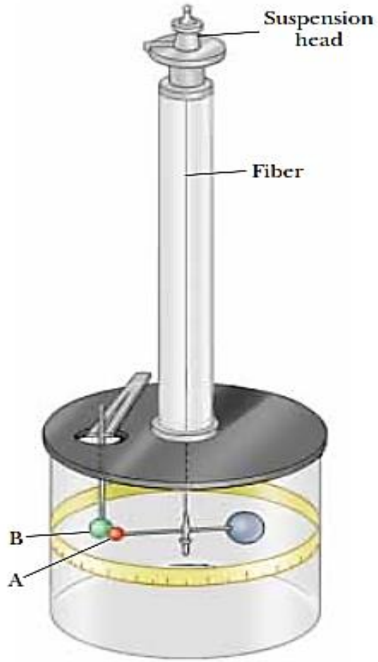
Coulomb’s Law

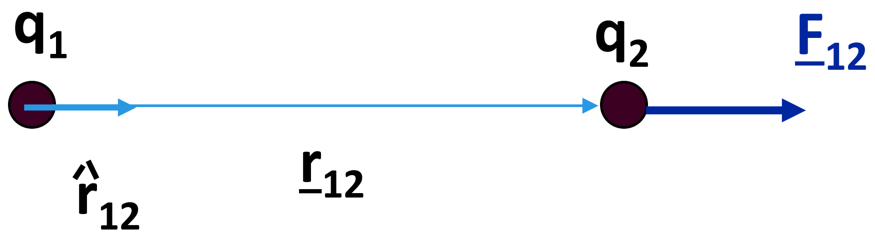
In 1785 Charles Coulomb (1736 –1806) experimentally established the fundamental law of electric force between two stationary charged particles. He measured the magnitudes of the electric forces between charged objects using the torsion balance, which he invented (Fig. (1)).

Coulomb confirmed that the electric force between two small charged spheres is proportional to the inverse square of their separation distance r that is, $F_e \propto 1/r^2$.



Coulomb's Law

$$\mathbf{F}_{12} = \frac{k q_1 q_2}{r_{12}^{2}} \hat{\mathbf{r}}_{12}$$



Force on 2 due to 1

Because the electric force obeys Newton's third law, the electric force exerted by q_2 on q_1 is equal in magnitude to the force exerted by q_1 on q_2 and in the opposite direction; that is, $\vec{F}_{21} = -\vec{F}_{12}$.

$k = (4\pi\varepsilon_0)^{-1} = 9.0 \times 10^{9}\ \mathrm{Nm^2/C^2}$

ε_0 = permitivity of free space

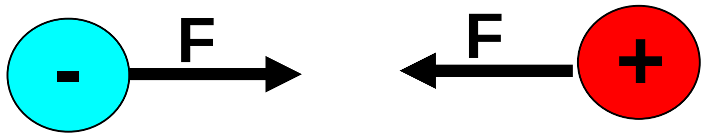
$= 8.86 \times 10^{-12}\ \mathrm{C^2/Nm^2}$

Coulomb's law describes the interaction between bodies due to their charges

Electric Forces

Like Charges - Repel

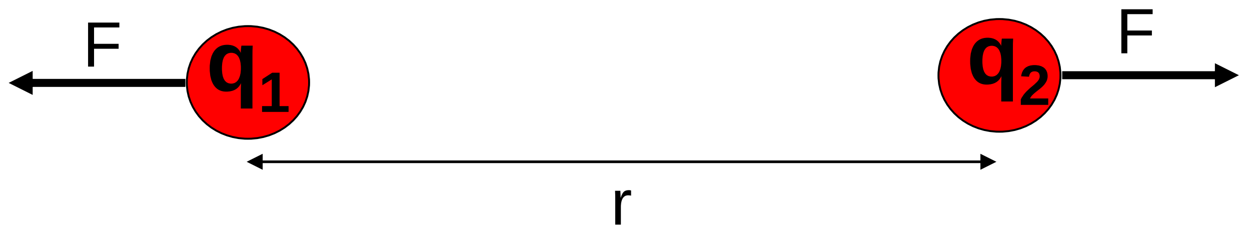
Unlike Charges - Attract



Example 1

Two charges are separated by a distance r and have a force F on each other.

$$F = k\frac{q_1 q_2}{r^2}$$



If r is doubled then F is : ¼ of F

If q_1 is doubled then F is : 2F

If q_1 and q_2 are doubled and r is halved then F is : 16F

Example 2

Two 40 gram masses each with a charge of 3μC are placed 50cm apart. Compare the gravitational force between the two masses to the electric force between the two masses. (Ignore the force of the earth on the two masses)

$$F_g = G\frac{m_1 m_2}{r^2}$$

$$= 6.67\times10^{-11}\frac{(.04)(.04)}{(0.5)^2} \approx 4.27\times10^{-13}N$$

$$F_E = k\frac{q_1 q_2}{r^2}$$

$$= 9.0\times10^{9}\frac{(3\times10^{-6})(3\times10^{-6})}{(0.5)^2} \approx 0.324N$$

The electric force is much greater than the gravitational force

Example 3:

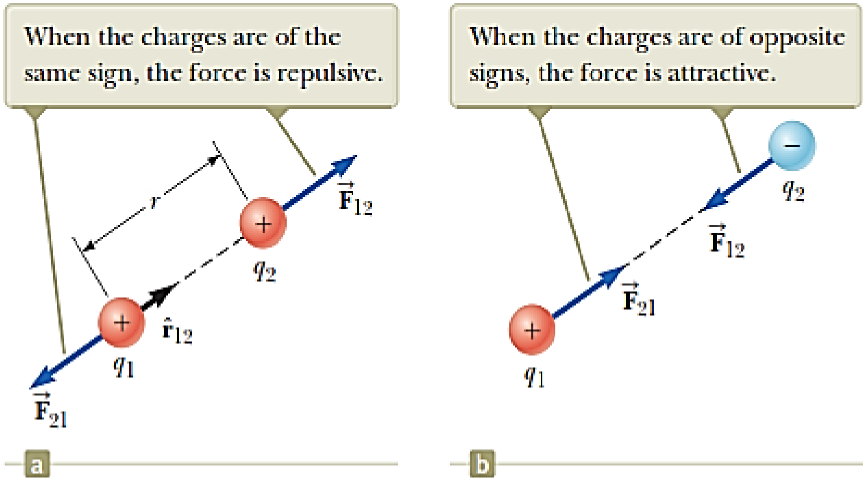
The electron and proton of a hydrogen atom are separated (on the average) by a distance of about 5.3×10^{-11} m.

