# GRAPHICS

# Three-dimensional plots Table List List plots

## Three Dimensional Plotting

For creating three dimensional plots, use

## Plot3D[]

The arguments of Plot3D[] are similar to those of the function Plot[].

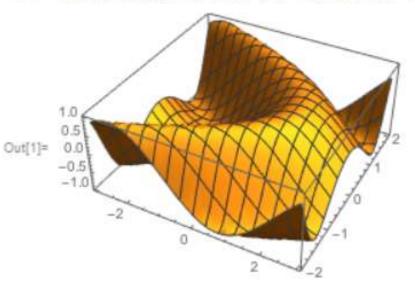
Plot3D[f, {x, x<sub>min</sub>, x<sub>max</sub>}, {y, y<sub>min</sub>, y<sub>max</sub>}]
generates a three-dimensional plot of f as a function of x and y.

Plot3D[ $\{f_1, f_2, ...\}, \{x, x_{min}, x_{max}\}, \{y, y_{min}, y_{max}\}$ ] plots several functions.

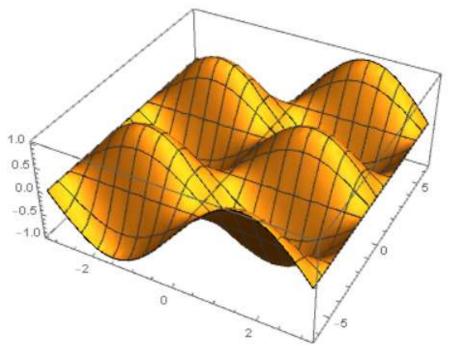
Plot3D[..., {x, y} ∈ reg]
takes variables {x, y} to be in the geometric region reg.

Plot a function:

 $\ln[1] = Plot3D[Sin[x + y^2], \{x, -3, 3\}, \{y, -2, 2\}]$ 



Plot3D[Sin[x] Cos[y], {x, -Pi, Pi}, {y, -2Pi, 2Pi}]



# PlotPoints

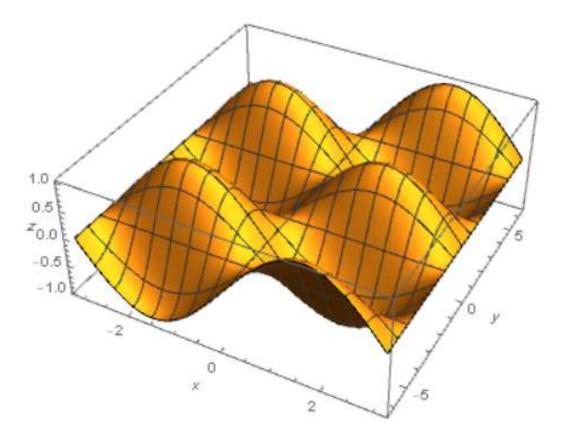
#### PlotPoints

is an option for plotting functions that specifies how many initial sample points to use.

## ✓ Details

- With a single variable, PlotPoints -> n specifies the total number of initial sample points to use.
- With more than one variable, PlotPoints -> n specifies that n initial points should be used in each direction.
- PlotPoints -> {n1, n2, ...} specifies different numbers of initial sample points for each successive direction.
- The initial sample points are usually equally spaced.

Plot3D[Sin[x] Cos[y], {x, -Pi, Pi}, {y, -2Pi, 2Pi}, AxesLabel -> {x, y, z}, PlotPoints -> 40]



## Mesh

#### Mesh

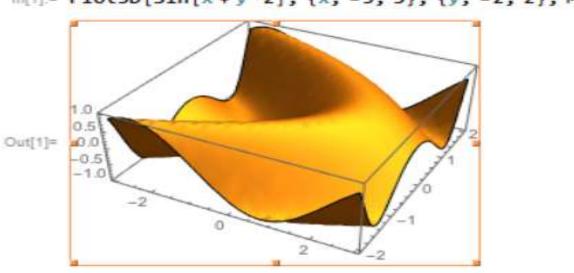
is an option for Plot3D, DensityPlot, and other plotting functions that specifies what mesh should be drawn.

## ✓ Details

The following settings can be given for Mesh:

None	no mesh drawn
п	n equally spaced mesh divisions
All	mesh divisions between all elements
Full	mesh divisions between regular data points
{ <i>spec</i> <sub>1</sub> , <i>spec</i> <sub>2</sub> ,}	separate specifications for each mesh function

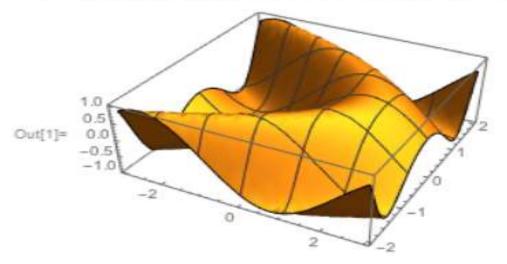
Use no mesh:



in[1]:= Plot3D[Sin[x + y^2], {x, -3, 3}, {y, -2, 2}, Mesh -> None]

