curves in space

4.1 parametric representation of curves
4.3 Arc length
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4.1 parametric representation of curves

Curves in two- and three-dimensional space are often represented as the image of a vector-valued function of a real variable. This is called aparametric representation.
A parametric representation in two dimensions is provided by two coordinate functions, $x(t)$ and $y(t)$, and the vector-valued function $t \to \big(x(t), y(t)\big)$. These curves
are very easy to plot with MATLAB.

Example:

plot the circle with center at $(1,3)$ and radius $r = 2$.

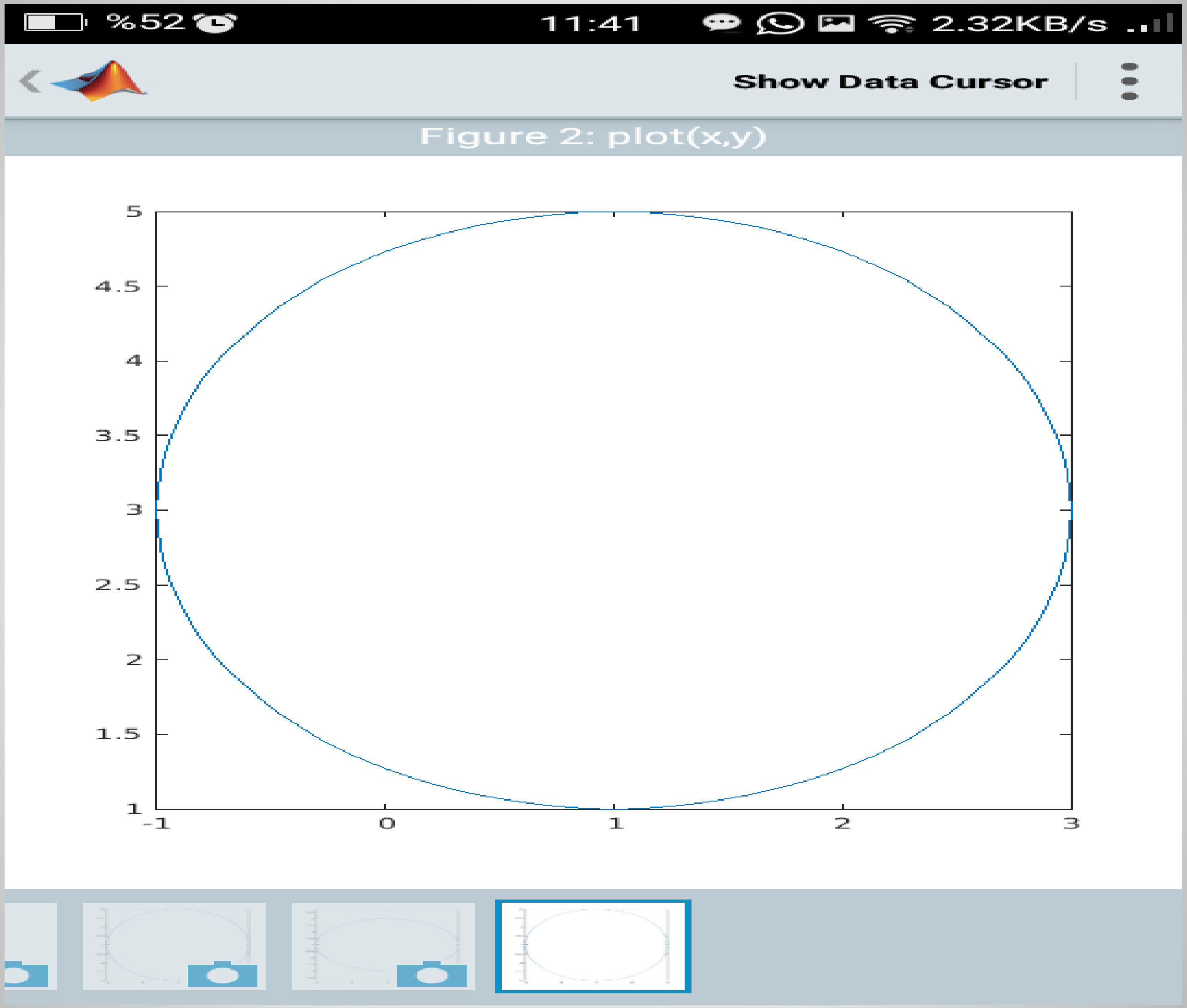
We must use this form :

$$x = x_0 + rcost$$
$$y = y_0 + rsint$$

Where (x_0, y_0) Is the center of the circle

Solution:

- $\gg t = linspace(0, 2 * pi, 101);$
- $\gg x = 1 + 2 * \cos(t)\,;$
- $\gg y = 3 + 2 * \sin(t)\,;$
- $\gg plot(x, y)$


%52
11:41
2.32KB/s
Show Data Cursor
Figure 2: plot(x,y)
5
4.5
4
3.5
3
2.5
2
1.5
1
-1
0
1
2
3

- We also can use the inline function to make such plots:

$$\gg x = inline('1 + 2 * \cos(t)');$$

$$\gg y = inline('3 + 2 * \sin(t)');$$

$$\gg t = linspace(0,2 * pi, 101);$$

$$\gg plot\big(x(t), y(t)\big)$$

- The third method of graphing these two dimensional curves is using symbolically defined functions:

$\gg$ $syms\ t$

$\gg$ $x = \cos(t)$

$\gg$ $y = \sin(t)$

$\gg$ $ezplot(x, y)$

- The defalt rang of the parameter t is $[0, 2 * pi], we\ can\ change\ it\ by\ ezplot(x, y, [1,5])$.

Similarly, curves in three-dimensional space are represented parametrically by vector valued functions

With three coordinates functions, $t \to (x(t), y(t), z(t))$

Example:

Plot the circular helix

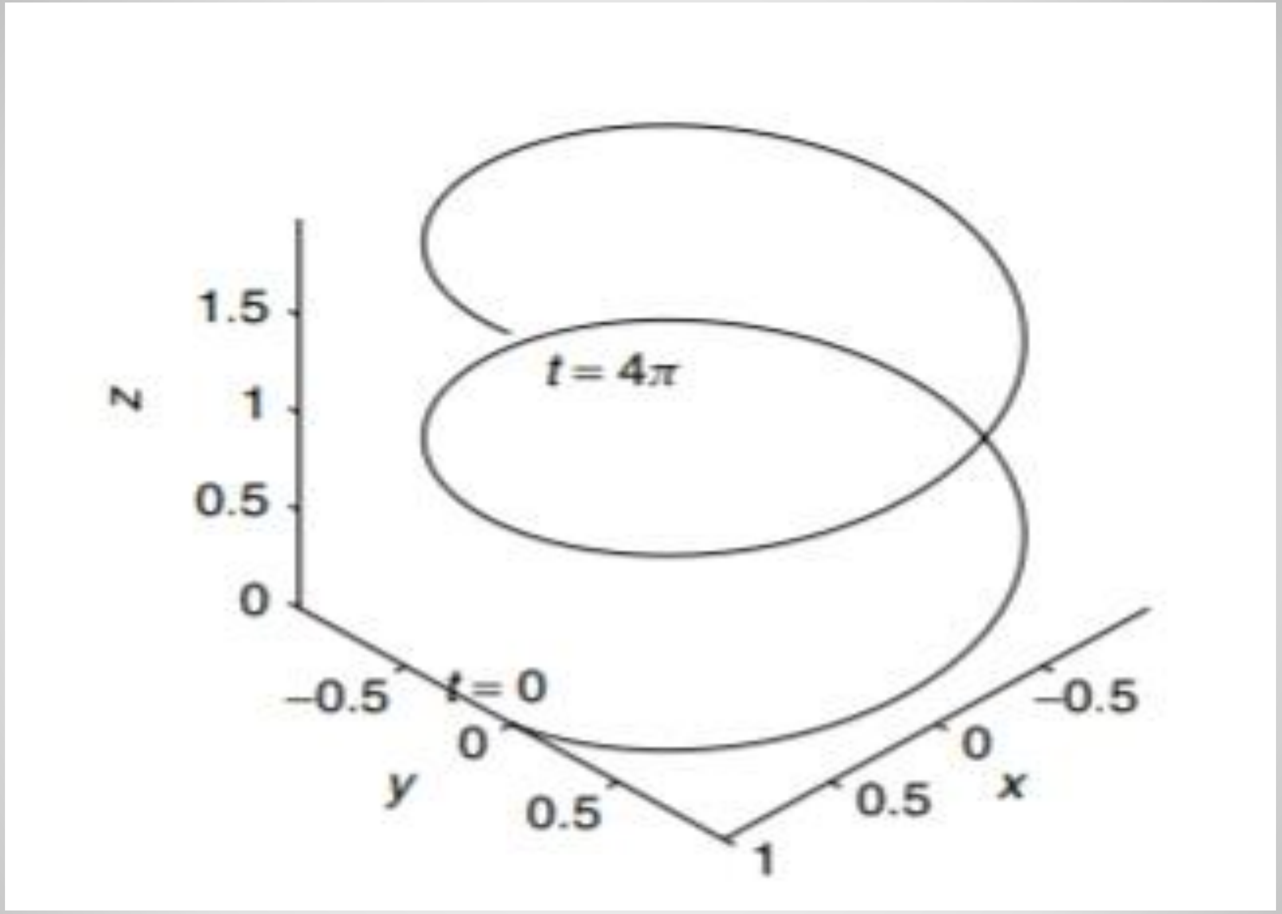
$x(t) = cost, y(t) = sint, z(t) = \frac{t}{2pi}, 0 \leq t$