(a) Find the magnitudes of the electric force and the gravitational force that each particle exerts on the other,

(b) and the ratio of the electric force F_e to the gravitational force F_g.

(c) Compute the acceleration caused by the electric force of the proton on the electron. Repeat for the gravitational acceleration.

Figure (1): Two point charges separated by a distance r exert a force on each other that is given by Coulomb's law. The force $F21$ exerted by q_2 on q_1 is equal in magnitude and opposite in direction to the force $F12$ exerted by q_1 on q_2.



Solution:

(a) Compute the magnitudes of the electric and gravitational forces, and find the ration F_e / F_g.

Substitute $|q_1| = |q_2| = e$ and the distance into Coulomb's law to find the electric force:

$$F_e = k_e \frac{|e|^2}{r^2} = 8.9876 \times 10^9 \times \frac{\left|1.6\times10^{-19}\right|^2}{(5.3\times10^{-11})^2} = 8.2 \times 10^{-8}\ N$$

Substitute the masses and distance into Newton's law of gravity to find the gravitational force:

$$F_g = G\frac{m_e\, m_p}{r^2} = 6.67 \times 10^{-11}\frac{N.m^2}{kg^2}\ \frac{9.11\times 10^{-31}kg \times 1.67\times 10^{-27}kg}{(5.3\times10^{-11})^2\ m^2} = 3.6 \times 10^{-47} N$$

Find the ratio of the two forces: $\frac{F_e}{F_g} = 2.3 \times 10^{39}$

(b) Compute the acceleration of the electron caused by the electric force. Repeat for the gravitational acceleration.

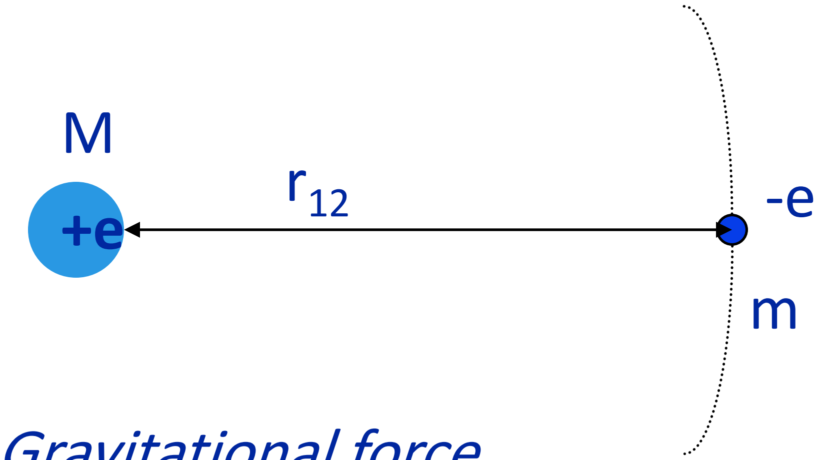
Use Newton's second law and the electric force found in part (a):

$$F_e = m_e a_e \rightarrow a_e = \frac{F_e}{m_e} = \frac{8.2\times10^{-8}\ N}{9.11\times 10^{-31}kg} = 9.0 \times 10^{22}\ \frac{m}{s^2}$$

Use Newton's second law and the gravitational force found in part (a):

$$F_g = m_e a_g \rightarrow a_g = \frac{F_g}{m_e} = \frac{3.6\times10^{-47}\ N}{9.11\times 10^{-31}kg} = 3.95 \times 10^{-17}\ \frac{m}{s^2}$$

Gravitational and Electric Forces in the Hydrogen Atom



m = 9.1 10^{-31} kg
M = 1.7 10^{-27} kg
r_{12} = 5.3 10^{-11} m

Gravitational force

$$\vec{F}_g = G\frac{Mm}{r_{12}^2}\hat{r}$$

F_g = 3.6 10^{-47} N

Electric Force

$$\vec{F}_e = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{Qq}{r_{12}^2}\hat{r}$$

F_e = 3.6 10^{-8}N

Question: If the distance between two charges is doubled, by what factor is the magnitude of the electric force changed?

Exercise: (a) Find the magnitude of the electric force between two protons separated by 1 femtometer (10^{-15} m), approximately the distance between two protons in the nucleus of a helium atom. (b) If the protons were not held together by the strong nuclear force, what would be their initial acceleration due to the electric force between them?

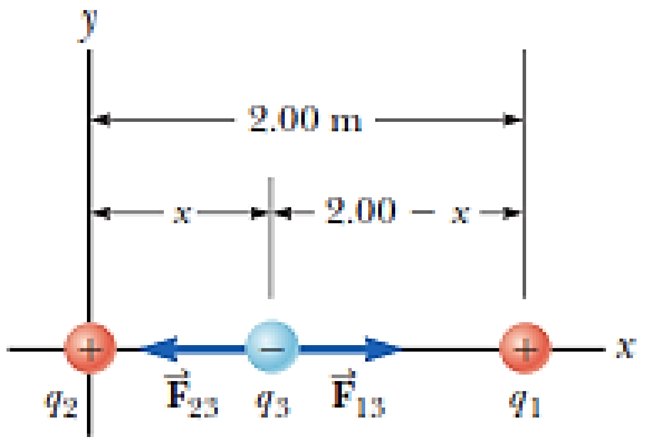
Answers: (a) 2×10^{2} N (b) 1×10^{29} m/s^{2}

Problem: Three charges lie along the x-axis as in Figure down.
The positive charge q_1 = 15 μC is at x = 2.0 m, and the positive charge q_2 = 6.0 μC is at the origin.
Where must a negative charge q_3 be placed on the x-axis so that the resultant electric force on it is zero?

Solution:

If q_3 is to the right or left of the other two charges, the net force on q_3 can't be zero because then F_{13} and F_{23} act in the same direction.

Figure: three point charges are placed along the x-axis. The charge q_3 is negative, whereas q_1 and q_2 are positive. If the resultant force on q_3 is zero, the force F*13* exerted by q_1 on q_3 must be equal in magnitude and opposite the force F*23* exerted by q_2 on q_3



on q_3.