#### Use 5 mesh lines in each direction:

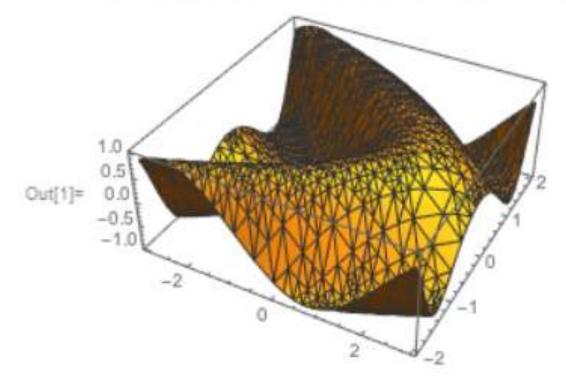
```
ln[1] = Plot3D[Sin[x + y^2], \{x, -3, 3\}, \{y, -2, 2\}, Mesh \rightarrow 5]
```



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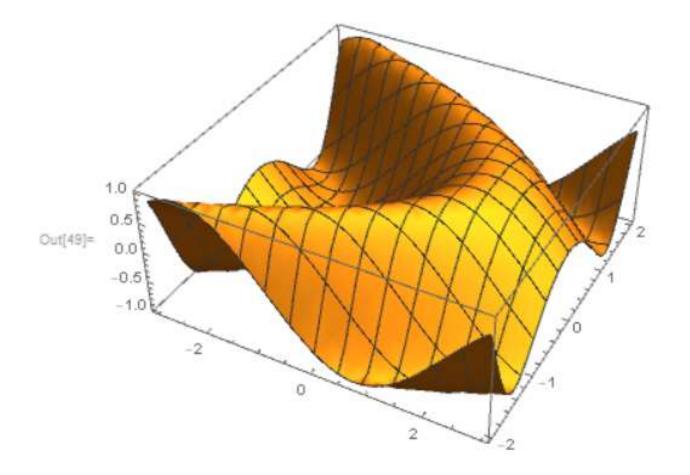
Show all mesh elements used in the sampling:

in[1]= Plot3D[Sin[x + y^2], {x, -3, 3}, {y, -2, 2}, Mesh -> All]



In[49]:=

#### Plot3D[Sin[x + y^2], {x, -3, 3}, {y, -2, 2}, Mesh → Full]



# MaxRecursion

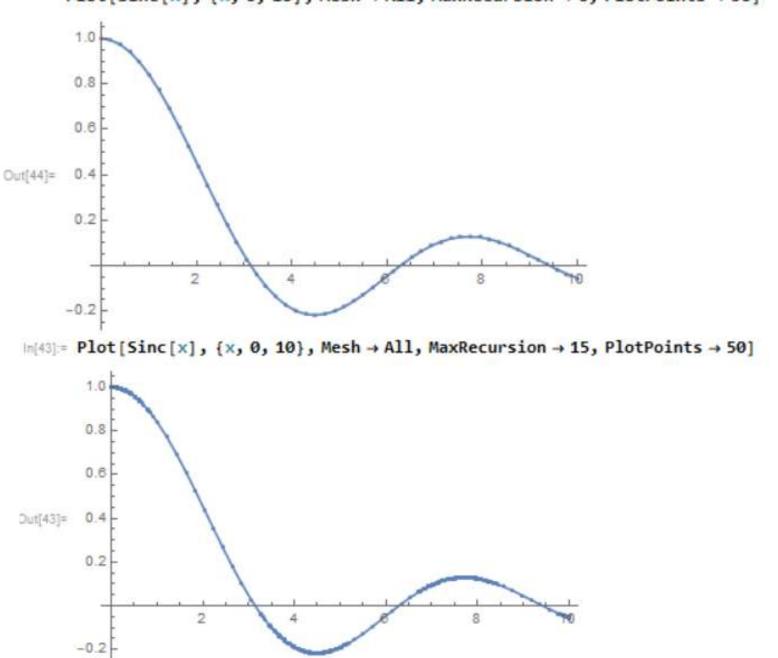
#### MaxRecursion

is an option for functions like NIntegrate and Plot that specifies how many recursive subdivisions can be made.