$\leq 4pi$

Solution:

$\gg t = linspace(0,4 * pi, 201);$

$\gg plot3(\cos(t), \sin(t), t/(2 * pi))$



1.5
z
1
0.5
0
t = 4π
t = 0
−0.5
0
y
0.5
1
−0.5
0
0.5
x

Note:

If we defined the symbolic parameter t
We must use ezplot3(x, y, z, [ ])instead of plot3.

4.3 Arc length

- The length of a line segment from point $p_0 = (x_0, y_0, z_0)$ to point $p_1 = (x_1, y_1, z_1)$ is simply the norm of the vector $p_1 - p_0 = [x_1 - x_0, y_1 - y_0, z_1 - z_0]$ and this is

$$\|p_1 - p_0\| = \sqrt[2]{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}$$

Similarly the length of a polygonal path Γ with vertices $p_1, p_2, \ldots, p_N$ is the sum of the length of the connecting line segments:

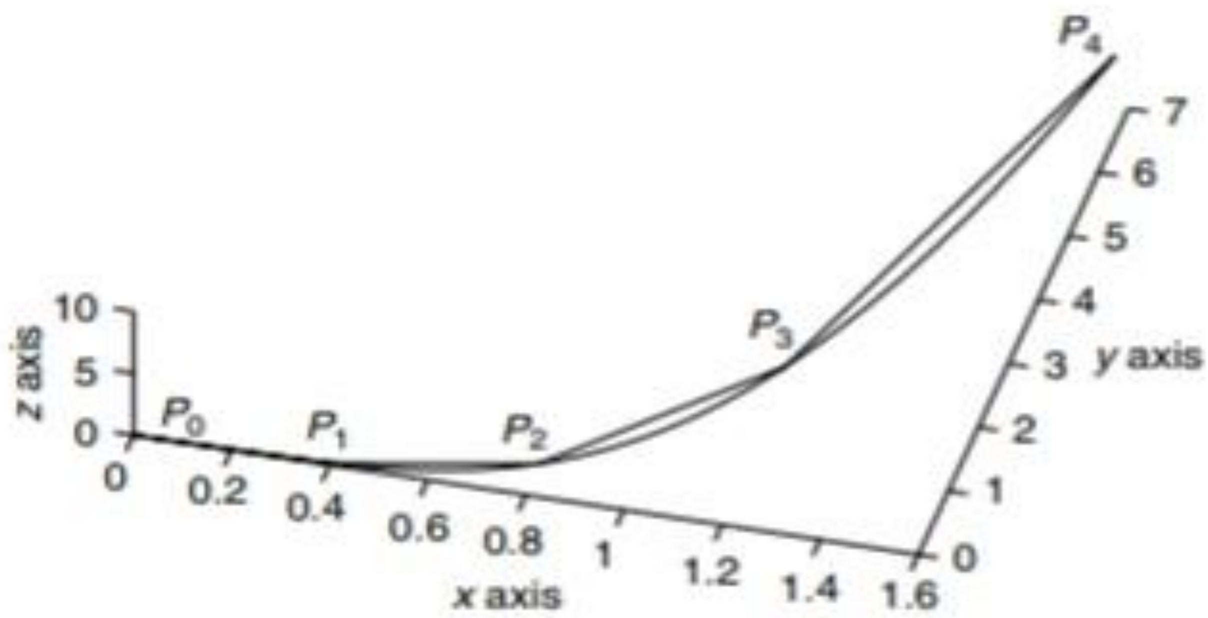


Figure 4.6 Polygonal path Γ approximating curve C.

Length(Γ)=$\|p_1 - p_0\| + \|p_2 - p_1\| + \cdots + \|p_N - p_{N-1}\|$

If C is paraneterized by $r(t)$, a$\leq t \leq b$, with velocity vector $v(t) = [x'(t), y'(t), z'(t)]$, the length $l(C)$ is given by the Arc length integral

$$l(C) = \int_a^b \|v(t)\| \, dt$$

$$= \int_a^b \sqrt[2]{x'(t)^2 + y'(t)^2 + z'(t)^2} \, dt.$$

Example:

- Let the curve C be parameterized by $r(t) = [2t,\ t^2, \ln(t)]$, $1 \leq t \leq 2$.

solution:

1. by using the sum of the length :

$\gg t = 1{:}.01{:}2;$

$\gg x = 2 * t;\ \ y = t.^2;\ \ z = \log(t);$

$\gg sum = 0;$

```
    for j=1:100
        dx=x(j+1)-x(j);