Consequently, q_3 must lie between the two other charges. Write F_{13} and F_{23} in terms of the unknown coordinate position x, then sum them and set them equal to zero, solving for the unknown. The solution can be obtained with the quadratic formula.

Write the x -component of $\vec{F}_{13}$:

$$F_{13x} = +k_e \frac{|15 \times 10^{-6}\ C||q_3|}{(2.0\ m - x)^2}$$

Write the x -component of $\vec{F}_{23}$:

$$F_{23x} = -k_e \frac{|6.0 \times 10^{-6}\ C||q_3|}{(x)^2}$$

Set the sum equal to zero:

$$k_e \frac{|15 \times 10^{-6}\ C||q_3|}{(2.0\ m - x)^2} + 2k_e \frac{|6.0 \times 10^{-6}\ C||q_3|}{(x)^2} = 0$$

Cancel k_e, 10^{-6}, and q_3 from the equation and rearrange terms (explicit significant figures and units are temporarily suspended for clarity):

$$6\,(2 - x)^2 = 15x^2$$

Put this equation into standard quadratic form, $ax^2 + bx + c = 0$

$$2(4 - 4x + x^2) = 5x^2 \xrightarrow{yields} 3x^2 + 8x - 8 = 0$$

Apply the quadratic formula

$$x = \frac{-8 \pm \sqrt{64 - |4||3||-8|}}{6}$$

Only the positive root makes sense: $x = 0.77$ m

Question: If q_1 has the same magnitude as before but is negative, in what region along the x -axis would it be possible for the net electric force on q_3 to be zero? (a) $x < 0$ (b) $0 < x < 2$ m (c) 2 m $< x$

Exercise: Three charges lie along the x -axis. A positive charge q_1 = 10.0 μC is at x = 1.00 m, and a negative charge q_2 = -2.0 μC is at the origin. Where must a positive charge q_3 be placed on the x -axis so that the resultant force on it is zero?

Answer: x = -0.809 m

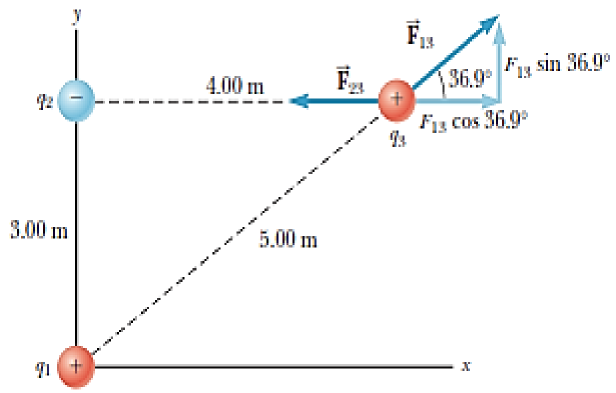
<u>Problem:</u> Consider three point charges at the corners of a triangle, as shown in Figure down, where $q_1 = 6.00 \times 10^{-9}$ C, $q_2 = -2.00 \times 10^{-9}$ C, and $q_3 = 5.00 \times 10^{-9}$ C.

(a) Find the components of the force F23 exerted by q_2 on q_3.
(b) (b) Find the components of the force F13 exerted by q_1 on q_3.
(c) Find the resultant force on q_3, in terms of components and also in terms of magnitude and direction.

<u>Solution:</u>

Coulomb's law gives the magnitude of each force, which can be split with right-triangle trigonometry into x – and y -components. Sum the vectors component wise and then find the magnitude and direction of the resultant vector.

Figure : The force exerted by q_1 on q_3 is F13. The force exerted by q_2 on q_3 is F23 . The resultant force F_3 exerted on q_3 is the *vector* sum F13 + F23.



(a) Find the components of the force exerted by q_2 on q_3.

Find the magnitude of $\vec{F}_{23}$ with Coulomb's law:

$$F_{23} = k_e \frac{|q_2||q_3|}{r^2}$$

$$= 8.9876 \times 10^9 \; (N.m^2/C^2) \times \frac{(2.00 \quad 10-9\;C)(5.00 \quad 10-9\;C)}{(4\;m)^2}$$

$$= 5.62 \times 10^{-9} \quad N$$

$$F_{23x} = -5.62 \times 10^{-9} \quad N, \quad F_{23y} = 0$$

Because $\vec{F}_{23}$ is horizontal and points in the negative x -direction, the negative of the magnitude gives the x -component, and the y -component is zero:

(b) Find the components of the force exerted by q_1 on q_3.

Find the magnitude of $\vec{F}_{13}$:

$$F_{13} = k_e \frac{|q_1||q_3|}{r^2} = 8.9876 \times 10^9 \ (N.m^2/C^2) \times \frac{(6.00 \quad 10-9\ C)(5.00 \quad 10-9\ C)}{(5\ m)^2}$$

$$= 1.08 \times 10^{-8} \quad N$$

Use the given triangle to find the components of $\vec{F}_{13}$:

$$F_{13x} = F_{13} \cos\theta = (1.08 \times 10^{-8}\ N)\cos(36.9^o) = 8.64 \times 10^{-9}\ N$$

$$F_{13y} = F_{13} \sin\theta = (1.08 \times 10^{-8}\ N)\sin(36.9^o) = 6.48 \times 10^{-9}\ N$$

(c) Find the components of the resultant vector.Sum the x -components to find the resultant F_x:

$$F_x = -5.62 \times 10^{-9}\ N + 8.64 \times 10^{-9}\ N = 3.02 \times 10^{-9}\ N$$

Sum the y -components to find the resultant F_y:

$$F_y = 0 + 6.48 \times 10^{-9}\ N = 6.48 \times 10^{-9}\ N$$

Find the magnitude of the resultant force on the charge q_3, using the Pythagorean theorem:

$$\vec{F} = \sqrt{F_x^2 + F_x^2} = \sqrt{(3.02 \times 10^{-9}\ N)^2 + (6.48 \times 10^{-9}\ N)^2} = 7.15 \times 10^{-9}\ N$$