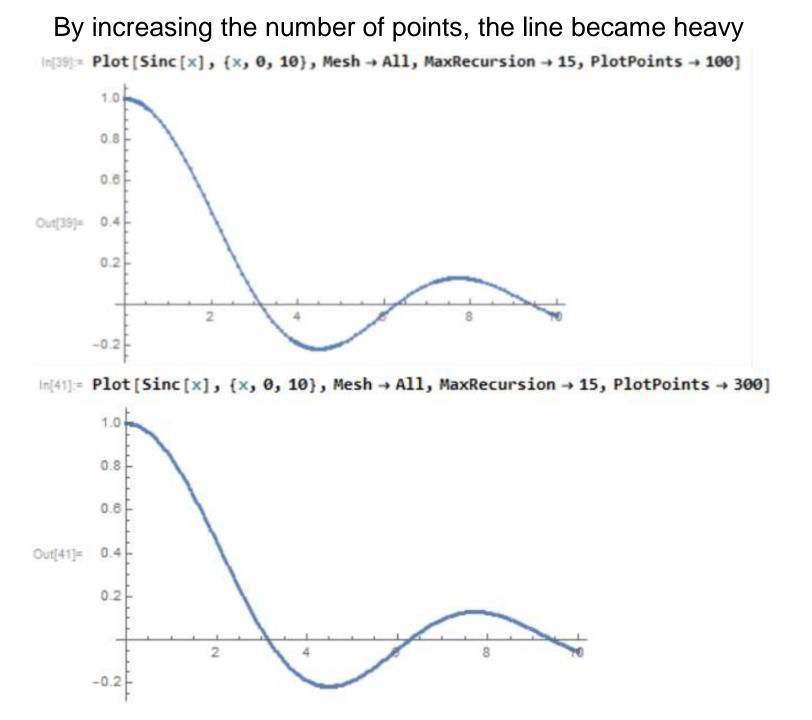
## ✓ Details

- MaxRecursion -> n specifies that up to n levels of recursion should be done.
- Recursive subdivision is done only in those places where more samples seem to be needed in order to achieve results with a
  certain level of quality.
- In d dimensions, each recursive subdivision increases the number of samples taken by a factor that increases roughly
  exponentially with d.

In[44];=

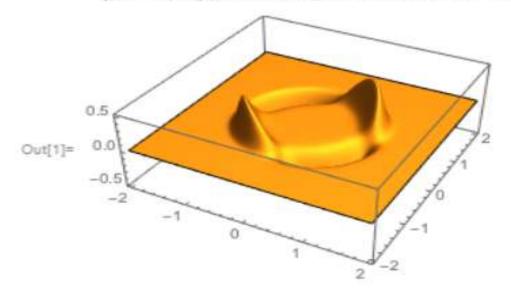


Plot[Sinc[x], {x, 0, 10}, Mesh → All, MaxRecursion → 0, PlotPoints → 50]



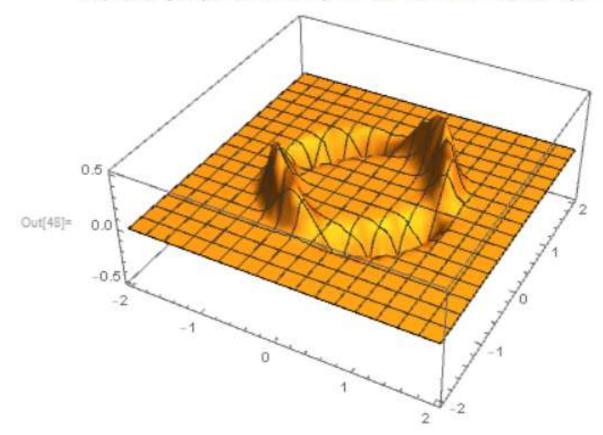
Get a very high-quality plot of a sharp feature:

```
 [1] = Plot3D[x y Exp[-10 (x^2 + y^2 - 1)^2], \{x, -2, 2\}, \\ \{y, -2, 2\}, PlotRange \rightarrow All, Mesh \rightarrow False, MaxRecursion \rightarrow 5]
```



In[48]:=

 $\label{eq:plot3D[xyExp[-10 (x^2 + y^2 - 1)^2], {x, -2, 2}, {y, -2, 2}, PlotRange \rightarrow All]}$ 



## Table

Table[*expr*, *n*] generates a list of *n* copies of *expr*.

Table[*expr*, {*i*, *i*<sub>max</sub>}] generates a list of the values of *expr* when *i* runs from 1 to *i*<sub>max</sub>.

Table[expr,  $\{i, i_{min}, i_{max}\}$ ] starts with  $i = i_{min}$ .

Table[*expr*, {*i*, *i*<sub>min</sub>, *i*<sub>max</sub>, *di*}] uses steps *di*.

Table[*expr*,  $\{i, \{i_1, i_2, ...\}\}$ ] uses the successive values  $i_1, i_2, ..., i_n$ 

Table[*expr*, {*i*, *i*<sub>min</sub>, *i*<sub>max</sub>}, {*j*, *j*<sub>min</sub>, *j*<sub>max</sub>}, ...] gives a nested list. The list associated with *i* is outermost. »

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### ✓ Details

- You can use Table to build up vectors, matrices, tensors, and other arrays.
- Table uses the standard Wolfram Language iteration specification.
- Table evaluates its arguments in a nonstandard way.
- Table[expr, spec] first evaluates spec, then localizes the variable specified and successively assigns values to it, each time
  evaluating expr.
- Table effectively uses Block to localize values or variables.
- Table[expr, spec1, spec2] is effectively equivalent to Table[Table[expr, spec2], spec1].

```
A table of the first 10 squares:
```

```
In[1]:= Table[i^2, {i, 10}]
Out[1]= {1, 4, 9, 16, 25, 36, 49, 64, 81, 100}
```

A table with *i* running from 0 to 20 in steps of 2:

```
In[1]= Table[f[i], {i, 0, 20, 2}]
Out[1]= {f[0], f[2], f[4], f[6], f[8], f[10], f[12], f[14], f[16], f[18], f[20]}
```

### Example

Use Mathematica to generate the list {1,2,3,4,5,6,7,8,9,10}.

