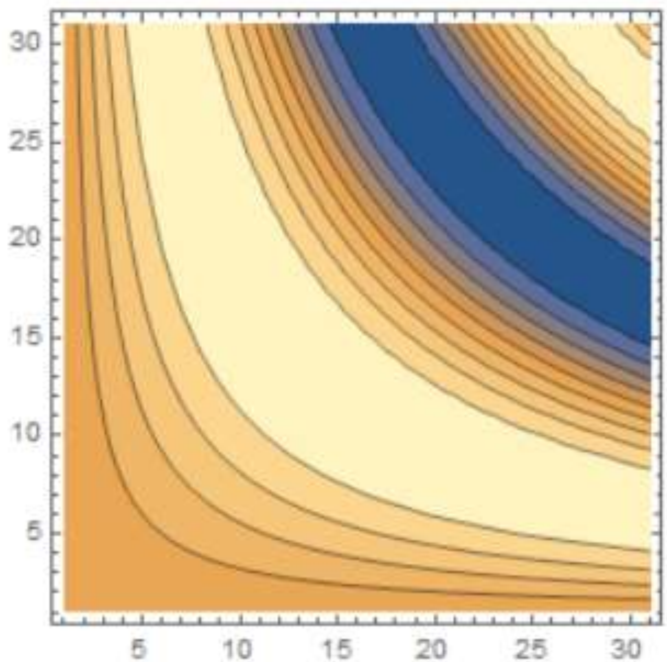
ListContourPlot[*array*]
generates a contour plot from an array of height values.

ListContourPlot[{{x_1, y_1, f_1}, {x_2, y_2, f_2}, ...}]
generates a contour plot from values defined at specified points.

```
In[1]:= data = Table[Sin[x y], {x, 0, 3, .1}, {y, 0, 3, .1}];

In[2]:= ListContourPlot[data]
```

Out[2]=




```
ListContourPlot[Table[Sin[x y], {x, 0, 3, .1}, {y, 0, 3, .1}]]
```

DensityPlot

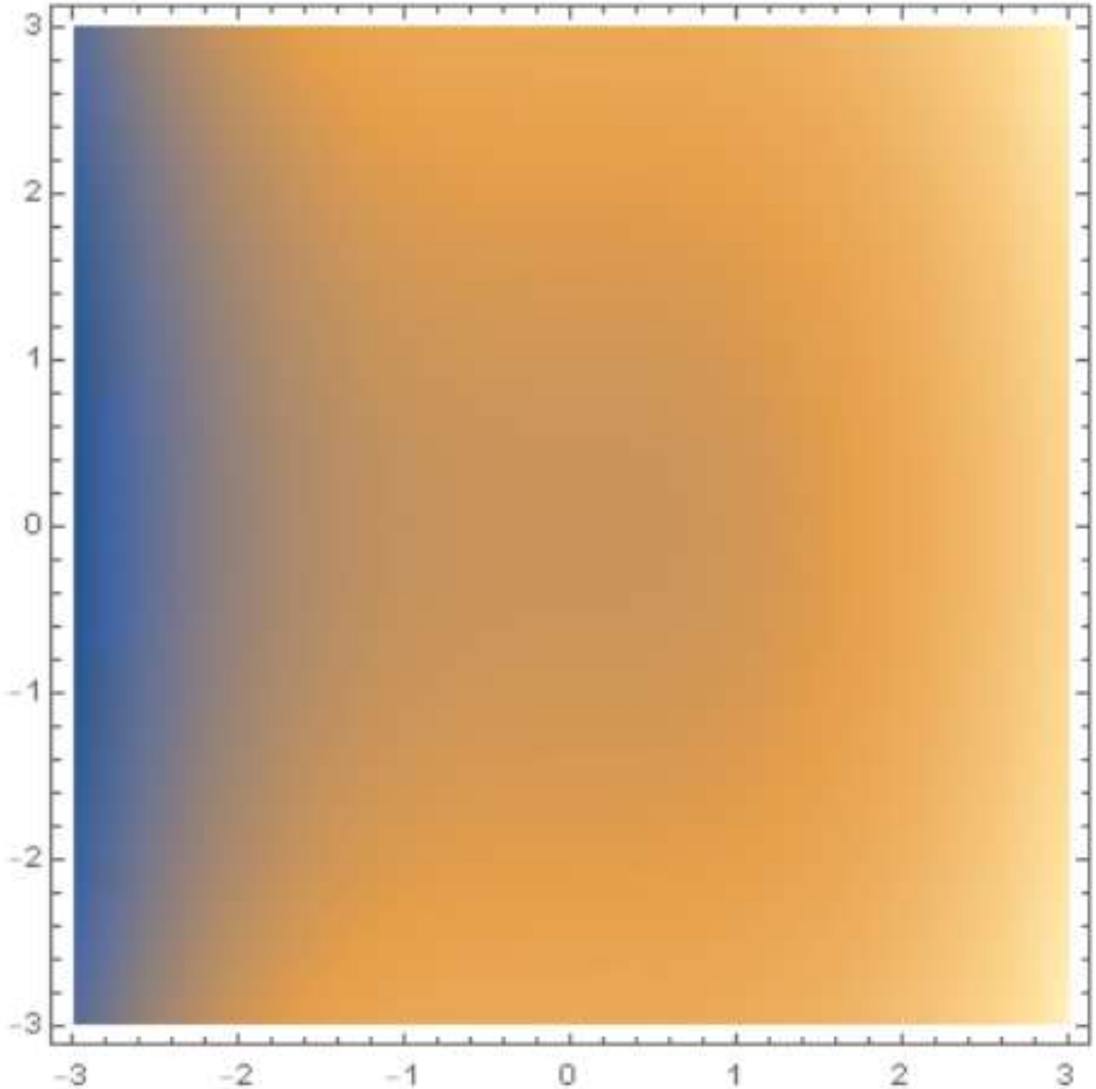
This has the syntax similar to that of the ContourPlot[ ],
but it has different options.
For an expression involving two variables,
this produces a shading of chosen rectangle.

DensityPlot

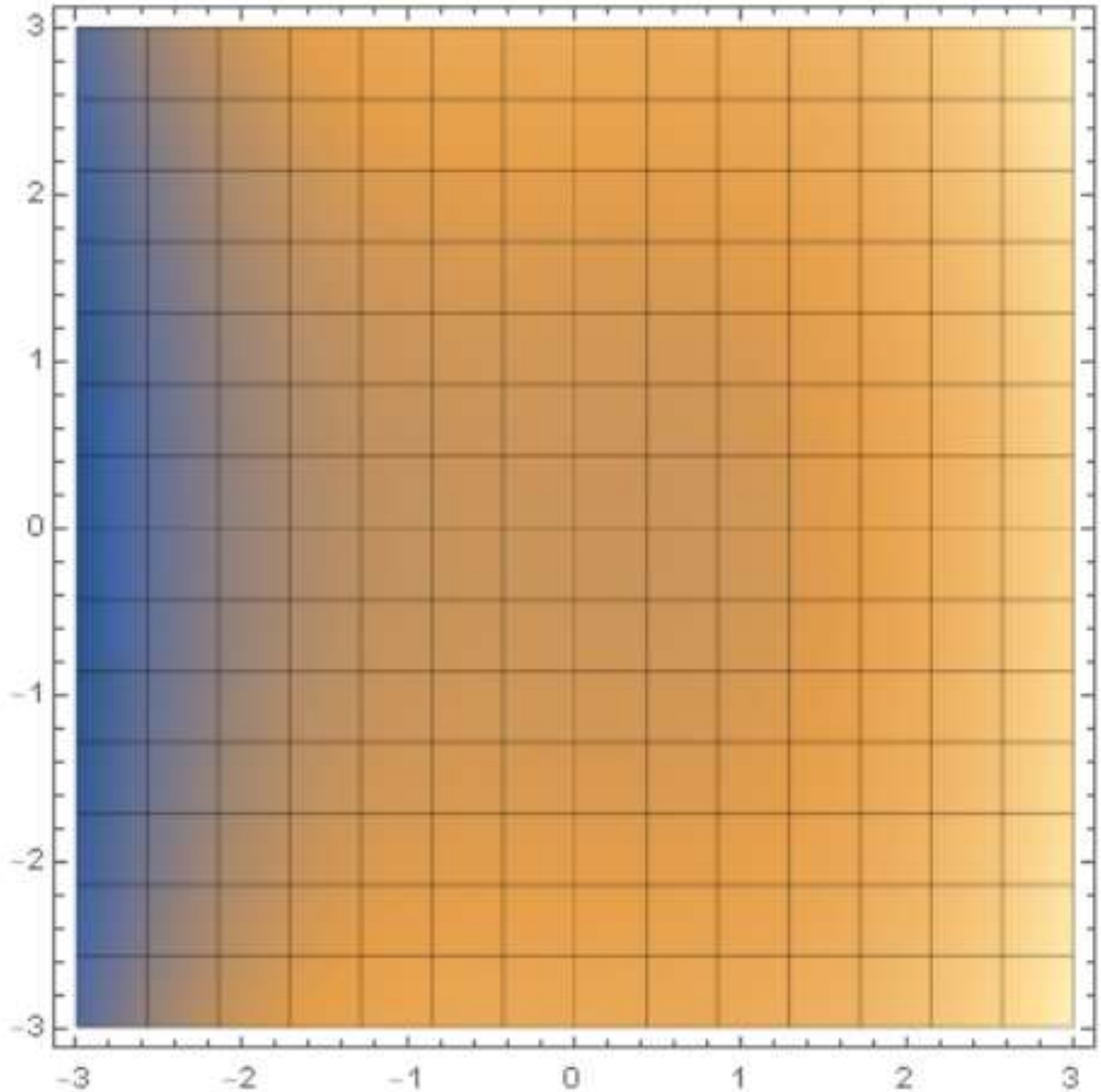
DensityPlot[f, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}]
makes a density plot of f as a function of x and y.