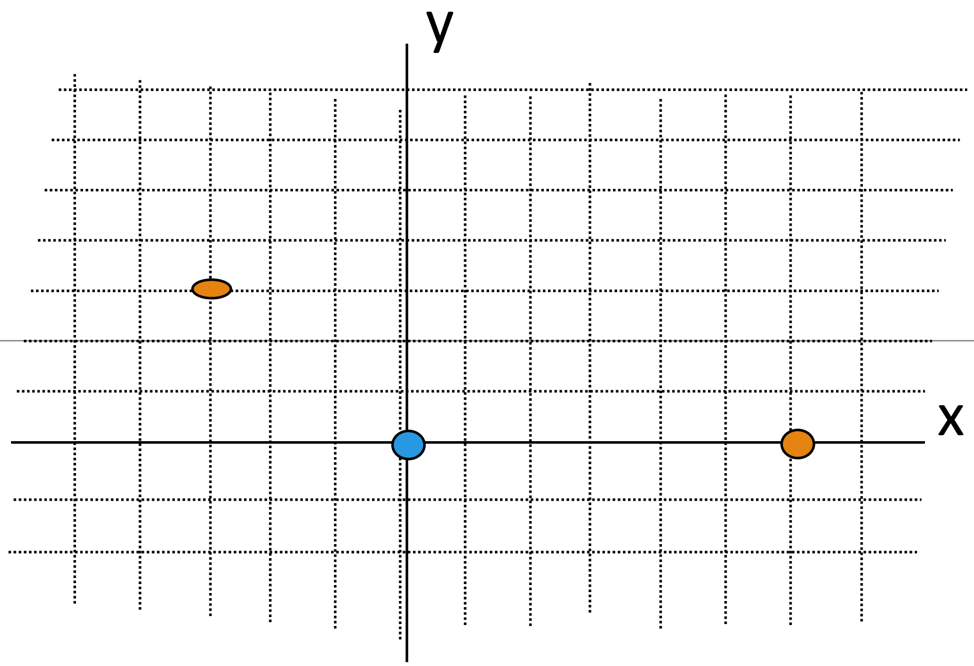
Find the angle the resultant force makes with respect to the positive x –axis:

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \theta = \tan^{-1}\left(\frac{6.48 \times 10^{-9}\ N}{3.02 \times 10^{-9}\ N}\right) = 65^o$$

Superposition of forces from two charges

Red charges fixed , negative, equal charge (-q)

What is force on positive Blue charge +q ?

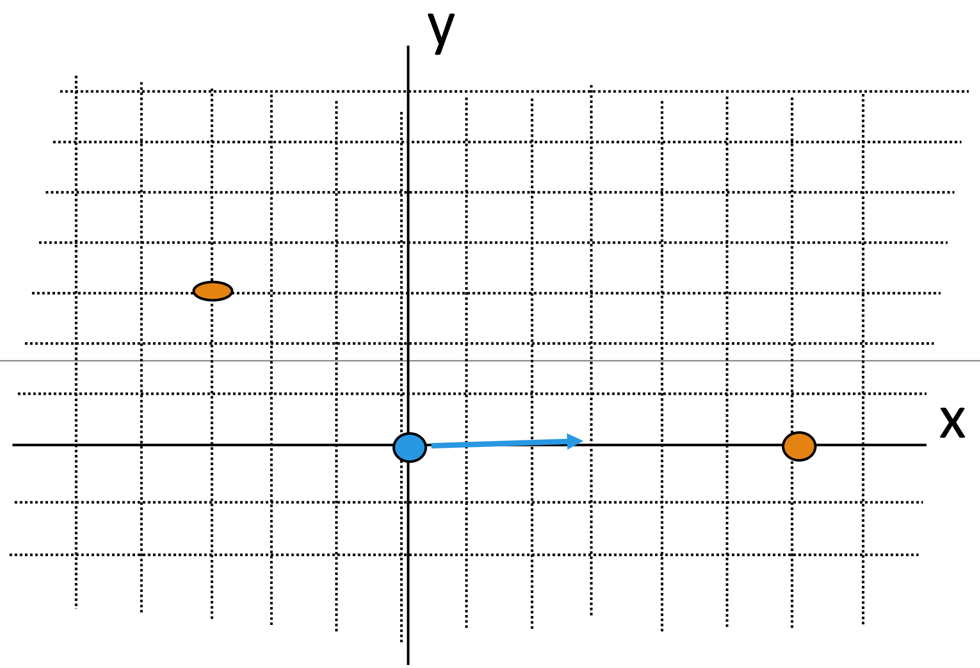


Superposition of forces from two charges

Red charges fixed , negative, equal charge (-q)

What is force on positive Blue charge +q ?

Consider effect of each charge separately:

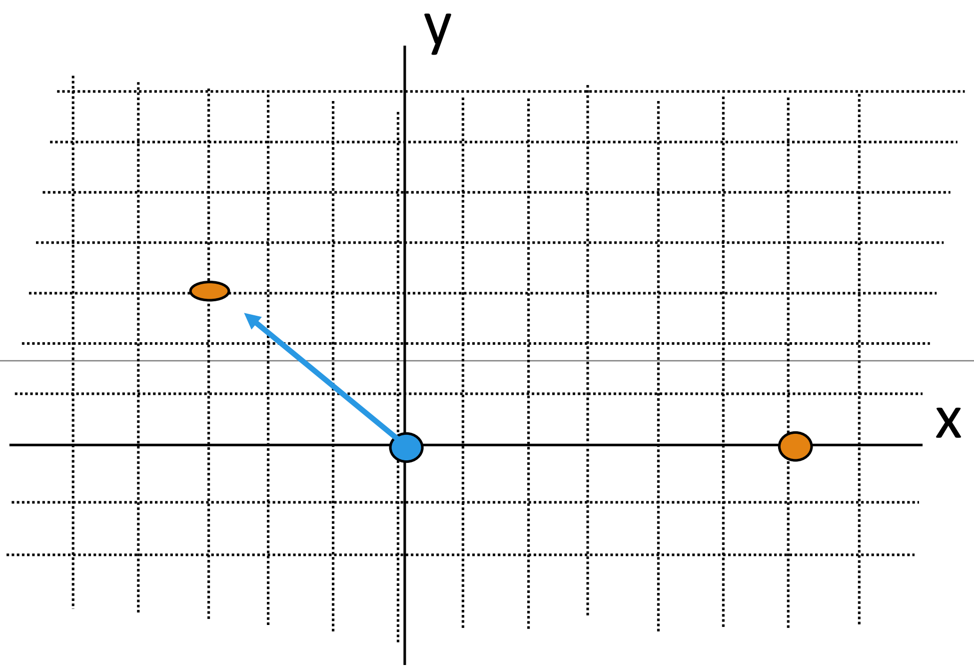


Superposition of forces from two charges

Red charges fixed , negative, equal charge (-q)

What is force on positive Blue charge +q ?