Table[i, {i, 10}]

 $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ 

Table[i, {i, 1, 10}]

 $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ 

#### Example

Use Mathematica to define listone to be the list of numbers {1, 3/2, 2, 5/2, 3, 7/2, 4}.

listone = 
$$\left\{1, \frac{3}{2}, 2, \frac{5}{2}, 3, \frac{7}{2}, 4\right\}$$
 listone = Table  $\left[i, \left\{i, 1, 4, \frac{1}{2}\right\}\right]$   
Last, we define  $i(n) = \frac{1}{2}n + \frac{1}{2}$  and use Array to create the table listone.  
 $i[n_1] = \frac{n}{2} + \frac{1}{2};$ 

listone = Array[i, 7]

### Example

Create a list of the first 25 prime numbers. What is the fifteenth prime number?

```
list = Table[{n, Prime[n]}, {n, 1, 25}];
```

#### Short[list]

 $\{\{1,2\},\{2,3\},\{3,5\},\{4,7\},\{5,11\},\{6,13\},\langle\langle 13\rangle\rangle,\{20,71\},\{21,73\},\{22,79\},\{23,83\},$ 

 $\{24, 89\}, \{25, 97\}\}$ 

## list[[15]]

{15, 47}

#### Example

(a) Generate a list consisting of five copies of the letter a. (b) Generate a list consisting of ten random integers between -10 and 10 and then a list of ten random real numbers between -10 and 10.

#### Clear[a]

Table[a, {5}]	RandomInteger[{-10, 10}, 10]
$\{a, a, a, a, a\}$	RandomReal[{-10, 10}, 10]

You can also use Take to extract elements of lists.

- 1. Take[list,n] returns the first *n* elements of list;
- 2. Take[list, -n] returns the last *n* elements of list; and
- 3. Take[list, {n,m}] returns the *n*th through *m*th elements of list.

## Take[t1, 5] Take[t1, -5]

## Example The Prime Difference Function and the Prime Number Theorem

In t1, we use Prime and Table to compute a list of the first 25,000 prime numbers.

 $t1 = Table[Prime[n], \{n, 1, 25000\}];$ 

Length[t1]

## Short[t1]

## Take[t1, {12501, 12505}]

In t2, we compute the difference,  $d_n$ , between the successive prime numbers in t1. The result is plotted with ListPlot in Fig. 4.4.

```
t_2 = Table[t_1[[i + 1]] - t_1[[i]], \{i, 1, Length[t_1] - 1\}];
```

## Short[t2]

#### $t1 = Table[{Sin[x + y], Cos[x - y]}, {x, 1, 5}, {y, 1, 5}]$

 $\{\{ Sin[2], 1\}, \{ Sin[3], Cos[1] \}, \{ Sin[4], Cos[2] \}, \{ Sin[5], Cos[3] \}, \{ Sin[6], Cos[4] \} \}, \\ \{ Sin[3], Cos[1] \}, \{ Sin[4], 1 \}, \{ Sin[5], Cos[1] \}, \{ Sin[6], Cos[2] \}, \{ Sin[7], Cos[3] \} \}, \\ \{ \{ Sin[4], Cos[2] \}, \{ Sin[5], Cos[1] \}, \{ Sin[6], 1 \}, \{ Sin[7], Cos[1] \}, \{ Sin[8], Cos[2] \} \}, \\ \{ \{ Sin[5], Cos[3] \}, \{ Sin[6], Cos[2] \}, \{ Sin[7], Cos[1] \}, \{ Sin[8], 1 \}, \{ Sin[9], Cos[1] \} \}, \\ \{ \{ Sin[6], Cos[4] \}, \{ Sin[7], Cos[3] \}, \{ Sin[8], Cos[2] \}, \{ Sin[9], Cos[1] \}, \{ Sin[10], 1 \} \} \}$ 

### Length[t1]

## 5 t1[[3]]

{{Sin[4], Cos[2]}, {Sin[5], Cos[1]}, {Sin[6], 1}, {Sin[7], Cos[1]}, {Sin[8], Cos[2]}}

and the 2nd element of the third level (or the second part of the third part) is

#### t1[[3,2]]

{Sin[5], Cos[1]}

## Example Dynamical Systems

A sequence of the form  $x_{n+1} = f(x_n)$  is called a **dynamical system**.

Sometimes, unusual behavior can be observed when working with dynamical systems. For example, consider the dynamical system with f(x) = x + 2.5x(1 - x) and  $x_0 = 1.2$ . Note that we define  $x_n$  using the form  $x[n_]:=x[n]=...$  so that Mathematica "remembers" the functional values it computes and thus avoids recomputing functional values previously computed. This is particularly advantageous when we compute the value of  $x_n$  for large values of n.