DensityPlot[f, {x, y} ∈ reg]
takes the variables {x, y} to be in the geometric region reg.

```
DensityPlot[x^3 + y^2, {x, -3, 3}, {y, -3, 3}]
```



```
DensityPlot[x^3 + y^2, {x, -3, 3}, {y, -3, 3}, Mesh → Full]
```



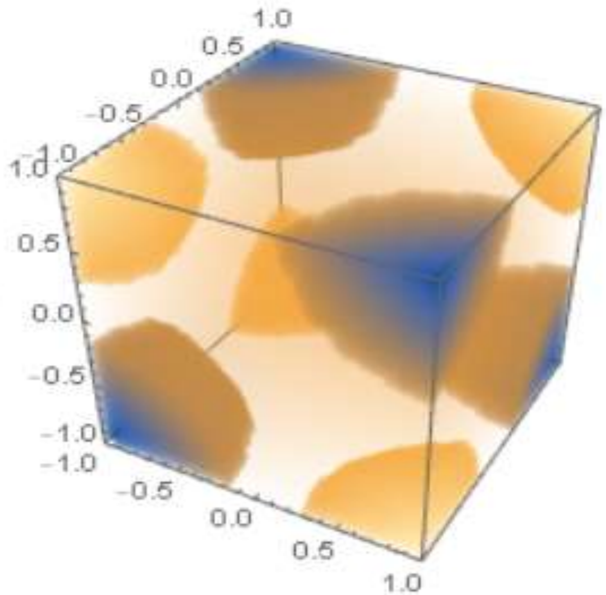
DensityPlot3D

DensityPlot3D[f, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}, {z, z_{min}, z_{max}}]
makes a density plot of f as a function of x, y, and z.

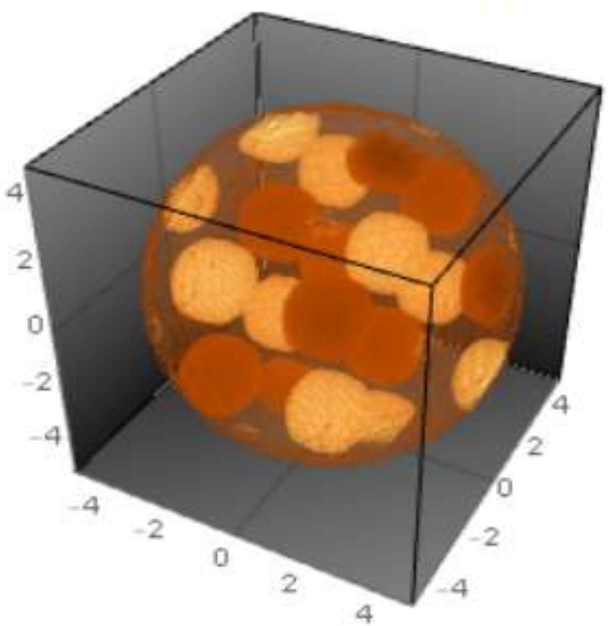
DensityPlot3D[f, {x, y, z} $\in$ *reg*]
takes the variables to be in the geometric region *reg*.

- DensityPlot3D by default generates colorized output.
- At positions where f does not evaluate to a real number, data is taken to be missing and is rendered transparently.
- DensityPlot3D treats the variables x, y, and z as local, effectively using Block.
- DensityPlot3D has attribute HoldAll, and evaluates f only after assigning specific numerical values to x, y, and z.

```
DensityPlot3D[x y z, {x, -1, 1}, {y, -1, 1}, {z, -1, 1}]
```




```
DensityPlot3D[Sin[x] Cos[y] Sin[z], {x, y, z} ∈ Ball[{0, 0, 0}, 5], PlotTheme → "Marketing"]
```



VectorPlot

VectorPlot[{v_x, v_y}, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}]
generates a vector plot of the vector field {v_x, v_y} as a function of x and y.

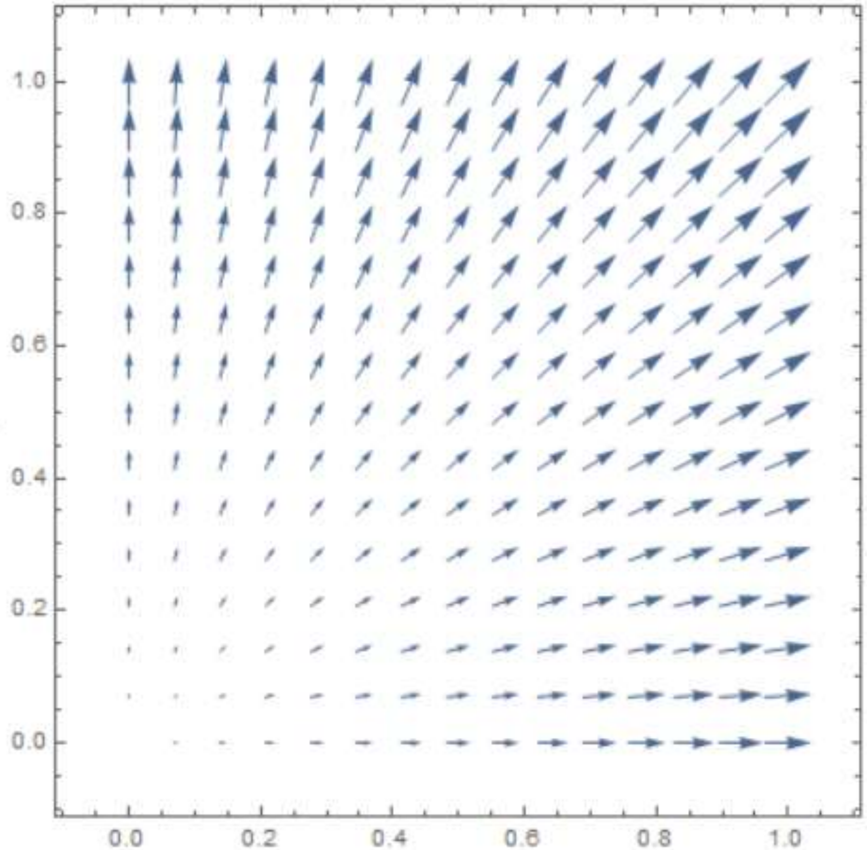
VectorPlot[{{v_x, v_y}, {w_x, w_y}, ...}, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}]
plots several vector fields.

VectorPlot[..., {x, y} ∈ *reg*]
takes the variables {x, y} to be in the geometric region *reg*.

- VectorPlot by default shows vectors from the vector field at a regular grid of positions.
- VectorPlot omits any vectors for which the v_i etc. do not evaluate to real numbers.
- VectorPlot treats the variables x and y as local, effectively using Block.

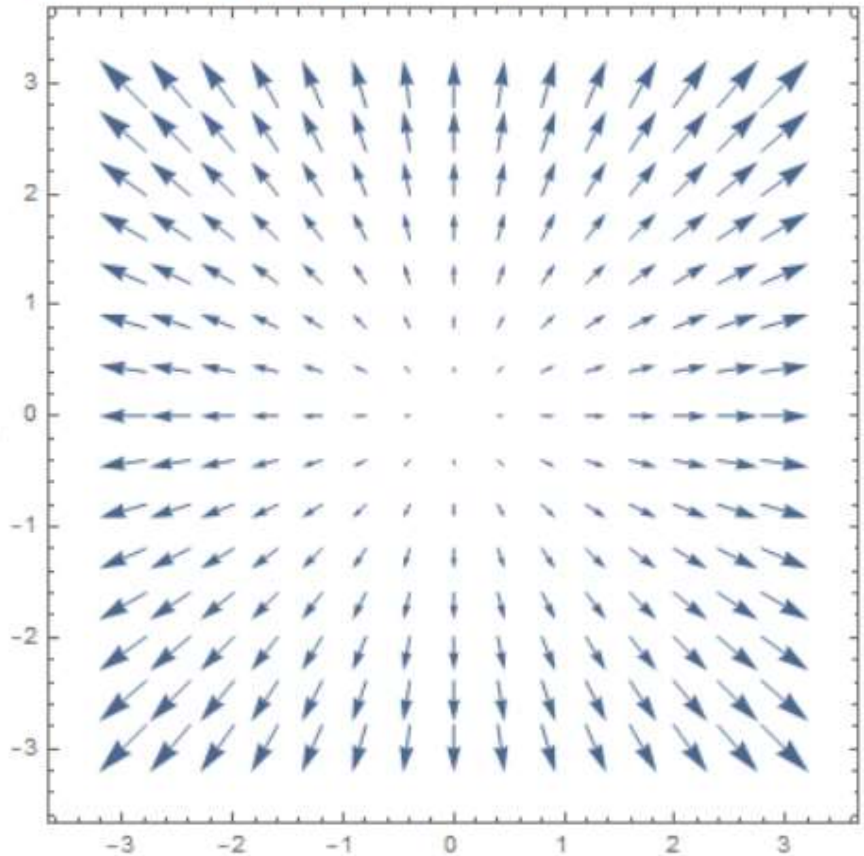
In[22]:= `VectorPlot[{x, y}, {x, 0, 1}, {y, 0, 1}]`