Take each charge in turn:

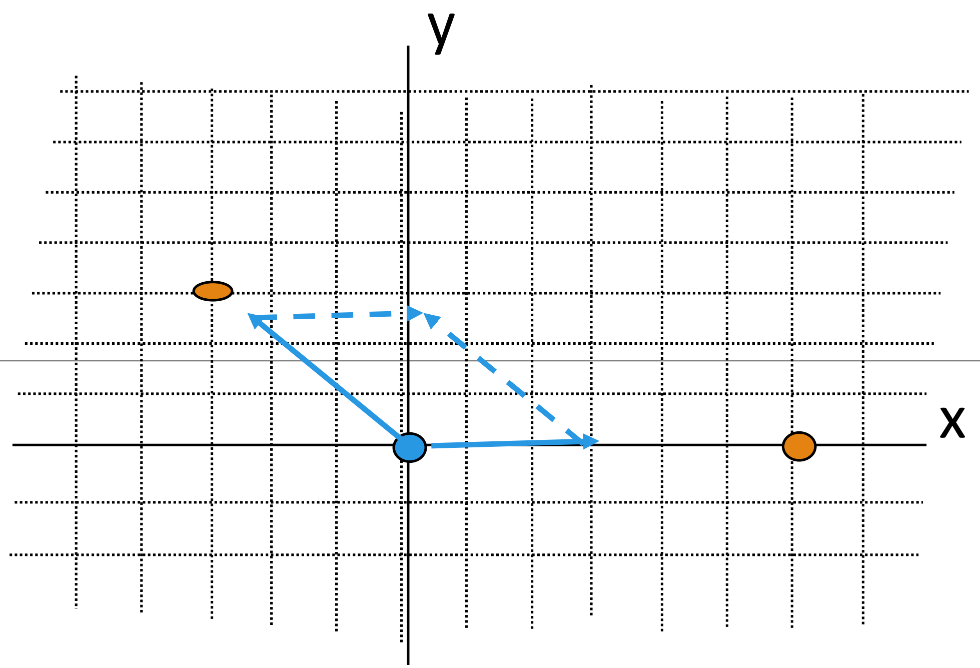


Superposition of forces from two charges

Red charges fixed , negative, equal charge (-q)

What is force on positive Blue charge +q ?

Create vector sum:

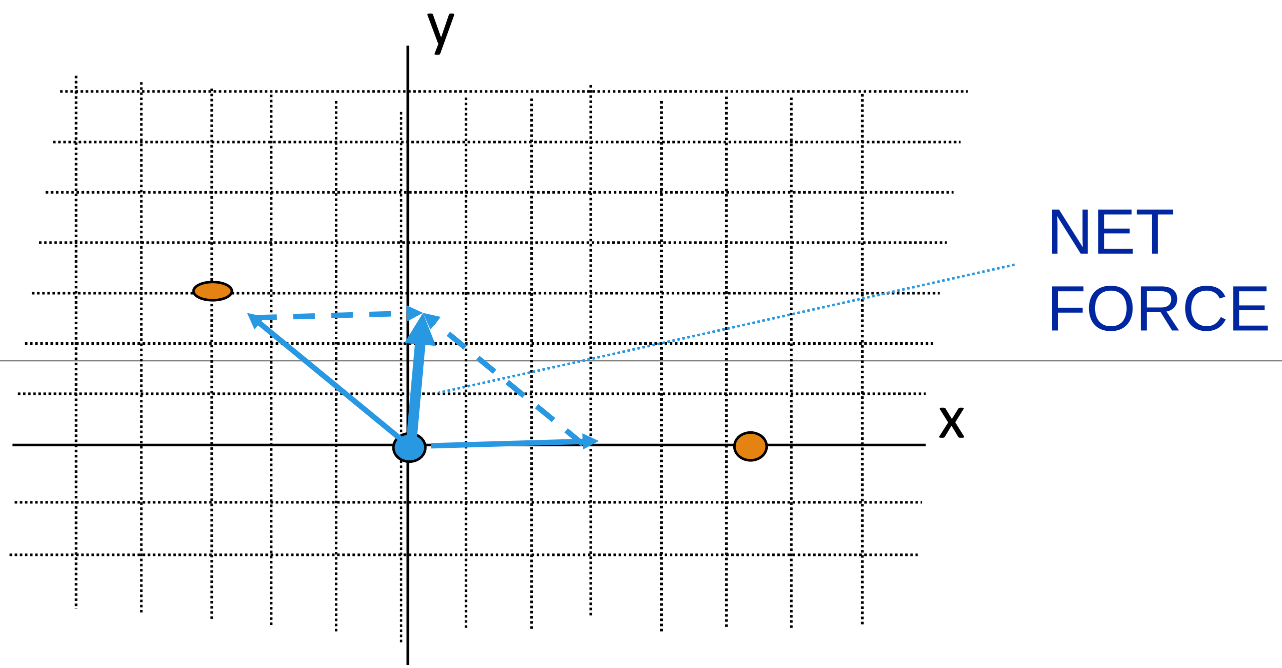


Superposition of forces from two charges

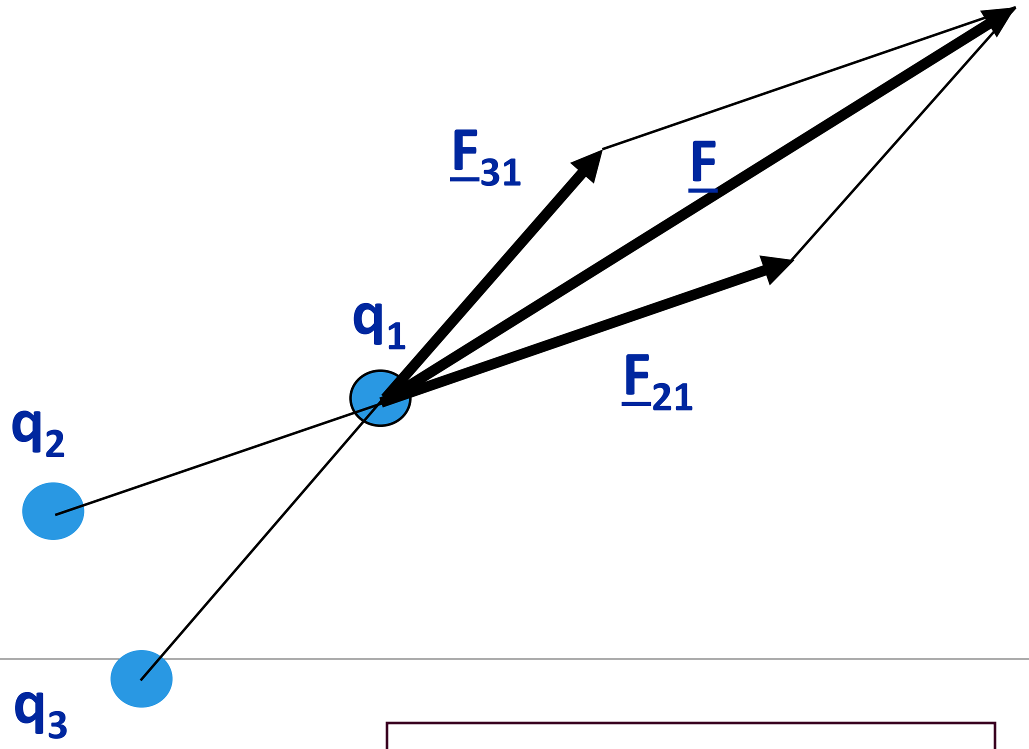
Red charges fixed , negative, equal charge (-q)

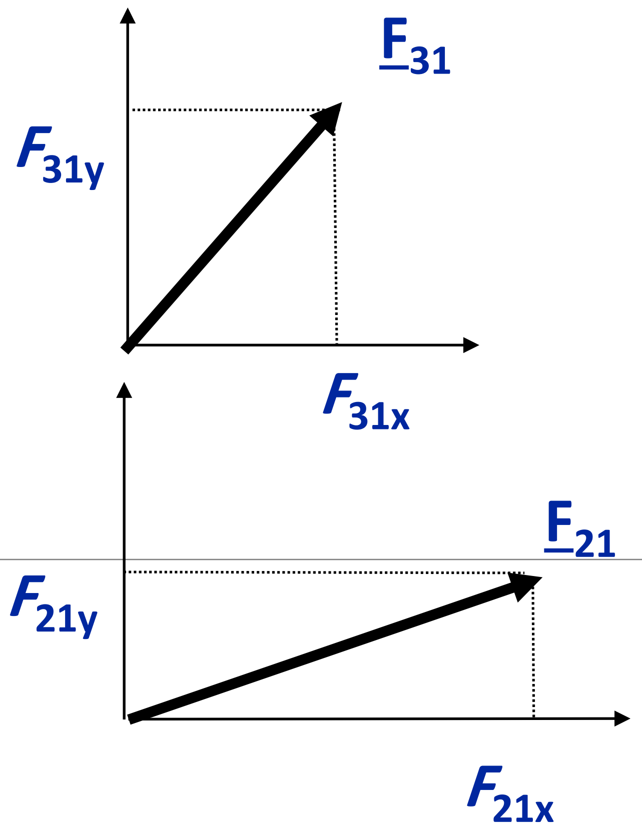
What is force on positive Blue charge +q ?

Find resultant:



Superposition Principle



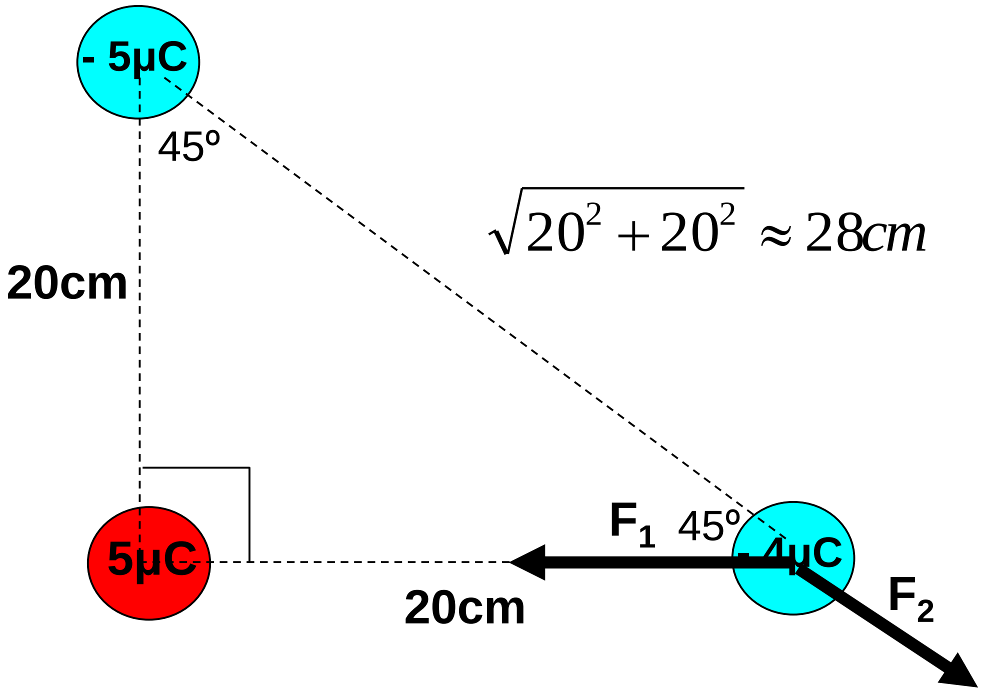


Forces add vectorially

$$\underline{F} = (F_{21x} + F_{31x})\,\underline{x} + (F_{21y} + F_{31y})\,\underline{y}$$

Example 4

Three charged objects are placed as shown. Find the net force on the object with the charge of -4μC.