## Clear[f, x]

 $f[x_]:=x+2.5x(1-x)$ 

 $x[n_{:=}x[n] = f[x[n-1]]$ 

x[0] = 1.2;

```
in[1]= Table[x, 10]
Out[1]= {x, x, x, x, x, x, x, x, x, x, x}
Make a 4×3 matrix:
```

```
in[1]:= Table[10i+j, {i, 4}, {j, 3}]
Out[1]= { {11, 12, 13}, {21, 22, 23}, {31, 32, 33}, {41, 42, 43} ]
```

```
In[2]= MatrixForm[%]
                 11 12 13
                 21 22 23
Out[2]//MatrixForm=
                 31 32 33
                 41 42 43
      Plot a table:
      In[1]:= ListPlot[Table[Prime[i], {i, 50}]]
            200
            150
      Out[1]= 100
             50
                      20
                   10
                           30
                               40
                                   50
       Arrange a table in a column:
       In[1]= Column[Table[Prime[i], {i, 5}]]
             2
             3
       Out[1]= 5
             7
```

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The index in the table can run backward:

```
In[1]:= Table[f[i], {i, 10, -5, -2}]
Out[1]= {f[10], f[8], f[6], f[4], f[2], f[0], f[-2], f[-4]}
```

```
Make a triangular array:
        In[1]:= Table[10i+j, {i, 5}, {j, i}]
       Out[1] = \{\{11\}, \{21, 22\}, \{31, 32, 33\}, \{41, 42, 43, 44\}, \{51, 52, 53, 54, 55\}\}
       In[2]:= TableForm[%]
              11
              21
                    22
Dut[2]//TableForm= 31
                    32
                         33
                                                                        TableForm[%]
                    42
                          43
              41
                                 44
              51
                    52
                           53
                                 54
                                        55
                                                                        2
                                                                        3
                                                                            4
    Table[j+i, {i, 5}, {j, i}]
                                                                               6
                                                                       4
                                                                            5
                                                                            6
                                                                        5
                                                                                 7
                                                                                       8
     \{2\}, \{3, 4\}, \{4, 5, 6\}, \{5, 6, 7, 8\}, \{6, 7, 8, 9, 10\}
                                                                       6
                                                                            7
                                                                                 8
                                                                                       9
```

Make a 3×2×4 array, or tensor:

```
In[1]= Table[100 i + 10 j + k, {i, 3}, {j, 2}, {k, 4}]
Out[1]= {{{111, 112, 113, 114}, {121, 122, 123, 124}},
{{211, 212, 213, 214}, {221, 222, 223, 224}}, {{311, 312, 313, 314}, {321, 322, 323, 324}}}
```

Iterate over an existing list:

```
in[1]= Table[Sqrt[x], {x, {1, 4, 9, 16}}]
Out[1]= {1, 2, 3, 4}
```

Make an array from existing lists:

 $In[1] = Table[j^{(1/i)}, \{i, \{1, 2, 4\}\}, \{j, \{1, 4, 9\}\}]$  $Out[1] = \{\{1, 4, 9\}, \{1, 2, 3\}, \{1, \sqrt{2}, \sqrt{3}\}\}$ 

Table evaluates the expression separately each time:

In[1]= Table[RandomInteger[10], 20]
Out[1]= {7, 7, 2, 7, 8, 5, 5, 4, 10, 6, 0, 4, 6, 8, 6, 1, 2, 1, 8, 9}

The table index can have symbolic values:

```
In[1] = Table[2^{x} + x, \{x, a, a + 5n, n\}]
Out[1] = \left\{2^{a} + a, 2^{a+n} + a + n, 2^{a+2n} + a + 2n, 2^{a+3n} + a + 3n, 2^{a+4n} + a + 4n, 2^{a+5n} + a + 5n\right\}
```

The variables need not just be symbols:

```
in[1]:= Table[a[x]!, {a[x], 6}]
```

```
Out[1]= {1, 2, 6, 24, 120, 720}
```

```
In[2]:= Table[x[1]^2 + x[2]^2, {x[1], 3}, {x[2], 3}]
```

```
Out[2]= { { 2, 5, 10 }, { 5, 8, 13 }, { 10, 13, 18 } }
```

```
In[1]:= Grid[Table[{i, Prime[i]}, {i, 10}]]
```

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Monitor the values by showing them in a temporary cell:

```
In[1]:= Monitor[Table[Pause[1]; i^i^i, {i, 3}], i]
```

```
Out[1]= {1, 16, 7625597484987}
```

Range gives the sequence of values of a table iterator:

```
in[1]:= Range[1, 10, 2]
```

Out[1]= {1, 3, 5, 7, 9}

```
In[2]:= Table[i, {i, 1, 10, 2}]
```

Out[2]= {1, 3, 5, 7, 9}

Do evaluates the same sequence of expressions as Table, but does not return them:

```
In[1]= Do[Print[i^i], {i, 3}]
1
4
27
In[2]= Table[i^i, {i, 3}]
Out[2]= {1, 4, 27}
```

Sum effectively applies Plus to results from Table:

```
ln[1] = Sum[x^i, \{i, 3\}]
Out[1] = x + x^2 + x^3
ln[2] = Table[x^i, \{i, 3\}]
Out[2] = \{x, x^2, x^3\}
ln[2] = Array[Function[\{x, y\}, x^y], \{3, 4\}]
Out[2] = \{\{1, 1, 1, 1\}, \{2, 4, 8, 16\}, \{3, 9, 27, 81\}\}
ln[3] = Table[x^y, \{x, 3\}, \{y, 4\}]
```

```
Out[3]= { {1, 1, 1, 1}, {2, 4, 8, 16 }, {3, 9, 27, 81 } }
```