Out[22]=



```
In[34]:= VectorPlot[{x, y}, {x, -3, 3}, {y, -3, 3}]
```

Out[34]=

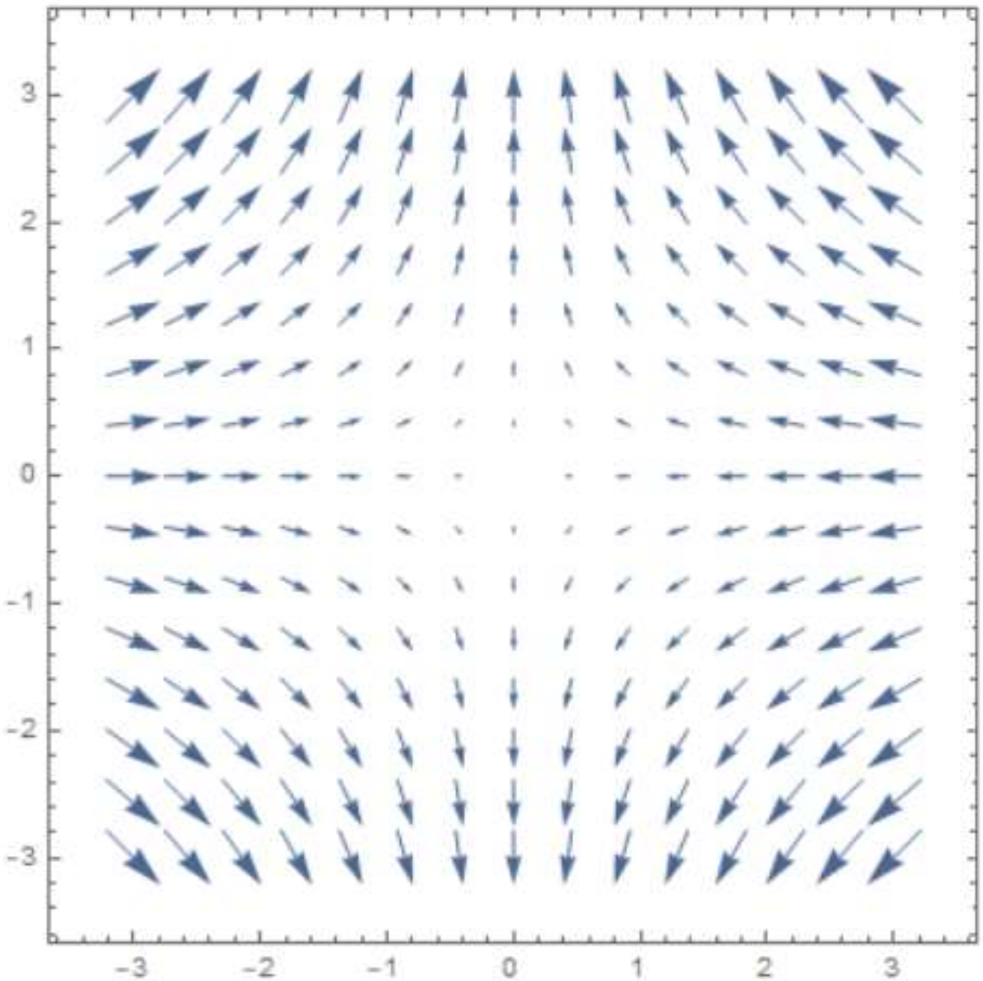



```
Table[{x, y}, {x, -3, 3}, {y, -3, 3}]; MatrixForm[%]
```

itrixForm=

$$\begin{pmatrix} \begin{pmatrix}-3\\-3\end{pmatrix} & \begin{pmatrix}-3\\-2\end{pmatrix} & \begin{pmatrix}-3\\-1\end{pmatrix} & \begin{pmatrix}-3\\0\end{pmatrix} & \begin{pmatrix}-3\\1\end{pmatrix} & \begin{pmatrix}-3\\2\end{pmatrix} & \begin{pmatrix}-3\\3\end{pmatrix} \\ \begin{pmatrix}-2\\-3\end{pmatrix} & \begin{pmatrix}-2\\-2\end{pmatrix} & \begin{pmatrix}-2\\-1\end{pmatrix} & \begin{pmatrix}-2\\0\end{pmatrix} & \begin{pmatrix}-2\\1\end{pmatrix} & \begin{pmatrix}-2\\2\end{pmatrix} & \begin{pmatrix}-2\\3\end{pmatrix} \\ \begin{pmatrix}-1\\-3\end{pmatrix} & \begin{pmatrix}-1\\-2\end{pmatrix} & \begin{pmatrix}-1\\-1\end{pmatrix} & \begin{pmatrix}-1\\0\end{pmatrix} & \begin{pmatrix}-1\\1\end{pmatrix} & \begin{pmatrix}-1\\2\end{pmatrix} & \begin{pmatrix}-1\\3\end{pmatrix} \\ \begin{pmatrix}0\\-3\end{pmatrix} & \begin{pmatrix}0\\-2\end{pmatrix} & \begin{pmatrix}0\\-1\end{pmatrix} & \begin{pmatrix}0\\0\end{pmatrix} & \begin{pmatrix}0\\1\end{pmatrix} & \begin{pmatrix}0\\2\end{pmatrix} & \begin{pmatrix}0\\3\end{pmatrix} \\ \begin{pmatrix}1\\-3\end{pmatrix} & \begin{pmatrix}1\\-2\end{pmatrix} & \begin{pmatrix}1\\-1\end{pmatrix} & \begin{pmatrix}1\\0\end{pmatrix} & \begin{pmatrix}1\\1\end{pmatrix} & \begin{pmatrix}1\\2\end{pmatrix} & \begin{pmatrix}1\\3\end{pmatrix} \\ \begin{pmatrix}2\\-3\end{pmatrix} & \begin{pmatrix}2\\-2\end{pmatrix} & \begin{pmatrix}2\\-1\end{pmatrix} & \begin{pmatrix}2\\0\end{pmatrix} & \begin{pmatrix}2\\1\end{pmatrix} & \begin{pmatrix}2\\2\end{pmatrix} & \begin{pmatrix}2\\3\end{pmatrix} \\ \begin{pmatrix}3\\-3\end{pmatrix} & \begin{pmatrix}3\\-2\end{pmatrix} & \begin{pmatrix}3\\-1\end{pmatrix} & \begin{pmatrix}3\\0\end{pmatrix} & \begin{pmatrix}3\\1\end{pmatrix} & \begin{pmatrix}3\\2\end{pmatrix} & \begin{pmatrix}3\\3\end{pmatrix} \end{pmatrix}$$

```
VectorPlot[{-x, y}, {x, -3, 3}, {y, -3, 3}]
```

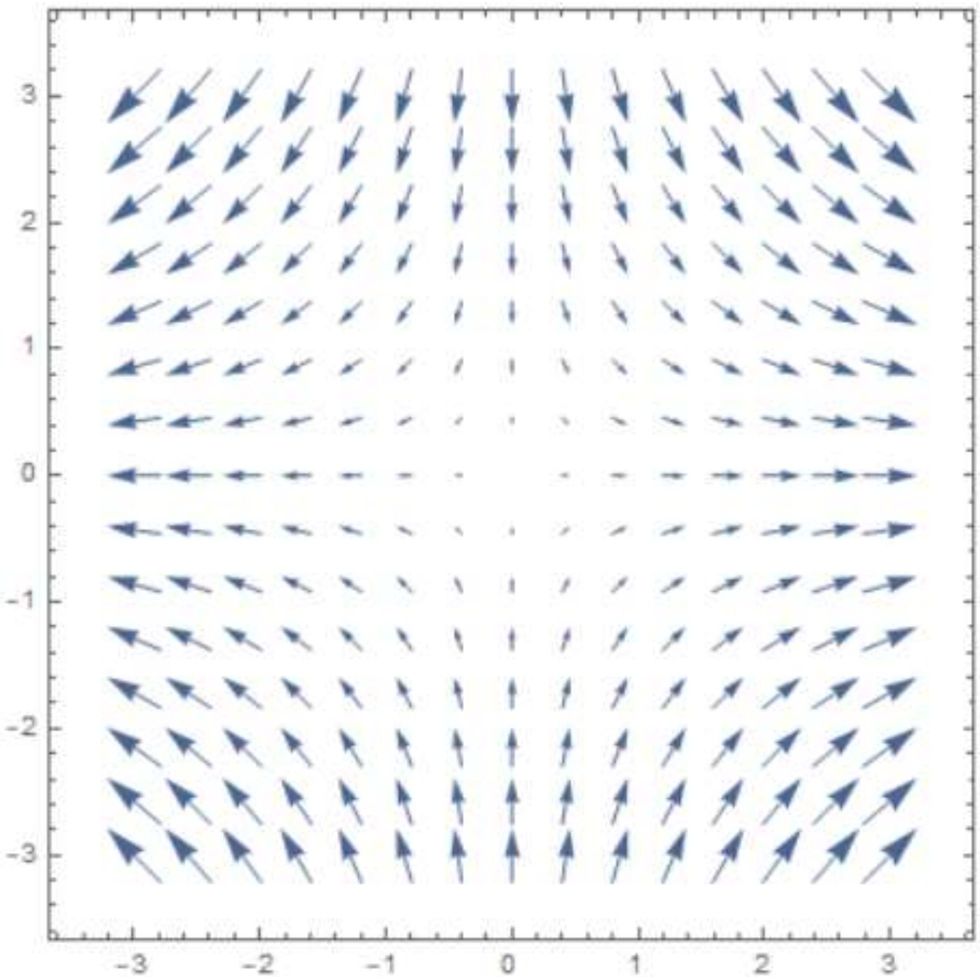



```
Table[{-x, y}, {x, -3, 3}, {y, -3, 3}]; MatrixForm[%]
```