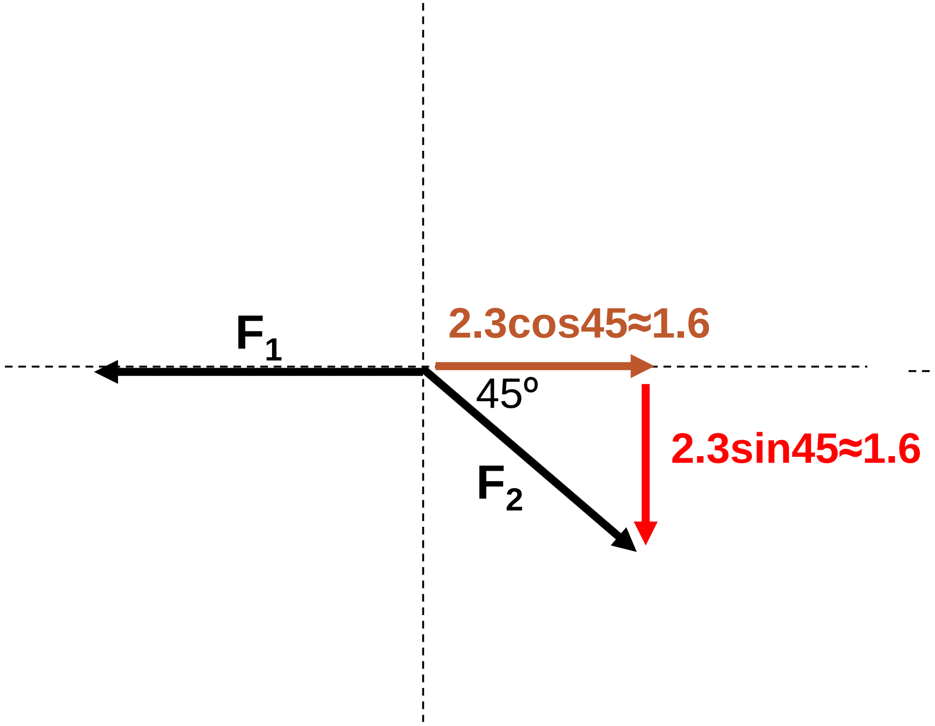
$$\sqrt{20^2+20^2} \approx 28cm$$

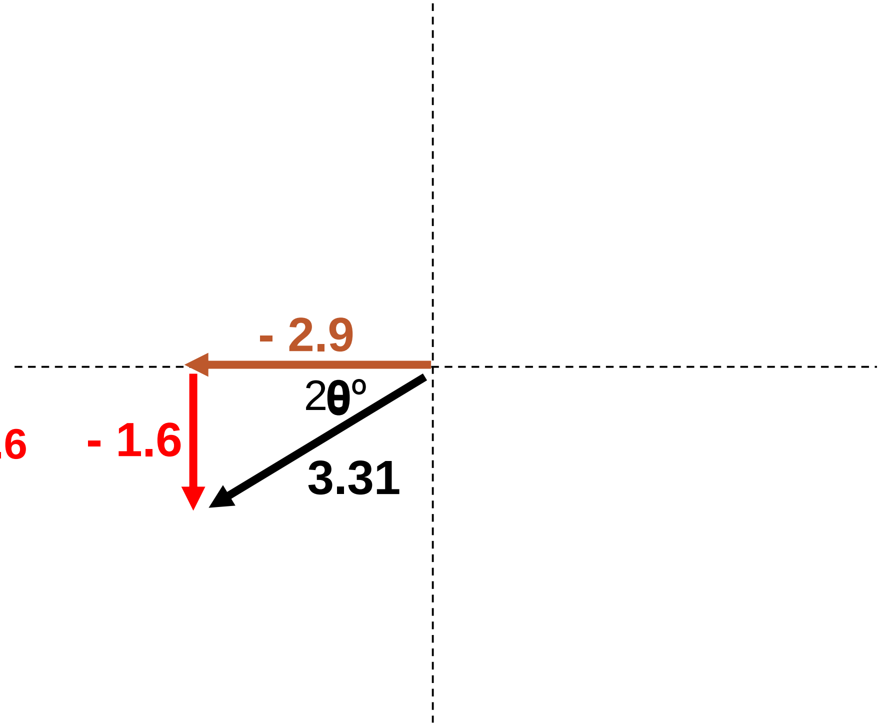
$$F = k\frac{q_1 q_2}{r^2}$$

$$F_1 = 9\times10^9 \frac{(5\times10^{-6})(4\times10^{-6})}{(0.20)^2} = 4.5N$$

$$F_2 = 9\times10^9 \frac{(5\times10^{-6})(4\times10^{-6})}{(0.28)^2} = 2.30N$$

F_1 and F_2 must be added together as vectors.





$\mathbf{F_1} = < -4.5\ ,\ \ 0.0 >$

$+ \quad \mathbf{F_2} = < 1.6\ ,\ -1.6 >$

$\mathbf{F_{net}} = < -2.9\ ,\ -1.6 >$

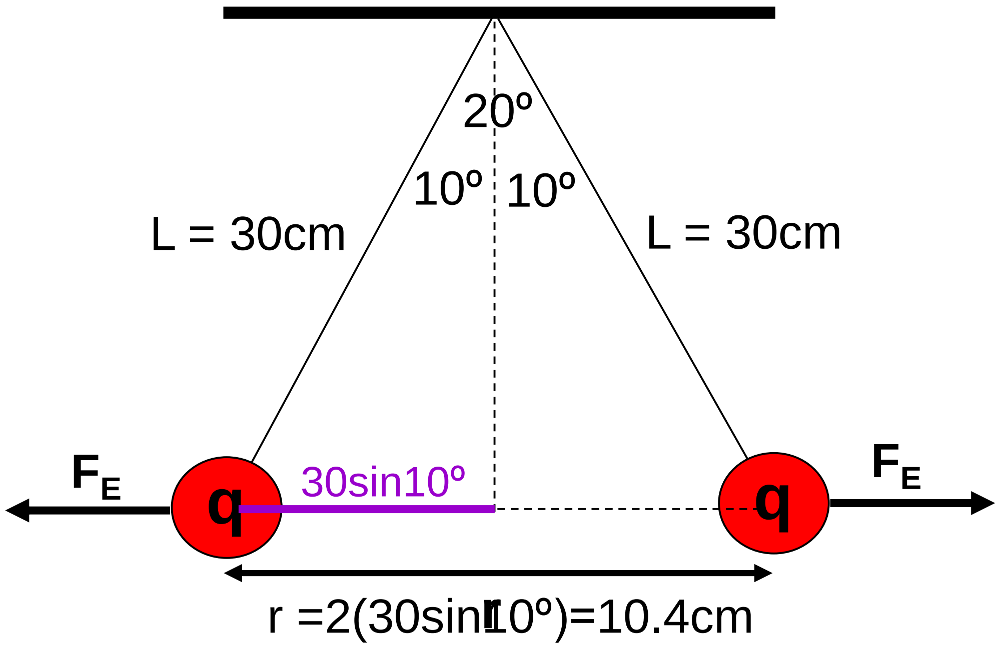
$$F_{net} = \sqrt{2.9^2 + 1.6^2} \approx 3.31N$$

$$\theta = \tan^{-1}\left(\frac{-1.6}{-2.9}\right) \approx 29^\circ$$

3.31N at 209º

Example 4 (Balloon Lab)

Two 8 gram, equally charged balls are suspended on earth as shown in the diagram below. Find the charge on each ball.