Using multiple iteration specifications is equivalent to nesting Table functions:

```
in[1]:= Table[i+j, {i, 3}, {j, i}]
```

```
Out[1]= \{\{2\}, \{3, 4\}, \{4, 5, 6\}\}
```

```
In[2]:= Table[Table[i+j, {j, i}], {i, 3}]
Out[2]= { {2}, {3, 4}, {4, 5, 6} }
```

Formatting wrappers such as Grid give expressions that are no longer lists:

```
In[1] = Grid[Table[ij, \{i, 4\}, \{j, 4\}]]
I = 2 3 4
2 4 6 8
3 6 9 12
4 8 12 16
In[2] = x + %
I = 2 3 4
2 4 6 8
Out[2] = x + %
I = 2 3 4
2 4 6 8
3 6 9 12
4 8 12 16
```

For some step sizes, output from Table may not include the upper limit given:

```
in[1]:= Table[x, {x, 0, 10, 3}]
Out[1]= {0, 3, 6, 9}
```

#### MatrixForm[%]

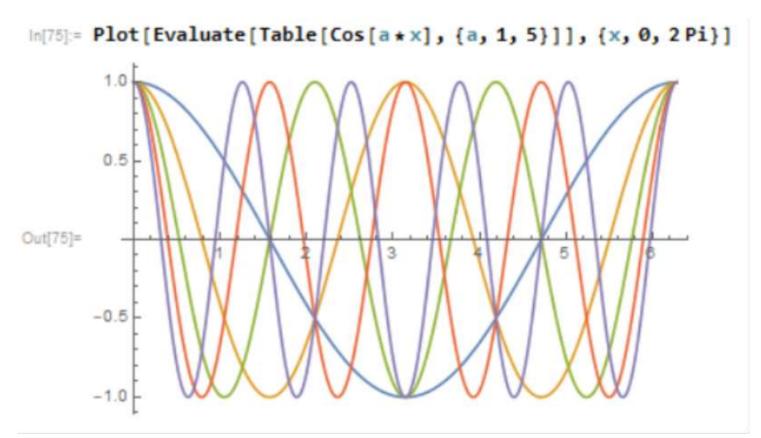
```
0
3
6
9
```

## **Evaluate command**

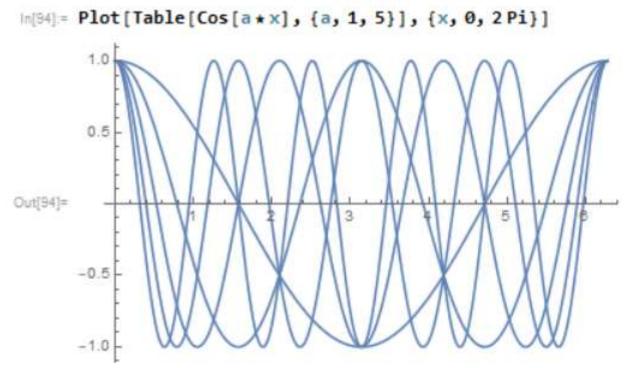
A list of expressions can be given to Mathematica to plot using the

### Table[]

In such case, Evaluate command forces evaluation of the command.



## The output without applying Evaluate



## The values of a and x:

```
Table[{a, x}, {a, 1, 5}, {x, 0, 2Pi}]
```

```
 \{ \{ \{1, 0\}, \{1, 1\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 5\}, \{1, 6\} \}, \\ \{ \{2, 0\}, \{2, 1\}, \{2, 2\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{2, 6\} \}, \\ \{ \{3, 0\}, \{3, 1\}, \{3, 2\}, \{3, 3\}, \{3, 4\}, \{3, 5\}, \{3, 6\} \}, \\ \{ \{4, 0\}, \{4, 1\}, \{4, 2\}, \{4, 3\}, \{4, 4\}, \{4, 5\}, \{4, 6\} \}, \\ \{ \{5, 0\}, \{5, 1\}, \{5, 2\}, \{5, 3\}, \{5, 4\}, \{5, 5\}, \{5, 6\} \} \}
```

## The output values Cos [a\*x]

## The numeric values Cos [a\*x]

## List ({...})

{e<sub>1</sub>, e<sub>2</sub>, ...} is a list of elements.

### ✓ Details

- Lists are very general objects that represent collections of expressions.
- Functions with attribute Listable are automatically "threaded" over lists, so that they act separately on each list element.
   Most built-in mathematical functions are Listable.
- {a, b, c} represents a vector.
- = {{a, b}, {c, d}} represents a matrix.
- Nested lists can be used to represent tensors.
- If Nothing appears in a list, it is automatically removed.

```
In[1]:= List[a, b, c, d]
Out[1]= {a, b, c, d}
In[2]:= FullForm[{a, b, c, d}]
Out[2]//FullForm= List[a, b, c, d]
```

Lists are very good for holding data since the elements can be anything:

Sine of successive squares:

```
In[1]= ssq = N[Sin[Range[10]^2]]
Out[1]= {0.841471, -0.756802, 0.412118, -0.287903,
        -0.132352, -0.991779, -0.953753, 0.920026, -0.629888, -0.506366}
```

## ListPlot

ListPlot[{y1, y2, ...}] plots points {1, y1}, {2, y2}, ....