trixForm=

$$\begin{pmatrix} \begin{pmatrix}3\\-3\end{pmatrix} & \begin{pmatrix}3\\-2\end{pmatrix} & \begin{pmatrix}3\\-1\end{pmatrix} & \begin{pmatrix}3\\0\end{pmatrix} & \begin{pmatrix}3\\1\end{pmatrix} & \begin{pmatrix}3\\2\end{pmatrix} & \begin{pmatrix}3\\3\end{pmatrix} \\ \begin{pmatrix}2\\-3\end{pmatrix} & \begin{pmatrix}2\\-2\end{pmatrix} & \begin{pmatrix}2\\-1\end{pmatrix} & \begin{pmatrix}2\\0\end{pmatrix} & \begin{pmatrix}2\\1\end{pmatrix} & \begin{pmatrix}2\\2\end{pmatrix} & \begin{pmatrix}2\\3\end{pmatrix} \\ \begin{pmatrix}1\\-3\end{pmatrix} & \begin{pmatrix}1\\-2\end{pmatrix} & \begin{pmatrix}1\\-1\end{pmatrix} & \begin{pmatrix}1\\0\end{pmatrix} & \begin{pmatrix}1\\1\end{pmatrix} & \begin{pmatrix}1\\2\end{pmatrix} & \begin{pmatrix}1\\3\end{pmatrix} \\ \begin{pmatrix}0\\-3\end{pmatrix} & \begin{pmatrix}0\\-2\end{pmatrix} & \begin{pmatrix}0\\-1\end{pmatrix} & \begin{pmatrix}0\\0\end{pmatrix} & \begin{pmatrix}0\\1\end{pmatrix} & \begin{pmatrix}0\\2\end{pmatrix} & \begin{pmatrix}0\\3\end{pmatrix} \\ \begin{pmatrix}-1\\-3\end{pmatrix} & \begin{pmatrix}-1\\-2\end{pmatrix} & \begin{pmatrix}-1\\-1\end{pmatrix} & \begin{pmatrix}-1\\0\end{pmatrix} & \begin{pmatrix}-1\\1\end{pmatrix} & \begin{pmatrix}-1\\2\end{pmatrix} & \begin{pmatrix}-1\\3\end{pmatrix} \\ \begin{pmatrix}-2\\-3\end{pmatrix} & \begin{pmatrix}-2\\-2\end{pmatrix} & \begin{pmatrix}-2\\-1\end{pmatrix} & \begin{pmatrix}-2\\0\end{pmatrix} & \begin{pmatrix}-2\\1\end{pmatrix} & \begin{pmatrix}-2\\2\end{pmatrix} & \begin{pmatrix}-2\\3\end{pmatrix} \\ \begin{pmatrix}-3\\-3\end{pmatrix} & \begin{pmatrix}-3\\-2\end{pmatrix} & \begin{pmatrix}-3\\-1\end{pmatrix} & \begin{pmatrix}-3\\0\end{pmatrix} & \begin{pmatrix}-3\\1\end{pmatrix} & \begin{pmatrix}-3\\2\end{pmatrix} & \begin{pmatrix}-3\\3\end{pmatrix} \end{pmatrix}$$

```
VectorPlot[{x, -y}, {x, -3, 3}, {y, -3, 3}]
```




```
Table[{x, -y}, {x, -3, 3}, {y, -3, 3}]; MatrixForm[%]
```

trixForm=

$$\begin{pmatrix} \begin{pmatrix} -3 \\ 3 \end{pmatrix} & \begin{pmatrix} -3 \\ 2 \end{pmatrix} & \begin{pmatrix} -3 \\ 1 \end{pmatrix} & \begin{pmatrix} -3 \\ 0 \end{pmatrix} & \begin{pmatrix} -3 \\ -1 \end{pmatrix} & \begin{pmatrix} -3 \\ -2 \end{pmatrix} & \begin{pmatrix} -3 \\ -3 \end{pmatrix} \\ \begin{pmatrix} -2 \\ 3 \end{pmatrix} & \begin{pmatrix} -2 \\ 2 \end{pmatrix} & \begin{pmatrix} -2 \\ 1 \end{pmatrix} & \begin{pmatrix} -2 \\ 0 \end{pmatrix} & \begin{pmatrix} -2 \\ -1 \end{pmatrix} & \begin{pmatrix} -2 \\ -2 \end{pmatrix} & \begin{pmatrix} -2 \\ -3 \end{pmatrix} \\ \begin{pmatrix} -1 \\ 3 \end{pmatrix} & \begin{pmatrix} -1 \\ 2 \end{pmatrix} & \begin{pmatrix} -1 \\ 1 \end{pmatrix} & \begin{pmatrix} -1 \\ 0 \end{pmatrix} & \begin{pmatrix} -1 \\ -1 \end{pmatrix} & \begin{pmatrix} -1 \\ -2 \end{pmatrix} & \begin{pmatrix} -1 \\ -3 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 3 \end{pmatrix} & \begin{pmatrix} 0 \\ 2 \end{pmatrix} & \begin{pmatrix} 0 \\ 1 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ -1 \end{pmatrix} & \begin{pmatrix} 0 \\ -2 \end{pmatrix} & \begin{pmatrix} 0 \\ -3 \end{pmatrix} \\ \begin{pmatrix} 1 \\ 3 \end{pmatrix} & \begin{pmatrix} 1 \\ 2 \end{pmatrix} & \begin{pmatrix} 1 \\ 1 \end{pmatrix} & \begin{pmatrix} 1 \\ 0 \end{pmatrix} & \begin{pmatrix} 1 \\ -1 \end{pmatrix} & \begin{pmatrix} 1 \\ -2 \end{pmatrix} & \begin{pmatrix} 1 \\ -3 \end{pmatrix} \\ \begin{pmatrix} 2 \\ 3 \end{pmatrix} & \begin{pmatrix} 2 \\ 2 \end{pmatrix} & \begin{pmatrix} 2 \\ 1 \end{pmatrix} & \begin{pmatrix} 2 \\ 0 \end{pmatrix} & \begin{pmatrix} 2 \\ -1 \end{pmatrix} & \begin{pmatrix} 2 \\ -2 \end{pmatrix} & \begin{pmatrix} 2 \\ -3 \end{pmatrix} \\ \begin{pmatrix} 3 \\ 3 \end{pmatrix} & \begin{pmatrix} 3 \\ 2 \end{pmatrix} & \begin{pmatrix} 3 \\ 1 \end{pmatrix} & \begin{pmatrix} 3 \\ 0 \end{pmatrix} & \begin{pmatrix} 3 \\ -1 \end{pmatrix} & \begin{pmatrix} 3 \\ -2 \end{pmatrix} & \begin{pmatrix} 3 \\ -3 \end{pmatrix} \end{pmatrix}$$

VectorPlot3D

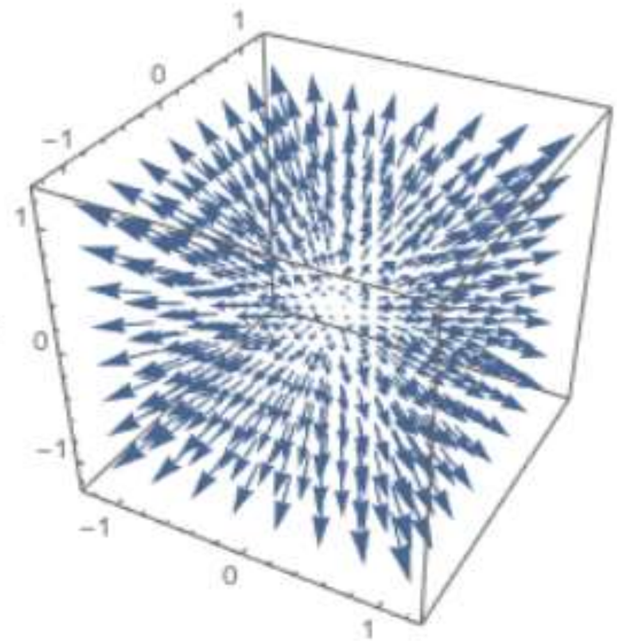
VectorPlot3D[{v_x, v_y, v_z}, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}, {z, z_{min}, z_{max}}]
generates a 3D vector plot of the vector field {v_x, v_y, v_z} as a function of x, y, and z.

VectorPlot3D[{$field_1$, $field_2$, ...}, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}, {z, z_{min}, z_{max}}]
plots several vector fields.

Plot a vector field:

```
In[1]:= VectorPlot3D[{x, y, z}, {x, -1, 1}, {y, -1, 1}, {z, -1, 1}]
```

Out[1]=



ParametricPlot

ParametricPlot[ ] draws the curve formed by a pair of expression

{x[t] , y[t]} as the parameter t varies.

ParametricPlot[{f_x, f_y}, {u, u_{min}, u_{max}}]
generates a parametric plot of a curve with x and y coordinates f_x and f_y as a function of u.

ParametricPlot[{{f_x, f_y}, {g_x, g_y}, ...}, {u, u_{min}, u_{max}}]
plots several parametric curves.

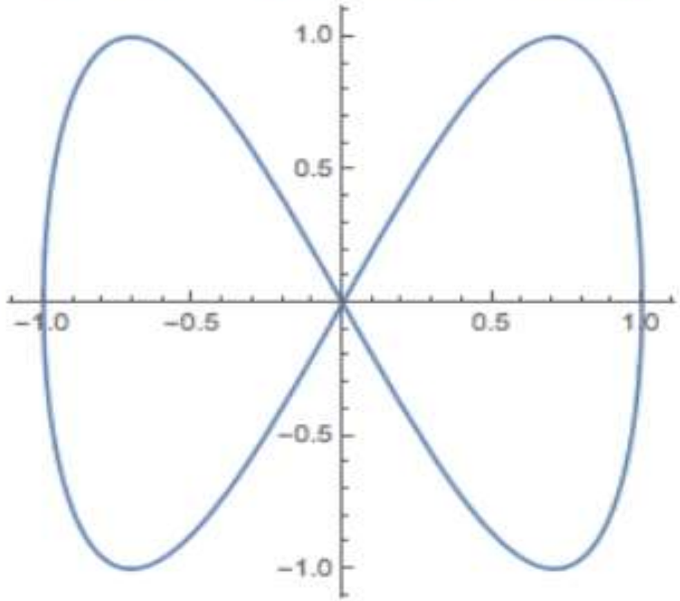
ParametricPlot[{f_x, f_y}, {u, u_{min}, u_{max}}, {v, v_{min}, v_{max}}]
plots a parametric region.