```

```
    dy=y(j+1)-y(j);
    dz=z(j+1)-z(j);
     dr=[dx, dy, dz];
    sum=sum+norm(dr);
  end
  disp('this is the length of the polygonal approx using 100 segments')
Sum
```

$\gg ans =$

3.6931

2. by using the Arc length integral:

first we must calculate the speed by hand. We have $\|v(t)\| = \sqrt{4+4t^2+(1/t)^2}$

$$= 2t + \frac{1}{t}.$$

$$\gg speed = inline('2 * t + 1./t\ )$$

$$\gg quad8(speed, 1,2)$$

ans=

3.6931

3. finally we make a symbolic calculation with the upper limit of integration being a parameter b:

$\gg syms\ t\ b$

$\gg r = [2 * t, t^2, \log(t)];$

$\gg v = diff(r);$

$\gg nospe = sqrt(v(1)^2 + v(2)^2 + v(3)\hat{}2);$

$\gg length = int(nospe, t, 1, b)$

4.5 Rotations in the plane

- let the point (x, y) be represented by the column vector $v = [x, y]$. This vector can be rotated about the origin by a matrix multiplication. Let θ be an angle, $0 \leq \theta \leq 2\pi$. The following 2×2 matrix is called a rotation matrix:

$$R = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}.$$

- the vector $w = Rv$ is the image of v produced by this rotation . Then $w = [x cos\theta - y sin\theta, x sin\theta + y cos\theta]$.

or

$$x = x_0 cos\theta - y_0 sin\theta$$
$$y = x_0 sin\theta + y_0 cos\theta$$

Example:

- let us parameterized the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ with $r(t)=[3cost, 2sint]$, $0\le t\le 2\pi$.

solution:

$\gg t = linspace(0,2*pi, 101);$

$\gg x_0 = 3*\cos(t);\ y_0 = 2*\sin(t);$

$\gg plot(x_0, y_0)$

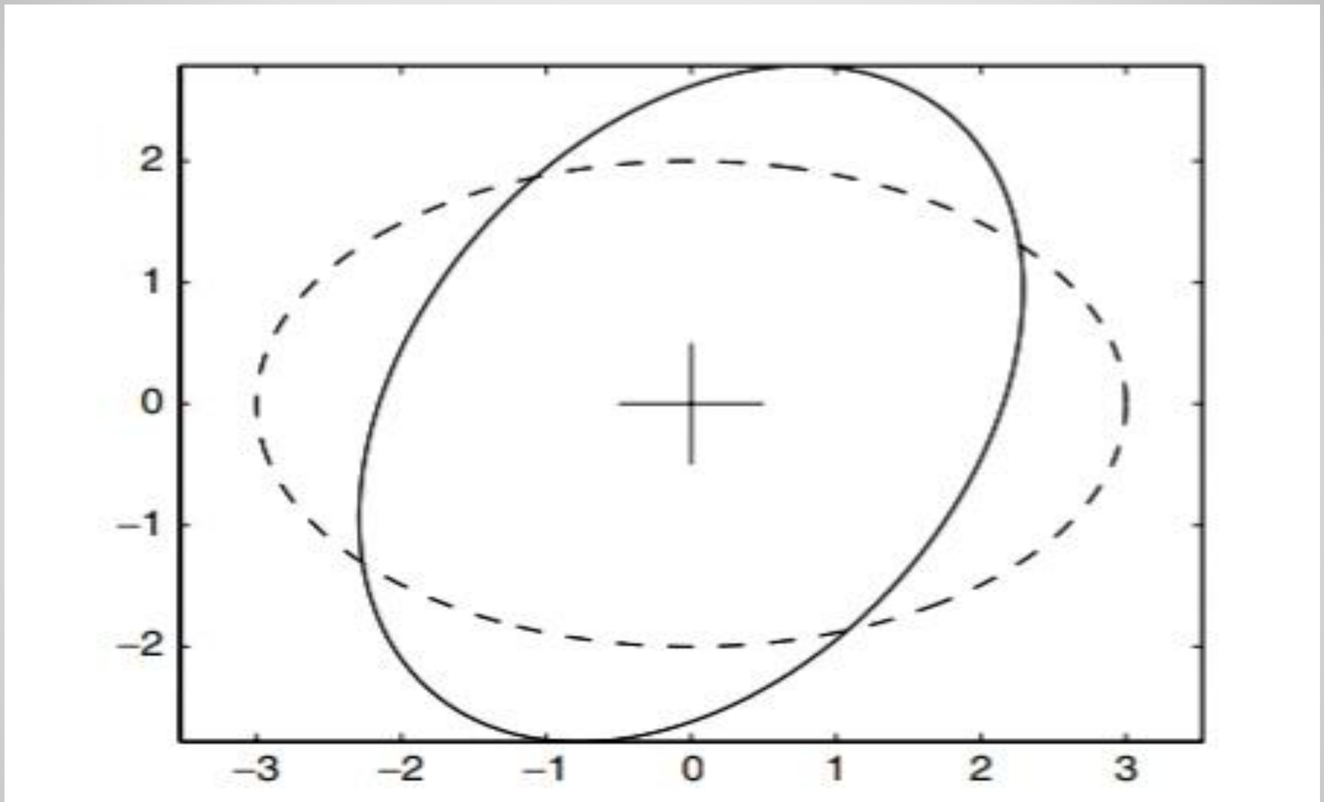
$\gg theta = input('entre\ the\ rotation\ angle')$

$\gg hold\ on$

$\gg x = \cos(theta) * x_0 - \sin(theta) * y_0$

$\gg y = \sin(theta) * x_0 + \cos(theta) * y_0$

$\gg plot(x, y)$; axis equal



2
1
0
−1
−2
−3
−2
−1
0
1
2
3

thank you