ListPlot[{{x1, y1}, {x2, y2}, ...}] plots a list of points with specified x and y coordinates.

```
ListPlot[{data1, data2, ...}]
plots data from all the data;.
```

ListPlot[{..., w[data<sub>i</sub>, ...], ...}] plots data<sub>i</sub> with features defined by the symbolic wrapper w.

Values x<sub>i</sub> and y<sub>i</sub> that are not of the form above are taken to be missing and are not shown.

The data; have the following forms and interpretations:

$< "k_1" \rightarrow y_1, "k_2" \rightarrow y_2, >$	values {y1, y2,}	
$\langle x_1 \rightarrow y_1, x_2 \rightarrow y_2, \dots \rangle$	key-value pairs {{x1, y1}, {x2, y2},}	
$\{y_1 \rightarrow "Ibh", y_2 \rightarrow "Ibh",\}, $ $\{y_1, y_2,\} \rightarrow \{"Ibh", "Ibh",\}$	values { y1, y2, } with labels { <i>lbh</i> , <i>lbh</i> , }	34

DataRange determines how values {y1, ..., yn} are interpreted into {{x1, y1}, ..., {xn, yn}}. Possible settings include:

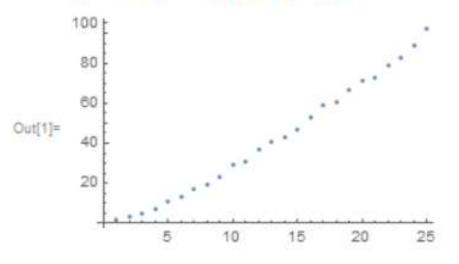
Automatic, All	uniform from 1 to n	
$\{X_{min}, X_{max}\}$	uniform from Xmin to Xmax	

- In general a list of pairs {{x1, y1}, {x2, y2}, ...} is interpreted as a list of points, but the setting DataRange → All forces it to be interpreted as multiple data; {{y11, y12}, {y21, y23}, ...}.
- LabelingFunction -> f specifies that each point should have a label given by f [value, index, Ibls], where value is the value associated with the point, index is its position in the data, and Ibls is the list of relevant labels.
- Typical settings for PlotLegends include:

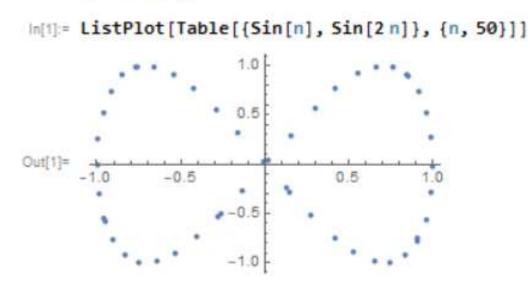
None	no legend
Automatic	automatically determine legend
$\{IbI_1, IbI_2,\}$	use <i>lbh</i> , <i>lbh</i> , as legend labels
Placed[Ispec,]	specify placement for legend

Plot a list of y values:

```
In[1]:= ListPlot[Prime[Range[25]]]
```



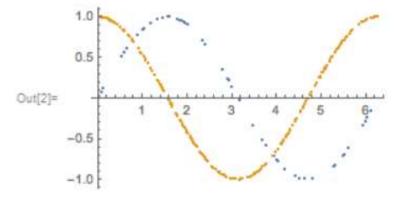
Plot a list of x, y pairs:



Plot multiple sets of irregular data:

```
in[1]:= data1 = Table[{x, Sin[x]}, {x, RandomReal[2 Pi, 50]}];
data2 = Table[{x, Cos[x]}, {x, RandomReal[2 Pi, 250]}];
```

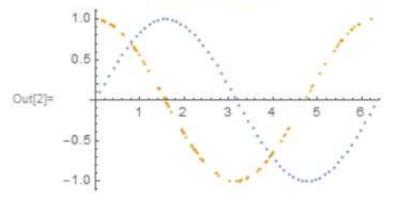
```
in[2]:= ListPlot[{data1, data2}]
```