ParametricPlot[{{f_x, f_y}, {g_x, g_y}, ...}, {u, u_{min}, u_{max}}, {v, v_{min}, v_{max}}]
plots several parametric regions.

ParametricPlot[..., {u, v} ∈ reg]
takes parameters {u, v} to be in the geometric region reg.

In[1]:= `ParametricPlot[{Sin[u], Sin[2 u]}, {u, 0, 2 Pi}]`

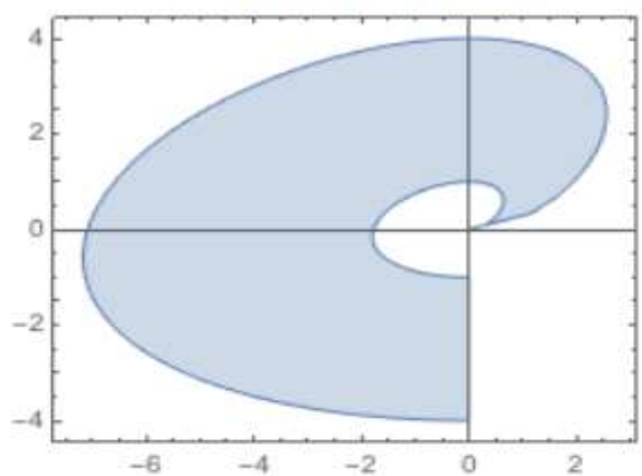
Out[1]=



Plot a parametric region:

In[1]:= `ParametricPlot[r^2 { Sqrt[t] Cos[t], Sin[t]}, {t, 0, 3 Pi / 2}, {r, 1, 2}]`

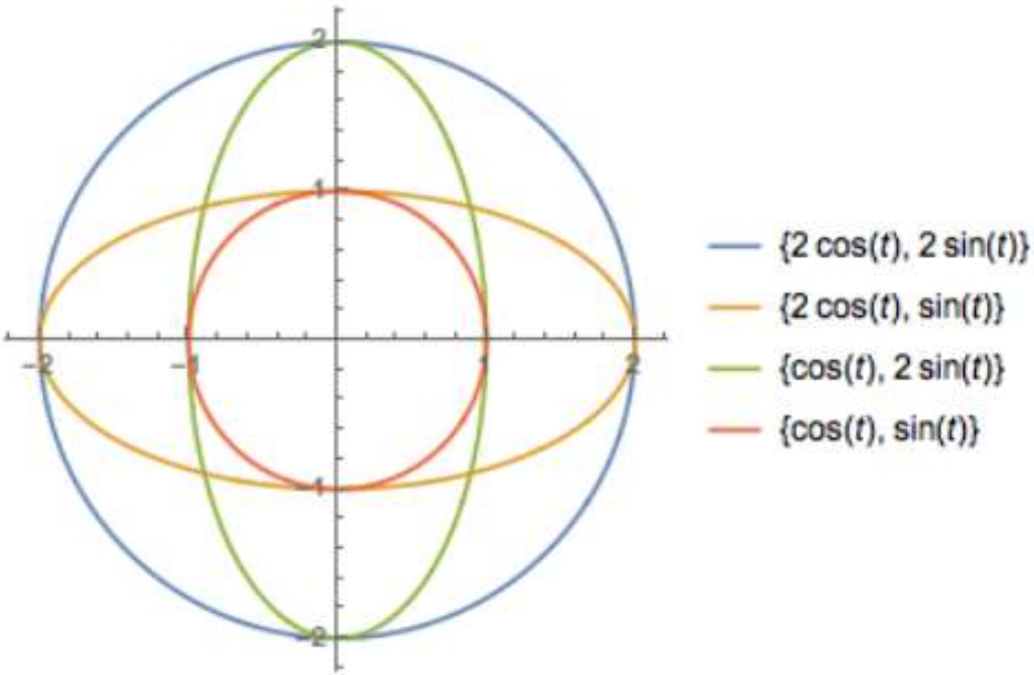
Out[1]=



Plot several parametric curves with a legend:

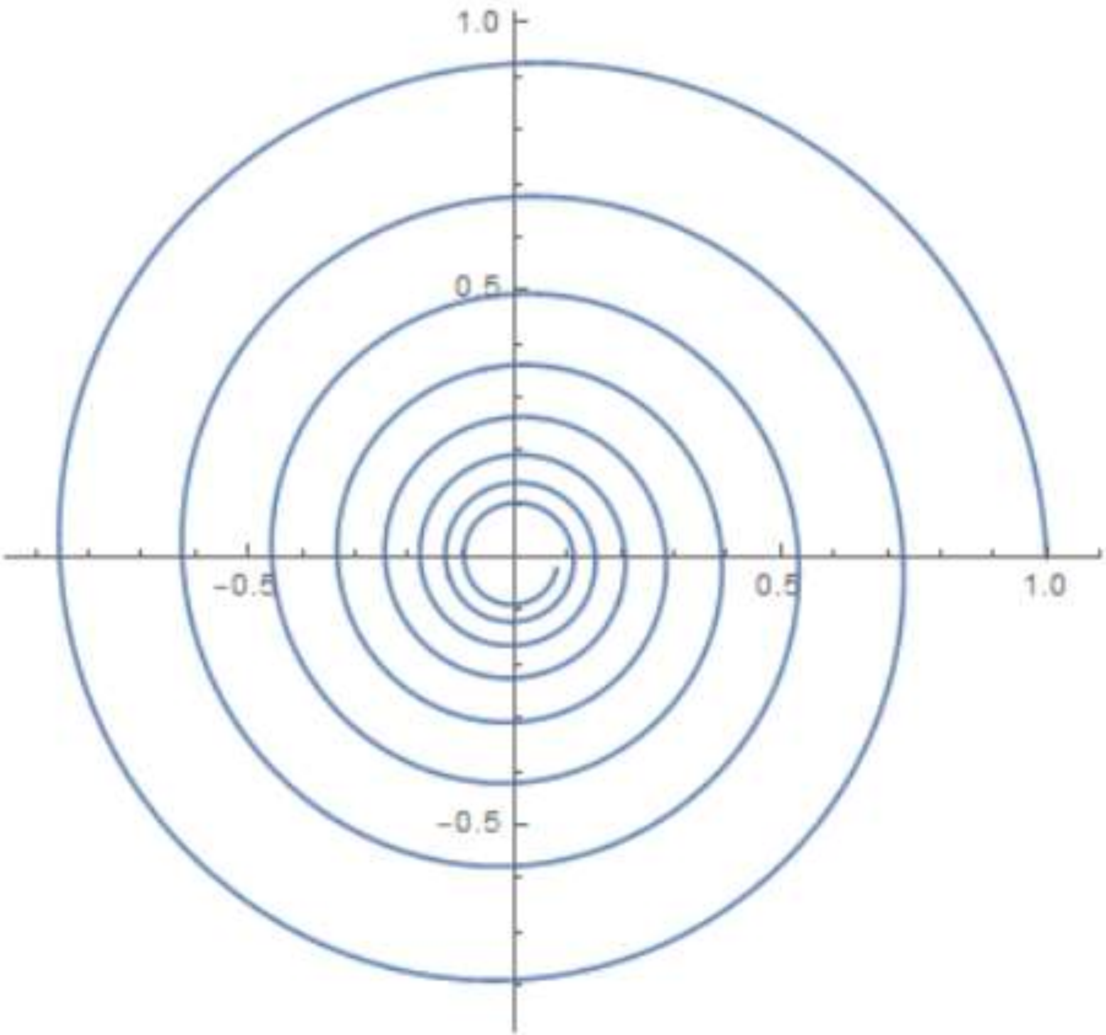
```
In[1]:= ParametricPlot[{{2 Cos[t], 2 Sin[t]}, {2 Cos[t], Sin[t]}, {Cos[t], 2 Sin[t]}, {Cos[t], Sin[t]}},
          {t, 0, 2 Pi}, PlotLegends → "Expressions"]
```

Out[1]=



In[58]:=
```
ParametricPlot[{Exp[-t / 20] Cos[t], Exp[-t / 20] Sin[t]}, {t, 0, 50}]
```

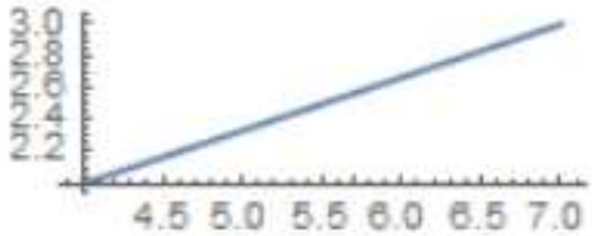
Out[58]=



Simple parametric curves including a line:

```
In[1]:= ParametricPlot[{3 u + 4, u + 2}, {u, 0, 1}]
```