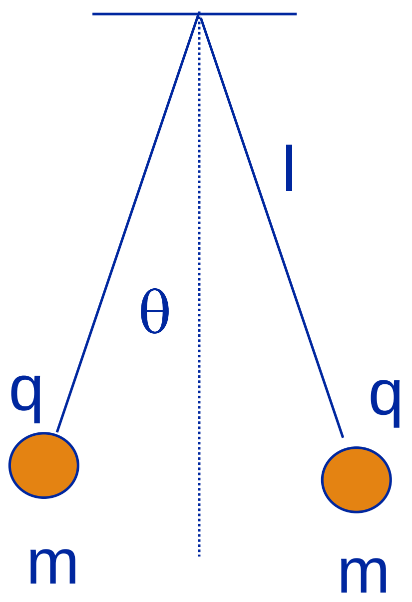


$$F_E = k\frac{q_1 q_2}{r^2} = k\frac{q^2}{r^2}$$

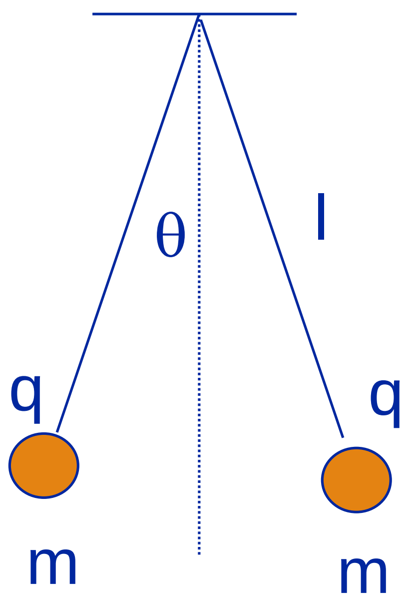
Example: electricity balancing gravity

Two identical balls, with mass m and charge q, hang from similar strings of length l.

After equilibrium is reached, find the charge q as a function of θ and l



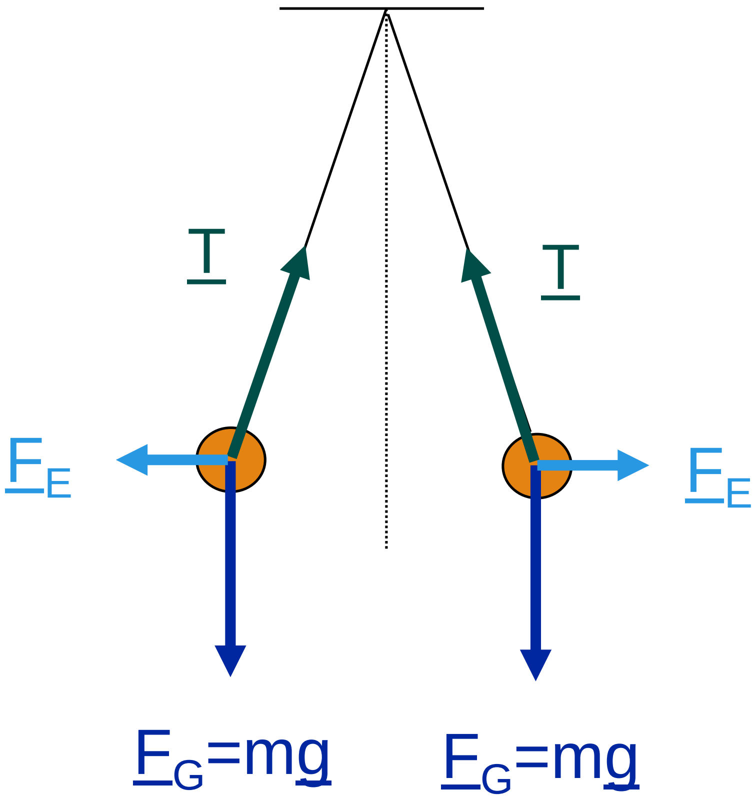
What forces are acting on the charged balls ?



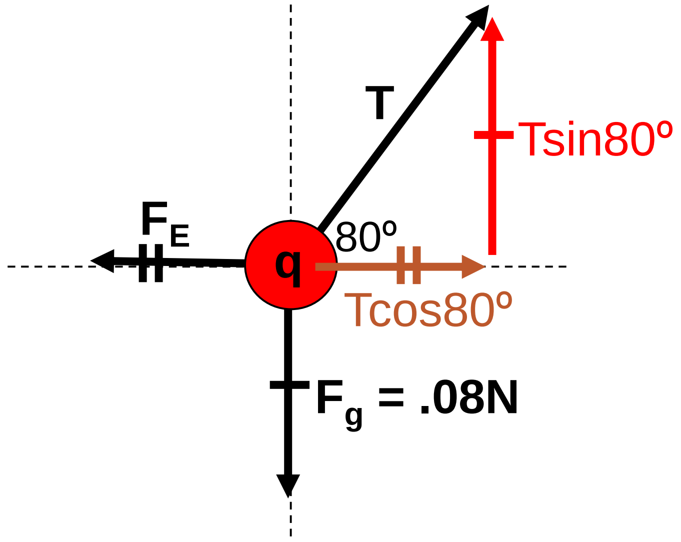
Draw vector force diagram while identifying the forces.

Apply Newton's 3rd Law, for a system in equilibrium, to the components of the forces.

Solve!



Draw a force diagram for one charge and treat as an equilibrium problem.



$$T\sin 80^\circ = .08$$

$$T = \frac{.08}{\sin 80^\circ} \approx .081N$$

$$F_E = T\cos 80^\circ$$

$$k\frac{q^2}{.104^2} = (.081)\cos 80^\circ$$

$$q^2 = \frac{.014}{k}(.104)^2$$

$$q = 1.3\times 10^{-7}C$$