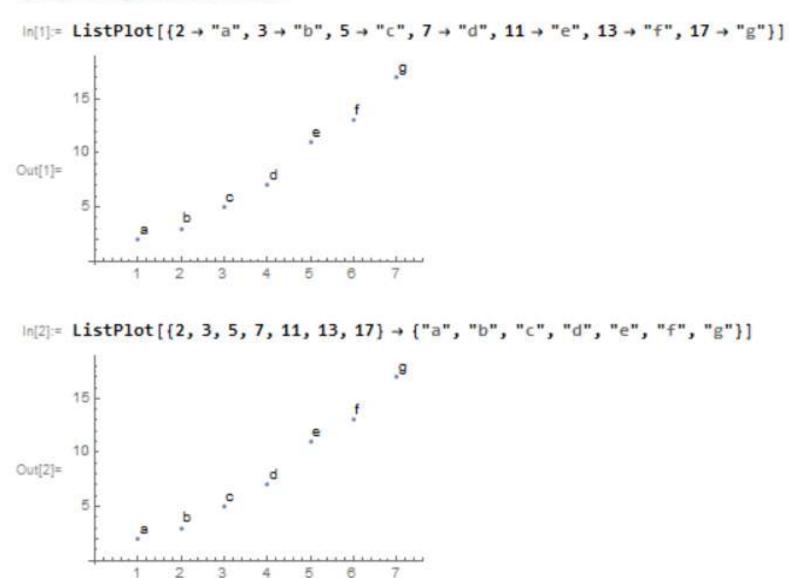
Plot multiple sets of data, regular or irregular, using DataRange to map them to the same x range:

```
in[1]:= data1 = Table[Sin[x], {x, 0, 2 Pi, 0.1}];
data2 = Table[{x, Cos[x]}, {x, RandomReal[2 Pi, 100]}];
```

```
in[2]= ListPlot[{data1, data2}, DataRange → {0, 2 Pi}]
```

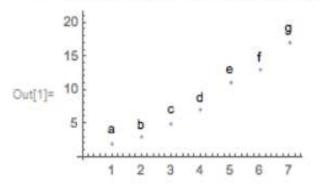


Specify strings to use as labels:



Specify a location for labels:

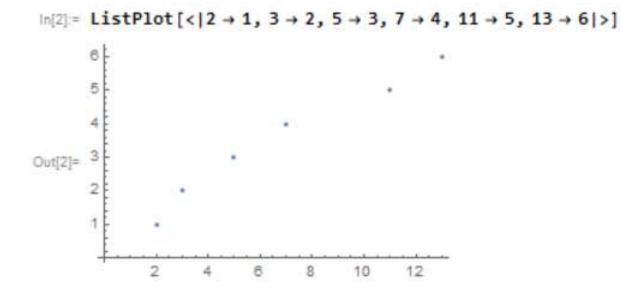
```
\label{eq:listPlot[{2, 3, 5, 7, 11, 13, 17} \rightarrow {"a", "b", "c", "d", "e", "f", "g"}, LabelingFunction \rightarrow Above]
```



Numeric values in an Association are used as the y coordinates:

```
in[1]:= ListPlot[<|"a" \rightarrow 2, "b" \rightarrow 3, "c" \rightarrow 5, "d" \rightarrow 7, "e" \rightarrow 11, "f" \rightarrow 13|>]
         14
                                                                  ,f
         12
                                                          e
          10
           8
                                                 d
Out[1]=
           6
                                         С
           4
                                b
           2
                               2
                                        3
                                                          5
                                                                  6
                                                 4
```

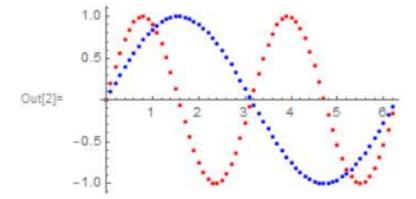
Numeric keys and values in an Association are used as the x and y coordinates:



Provide explicit styling to different sets:

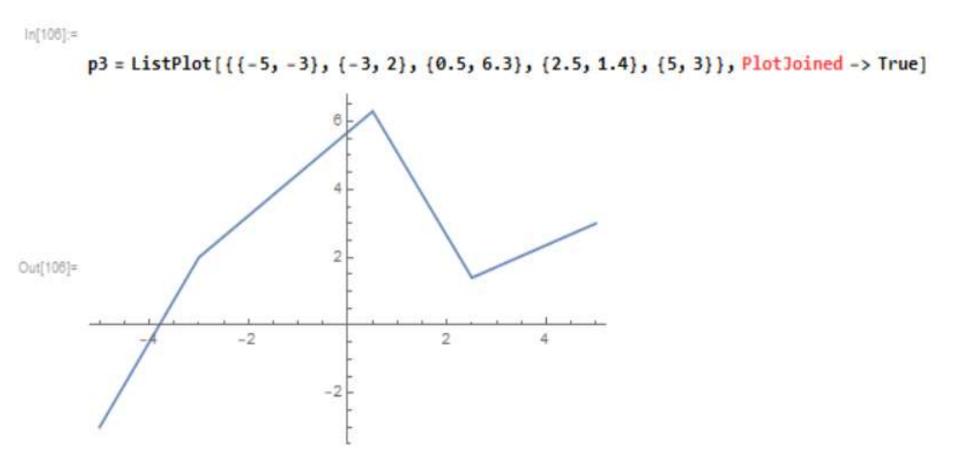
in[1]:= data = Table[Table[{x, f}, {x, 0, 2 Pi, 0.1}], {f, {Sin[x], Sin[2x]}}];

```
in[2]:= ListPlot[data, PlotStyle 	o {Blue, Red}]
```



### ListPlot

ListPlot[] command draws a list of points, given as coordinate pairs (x,y)



To draw a plot joining the points (1, y1), (2,y2),...,  $(n, y_n)$ .

In[107]= ListPlot[{2.5, 3.7, -1.2, 7.0, 9.1, -2.3}, PlotJoined → True]