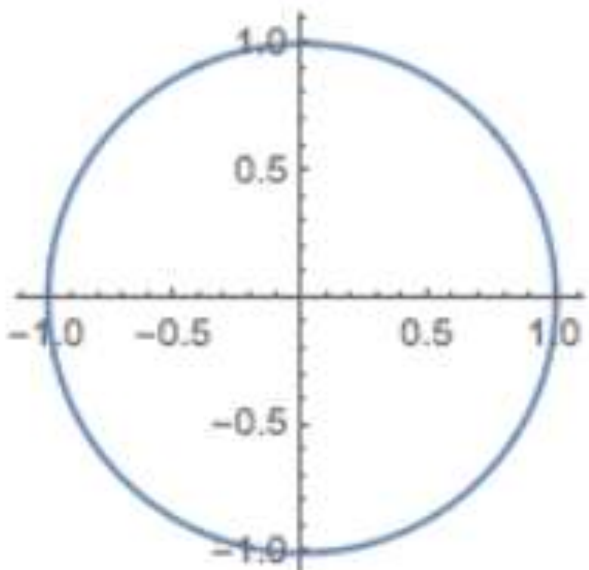
Out[1]=



Circle:

```
In[2]:= ParametricPlot[{Cos[u], Sin[u]}, {u, 0, 2 Pi}]
```

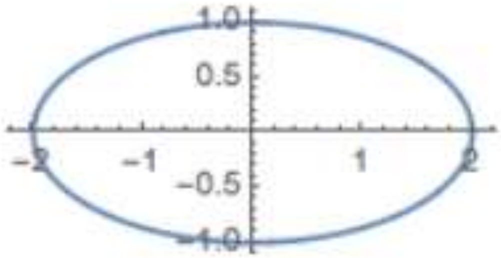
Out[2]=



Ellipse:

```
In[3]:= ParametricPlot[{2 Cos[u], Sin[u]}, {u, 0, 2 Pi}]
```

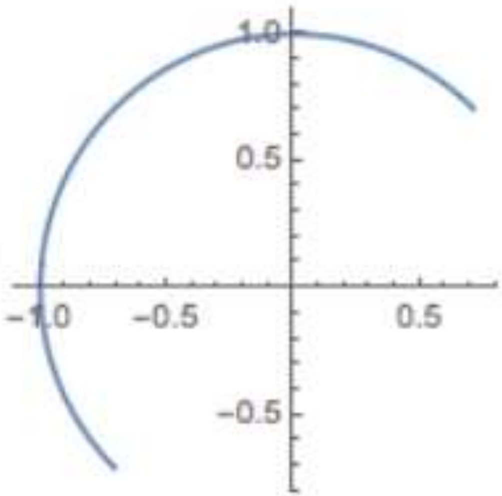
Out[3]=



Circle segment:

```
In[4]:= ParametricPlot[{Cos[u], Sin[u]}, {u, Pi / 4, 5 Pi / 4}]
```

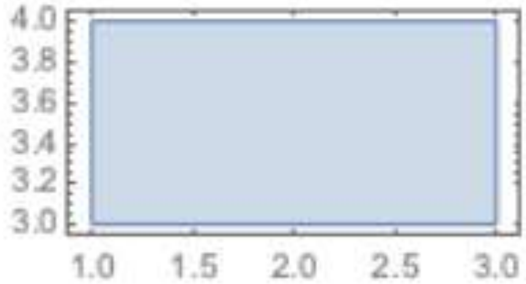
Out[4]=



Simple parametric regions including a rectangle:

```
In[1]:= ParametricPlot[{u, v}, {u, 1, 3}, {v, 3, 4}]
```

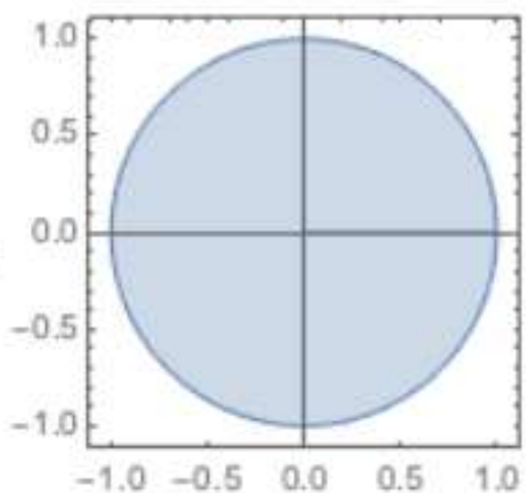
Out[1]=



Disk:

```
In[2]:= ParametricPlot[{v Cos[u], v Sin[u]}, {u, 0, 2 Pi}, {v, 0, 1}]
```

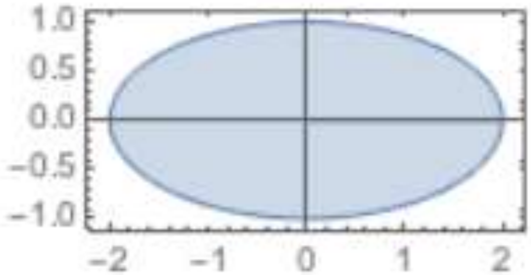
Out[2]=



Ellipse:

```
In[3]:= ParametricPlot[{2 v Cos[u], v Sin[u]}, {u, 0, 2 Pi}, {v, 0, 1}]
```

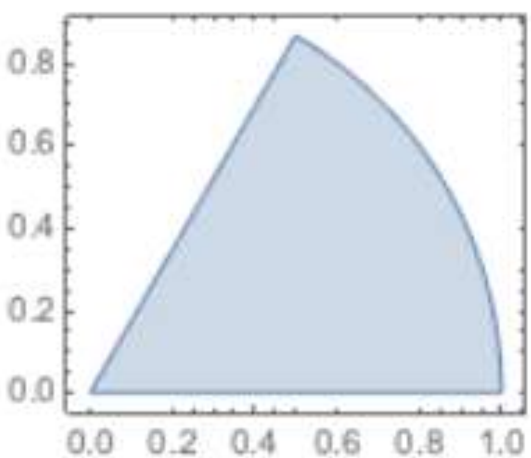
Out[3]=



Disk segment:

```
In[4]:= ParametricPlot[{v Cos[u], v Sin[u]}, {u, 0, Pi / 3}, {v, 0, 1}]
```

Out[4]=



ParametricPlot3D

This plots a parametrically defined three-dimensional curve (or a parametrically defined three-dimensional surface). It is part of the collection of Graphics packages, which must be loaded explicitly before it can be used,

ParametricPlot3D[{f_x, f_y, f_z}, {u, u_{min}, u_{max}}]

produces a three-dimensional space curve parametrized by a variable u which runs from u_{min} to u_{max}.

ParametricPlot3D[{f_x, f_y, f_z}, {u, u_{min}, u_{max}}, {v, v_{min}, v_{max}}]

produces a three-dimensional surface parametrized by u and v.

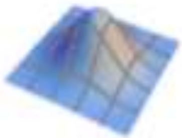
ParametricPlot3D[{{f_x, f_y, f_z}, {g_x, g_y, g_z} …} …]

plots several objects together.

ParametricPlot3D[…, {u, v} ∈ *reg*]

takes parameters {u, v} to be in the geometric region *reg*.

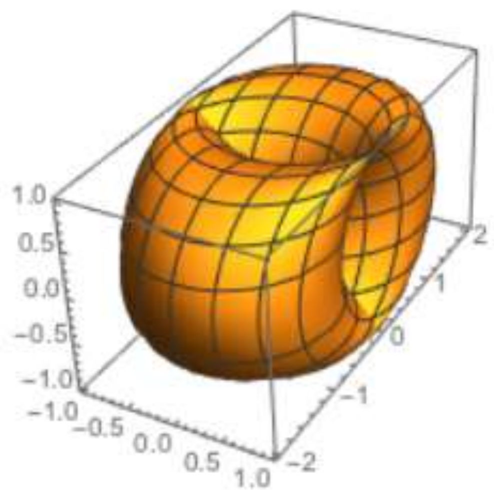
Themes that affect 3D surfaces include:

	"DarkMesh"	dark mesh lines
	"GrayMesh"	gray mesh lines
	"LightMesh"	light mesh lines
	"ZMesh"	vertical mesh lines
	"ThickSurface"	add thickness to surfaces

Plot a parametric surface:

```
In[1]:= ParametricPlot3D[{Cos[u], Sin[u] + Cos[v], Sin[v]}, {u, 0, 2 π}, {v, -π, π}]
```

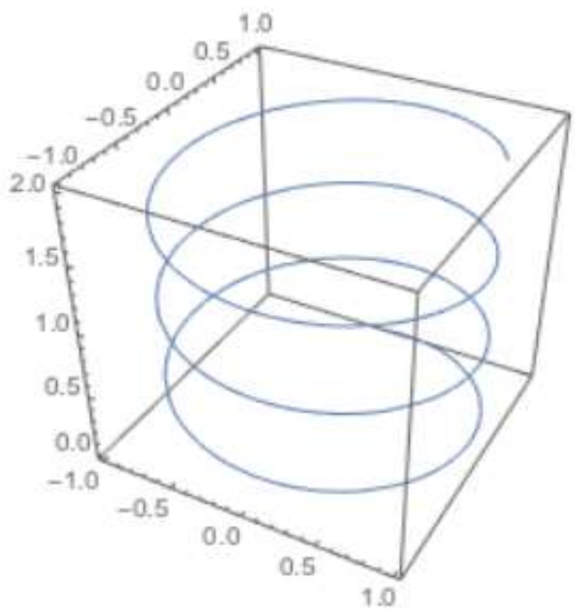
Out[1]=



Plot a parametric space curve:

```
In[1]:= ParametricPlot3D[{Sin[u], Cos[u], u / 10}, {u, 0, 20}]
```

