1- L8,9 4th year Math - 30-3-

Def: A top. sp. X is said to be connected if X can not be represented as a union of two disjoint nonempty open sets

Def: In a top. space X, two subsets A, B are said to form a separation of X, if $X = A \cup B$ and $\bar{A} \cap B = A \cap \bar{B} = \varphi$. A, B are called separated sets.

Theorem: A top. sp. X is connected iff X can not be expressed as the union of two nonempty separated sets in X.

Note $X = U \cup V$, U, V - disjoint open $\Longrightarrow$ U, V form a separation of X, since $\bar{U} \cap V = U \cap \bar{V} = \varphi$.

and if A, B are a separation of X $\Rightarrow$ both A and B open and closed which implies that X is not connected.

The connectedness for subspaces:

Theorem: Let (X, τ) - top. sp. and let $A \subset X$. Then the following conditions are equivalent:

(i) A is connected

(ii) A can not be expressed as the union of two non-empty sets that are separated in X

(iii) $\nexists$ $U, V \in \tau$ s.t $U \cap A \neq \varphi$, $V \cap A \neq \varphi$, $U \cap V \cap A = \varphi$, $A \subseteq U \cup V$.

proof: (see the book).

أثبت: لأى X ومعرف عليه التوبولوجي اللانهائي فإن X فضاء مترابط.

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٢- بفرض ان $X=[-1,0)\cup(0,1]$ في R مع التبولوجي الاعتيادي.
فان كل من $[-1,0)$ و $(0,1]$ مفتوحة ومغلقة في X وهما
تكونان separation لـ X لذلك X غير مترابط.

٣- بفرض ان Q مجموعة الاعداد النسبية في R فان Q ليست مترابطة
لانه على سبيل المثال $U=(-\infty,\sqrt{2})$, $V=(\sqrt{2},\infty)$
مجموعات مفتوحة تحقق

$Q\cap U\cap V=\phi$,

$U\cap Q\neq\phi\neq V\cap Q$,

$Q\subseteq U\cup V$.

$Q=(U\cup V)\cap Q$. اذا بالضبط

اي مجموعة جزئية من Q تكون اكثر من نقطة

والاكثر من ذلك تكون غير مترابطة. a,b بين $Y=\{a,b\}$ و $Y\subseteq Q$ لانه لاي ، وتكون
عليه اختيار اي عدد غير قياسي i

$Y=(-\infty,i)\cup(i,\infty)\cap Y$. حيث $(-\infty,i)$, (i,∞)

٤- في R^2 الفضاء

$X=A\cup B$,

$A=\{(x,y): y=0\}$, $B=\{(x,y): x>0, y=\frac{1}{x}\}$

A, B form a separation of X.

$\bar{A}\cap B=\phi=A\cap\bar{B}$, $X=A\cup B$. كما ان

مترابطات (R,τ_∞) ، (R,τ_c) —

٥- كل من (R,τ_{c}) (اي ثنت) مترابطات ، غير مترابط (X,e^t) حيث $|X|>1$ ، مترابطة

٦- (R,τ_{E}) totally disconnected نعني فقط الفضاءات المترابطة هي مجموعات نقطية.

$(-\infty,0)\cup(0,\infty)$

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Theorem: If the sets C, D form a separation of a top. space X, and if Y is any connected subspace of X, then Y is contained entirely either in C or in D.

Proof: Since C and D are both open in X, then $C \cap Y$ and $D \cap Y$ are open in Y, . $C \cap Y \cap D \cap Y = \varphi$.
If $C \cap Y$ and $D \cap Y$ are both nonempty, they will form a separation of Y. Therefore, one of them is empty. Hence Y must lie in C or D.

Using the fact that the property of being connected is preserved under continuous mapping, prove that: If $f: X \longrightarrow R$ is continuous real valued function, where X is connected. If $a, b \in X$ s.t $f(a) < f(b)$ and $d \in R$ s.t $f(a) < d < f(b)$. Then there exists $c \in X$ such that $f(c) = d$.

The following theorem is used to show that the finite product space of connected spaces is connected.

Theorem: Let $\{A_\alpha : \alpha \in \Delta\}$ be an indexed family of connected subsets of a topological sp. X. Let $\bigcap_{\alpha \in \Delta} A_\alpha \neq \varphi$ then $\bigcup_{\alpha \in \Delta} A_\alpha$ is connected

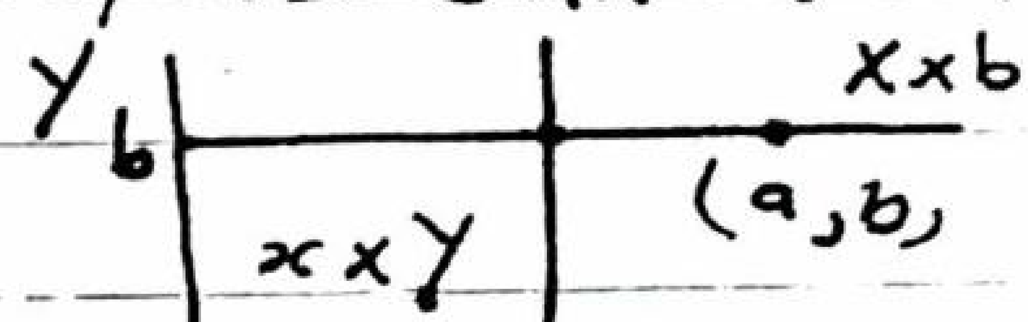
Proof: Let $\cap A_\alpha \neq \varphi$ and $A = \cup A_\alpha$ is not connected
Let U, V nonempty open subsets of A s.t $A = U \cup V$,
Let A_β any member of the family $\Rightarrow A_\beta \subseteq U \cup V$
Since A_β is connected then $A_\beta \subseteq U$ or $A_\beta \subseteq V$

suppose $A_\beta \subseteq U$. Then, since $A_\alpha \cap A_\beta \neq \varphi \ \forall \alpha \in \Delta$
$A_\alpha \subseteq U \Rightarrow \bigcup_{\alpha \in \Delta} A_\alpha \subseteq U$ and hence $V = \varphi$.

contradiction to the assumpt. that $V \neq \varphi$.

Theorem: The product of two connected spaces X and Y is connected.

proof: choose a point (a,b) in $X \times Y$.
The horizontal slice $X \times \{b\}$ is connected, being homeomorphic to X and each vertical slice $\{x\} \times Y$ is connected. The subset $A_x = X \times \{b\} \cup \{x\} \times Y$ is connec being the union of two connected spaces that have the point (x,b) in common.
form the union $\bigcup_{x \in X} A_x$ of all these connected sets. This union is connected because it is the union of a collection of connected sets that have the point (a,b) in common. since the union is $X \times Y$, then the space $X \times Y$ is connected.



using the fact that
$X_1 \times X_2 \times \cdots \times X_n$ is homeomorphic
with $(X_1 \times X_2 \times \cdots \times X_{n-1}) \times X_n$ and
By induction prove that any finite product of connected spaces is connected.

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Define the fixed point property and prove that any closed bounded interval $[a,b]$ with the standard relative top. has the fixed p. p.

Give an example of connected but not locally connected sp. and of locally connect but not connected sp.

Let τ be the lower limit top. on $\mathbb{R}$. Is $(\mathbb{R},\tau)$ connected? prove your answer.

what are the continuous map $f:(\mathbb{R},\tau_{N})\to(\mathbb{R},\tau_{[\,)})$