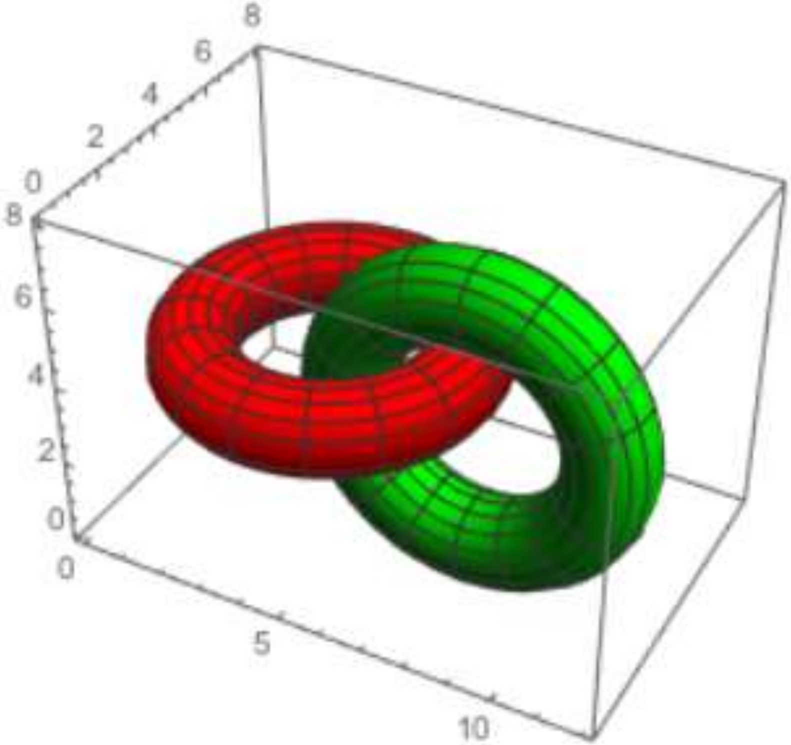
Out[1]=



Plot multiple parametric surfaces:

```
In[1]:= ParametricPlot3D[{{4 + (3 + Cos[v]) Sin[u], 4 + (3 + Cos[v]) Cos[u], 4 + Sin[v]},
          {8 + (3 + Cos[v]) Cos[u], 3 + Sin[v], 4 + (3 + Cos[v]) Sin[u]}},
         {u, 0, 2 Pi}, {v, 0, 2 Pi}, PlotStyle → {Red, Green}]
```

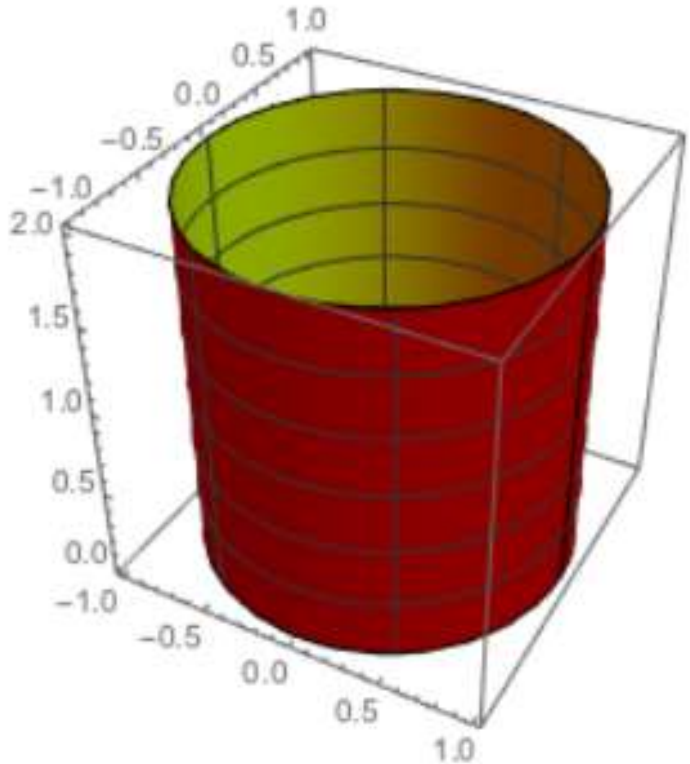
Out[1]=



Cylinder:

In[3]:=
```
ParametricPlot3D[{Cos[u], Sin[u], 2 v}, {u, 0, 2 Pi}, {v, 0, 1},
 Mesh → 5, BoundaryStyle → Black, PlotStyle → FaceForm[Red, Yellow]]
```

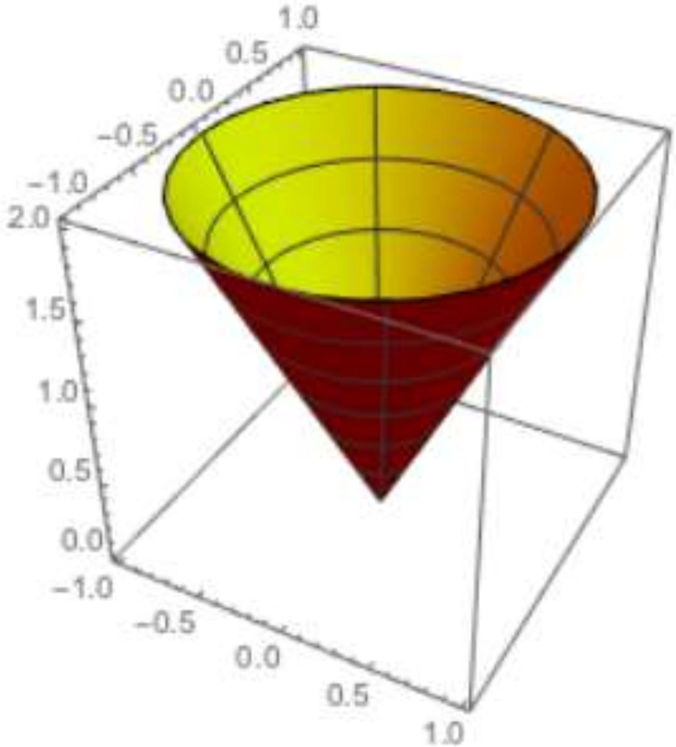
Out[3]=



Cone:

```
In[4]:= ParametricPlot3D[{v Cos[u], v Sin[u], 2 v}, {u, 0, 2 Pi},
  {v, 0, 1}, Mesh → 5, BoundaryStyle → Black, PlotStyle → FaceForm[Red, Yellow]]
```

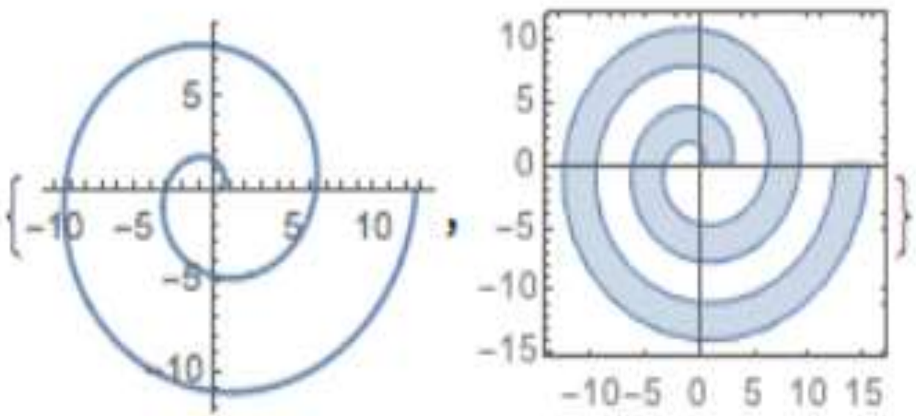
Out[4]=



Use ParametricPlot for curves and regions in two dimensions:

```
In[1]:= {ParametricPlot[{u Cos[u], u Sin[u]}, {u, 0, 4 Pi}],
         ParametricPlot[{(u + v) Cos[u], (u + v) Sin[u]}, {u, 0, 4 Pi}, {v, 0, 3}]}
```

Out[1]=



SphericalPlot3D

SphericalPlot3D[r, θ, ϕ]
generates a 3D plot with a spherical radius r as a function of spherical coordinates θ and ϕ.

SphericalPlot3D[r, {θ, θ_{min}, θ_{max}}, {ϕ, ϕ_{min}, ϕ_{max}}]
generates a 3D spherical plot over the specified ranges of spherical coordinates.

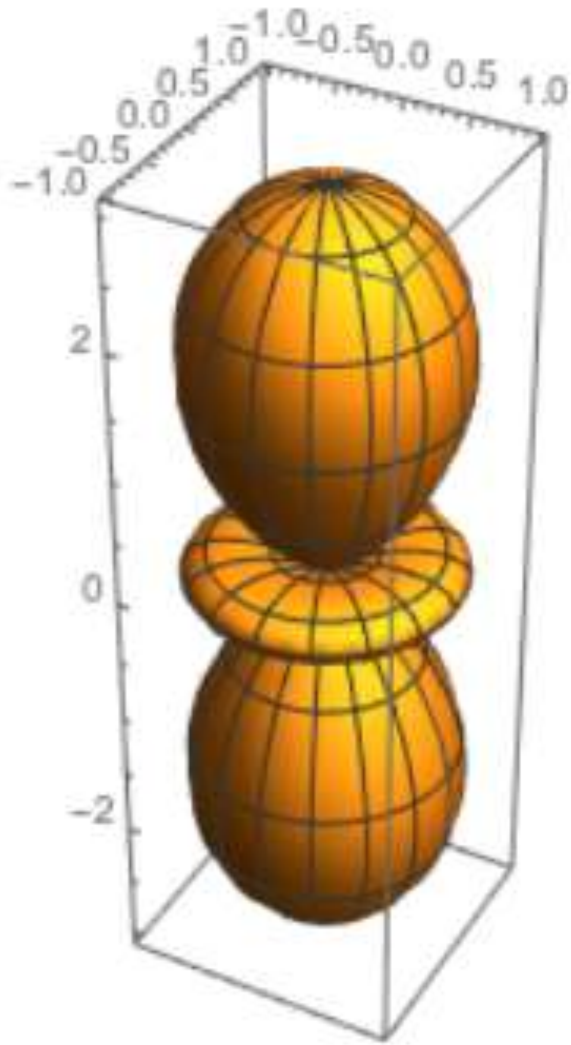
SphericalPlot3D[{r_1, r_2, …}, {θ, θ_{min}, θ_{max}}, {ϕ, ϕ_{min}, ϕ_{max}}]
generates a 3D spherical plot with multiple surfaces.

- The angles θ and ϕ are measured in radians.
- $\pi/2-\theta$ corresponds to "latitude"; θ is 0 at the "north pole", and π at the "south pole".
- ϕ corresponds to "longitude", varying from 0 to 2π counterclockwise looking from the north pole.
- SphericalPlot3D[r, θ, ϕ] takes θ to have range 0 to π, and ϕ to have range 0 to 2π.
- The x, y, z position corresponding to r, θ, ϕ is $r\sin(\theta)\cos(\phi)$, $r\sin(\theta)\sin(\phi)$, $r\cos(\theta)$. The variables θ and ϕ can have any values. The surfaces they define can overlap radially.

Plot a spherical surface:

```
In[1]:= SphericalPlot3D[1 + 2 Cos[2 θ], {θ, 0, Pi}, {ϕ, 0, 2 Pi}]
```

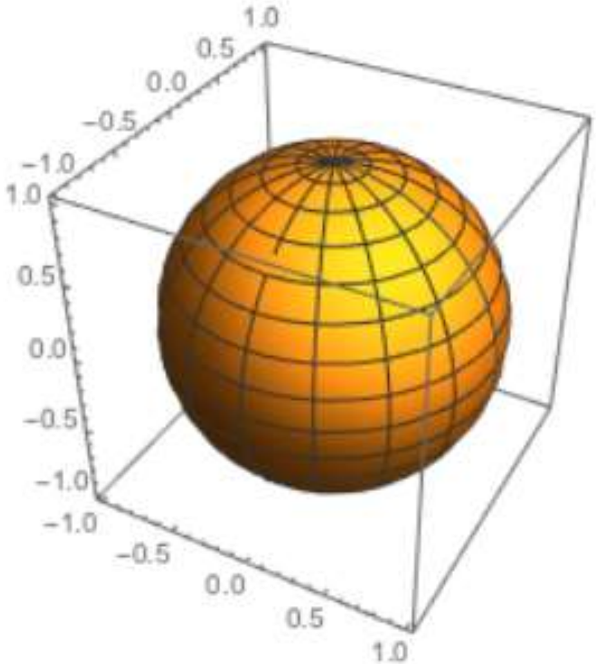
Out[1]=



Plot a sphere:

```
In[1]:= SphericalPlot3D[1, {θ, 0, Pi}, {ϕ, 0, 2 Pi}]
```

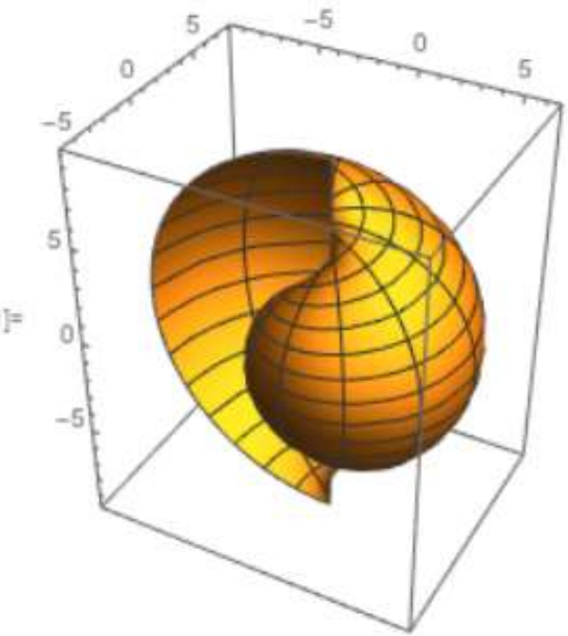
Out[1]=



A spiraling shell:

```
In[1]:= SphericalPlot3D[ϕ, {θ, 0, Pi}, {ϕ, 0, 3 Pi}]
```

Out[1]=



SphericalHarmonicY

SphericalHarmonicY[l, m, θ, ϕ]
gives the spherical harmonic $Y_l^m(\theta, \phi)$.

- Mathematical function, suitable for both symbolic and numerical manipulation.
- The spherical harmonics are orthonormal with respect to integration over the surface of the unit sphere.
- For $l \geq 0$, $Y_l^m(\theta, \phi) = \sqrt{(2l+1)/(4\pi)}\ \sqrt{(l-m)!/(l+m)!}\ P_l^m(\cos(\theta))\ e^{i m \phi}$ where P_l^m is the associated Legendre function.
- For $l \leq -1$, $Y_l^m(\theta, \phi) = Y_{-(l+1)}^m(\theta, \phi)$.

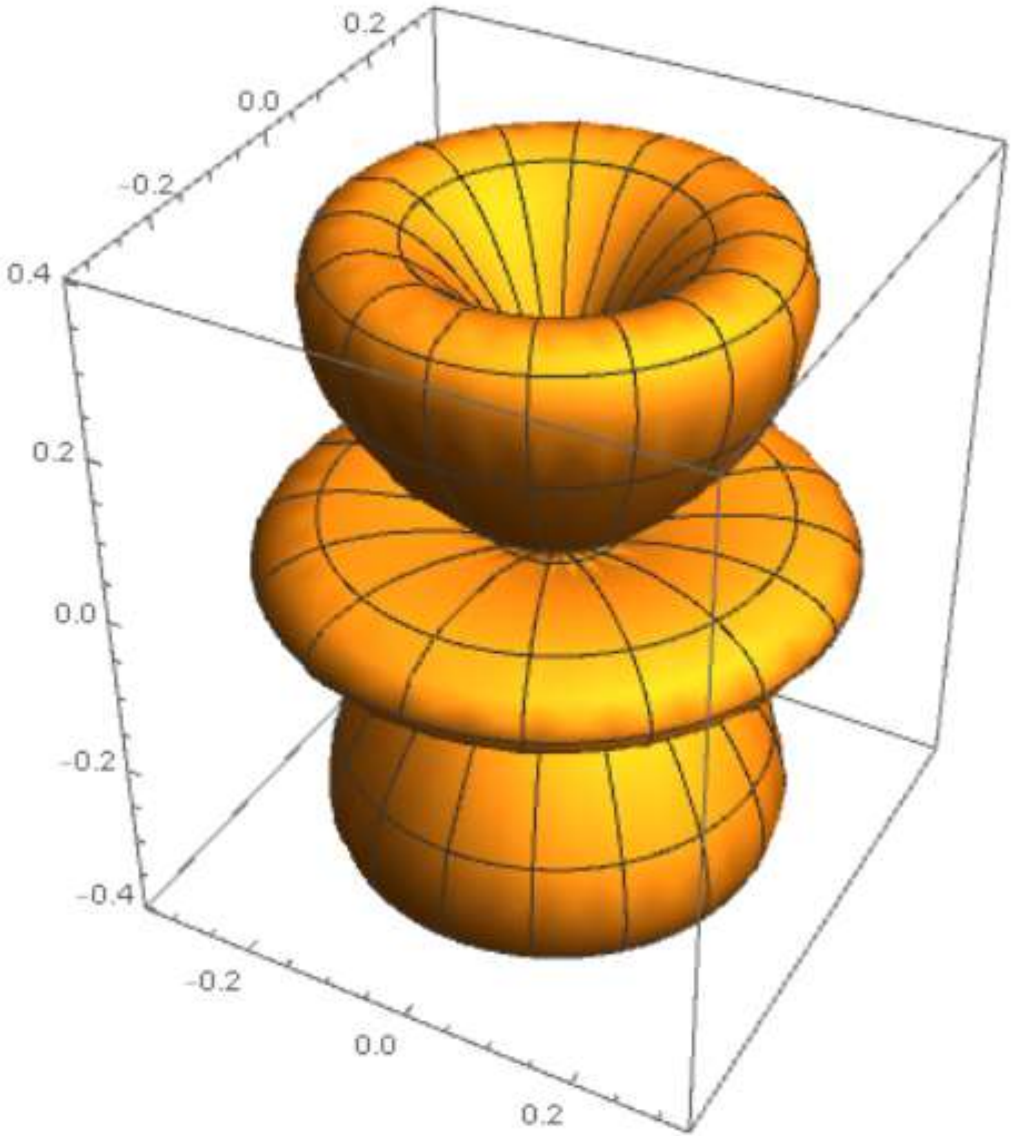
In[1]:= `SphericalHarmonicY[3, 1, θ, ϕ]`

Out[1]= $-\frac{1}{8} e^{i \phi} \sqrt{\frac{21}{\pi}} \left(-1 + 5 \operatorname{Cos}[\theta]^2\right) \operatorname{Sin}[\theta]$

In[1]:= `SphericalHarmonicY[n, m, θ, ϕ] // TraditionalForm`

Out[1]//TraditionalForm=
$Y_n^m(\theta, \phi)$

```
SphericalPlot3D[Abs[SphericalHarmonicY[3, 1, theta, phi]], {theta, 0, Pi}, {phi, 0, 2 Pi}]
```



```
SphericalPlot3D[Abs[SphericalHarmonicY[5, 1, theta, phi]], {theta, 0, Pi}, {phi, 0, 2 Pi}]
```