نظرية الحاسبات

المحاضرة السابعة

الزمن: ساعة

16.6. Finite – State Machine

A finite-state machine (FSM) is an abstract model of a machine with a primitive internal memory. It has a finite set of states, an input alphabet, and an output alphabet. The machine has an internal clock, that "ticks" at regular intervals. The machine is initiated by placing it in a fixed initial state before the clock is started. Just prior to each tick, the machine will be in one of its states. At the tick, the machine will accept an input and in response move to another state and produce an output.

What the next stage is will depend on what the input was as well as what the previous state was. If these two factors are given, the change in predictable and non random. In this way, over a succession of clock pulse, the machine produce a sequence of outputs in response to a sequence of inputs.

Definition. A finite – state machine M consists of

1. A finite set A of input symbols.
2. A finite set O of output.
3. A finite set S of states.
4. A **next-state function (or transition function)** f from S × A into S. i.e., $f : S \times A \to S$
5. An output function g from S × A into O.
6. An initial state $s_0 \in S$.

It is written as $M = \{A, O, S, f, g, s_0\}$

Example 25. The following defines a finite state machine M with two input symbols, tw
internal states, and two output symbols.

(1) $A = \{a, b\}$ (2) $O = \{0, 1\}$ (3) $S = \{s_0, s_1\}$

(4) Next state function $f : S \times A \rightarrow S$ defined by

$$f(s_0, a) = s_1$$
$$f(s_0, b) = s_1$$
$$f(s_1, a) = s_0$$
$$f(s_1, b) = s_1$$

(5) Output function $g : S \times A \to O$ defined by

$$g(s_0, a) = 1$$
$$g(s_0, b) = 1$$
$$g(s_1, a) = 0$$
$$g(s_1, b) = 1$$

(6) Initial state s_0.

State table and State Diagram of a Finite – State Machine

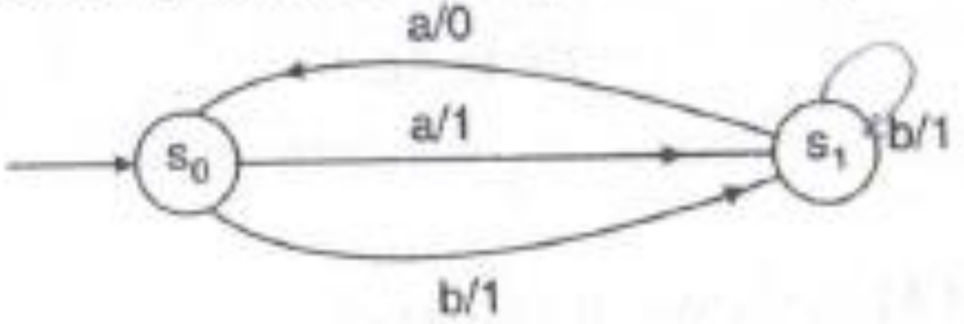
There are two ways of representing a finite state machine in compact form

1. State table, also known as **transition table** of the machine M.
2. State diagram, also known as **transition diagram** of the machine M

The state table is conventional tabular representation of the values of the next state function f and the output g for all pairs of states and inputs. The first column of the table lists the (present) stages for the machine. The entries in the second row are the elements of the input A, listed once under f and then again under g. The elements in the columns under g below the inputs are the elements of the output O. The state table of example 25 is shown in Table 16.2

	f		g	
S \ A	a	b	a	b
s_0	s_1	s_1	1	1
s_1	s_0	s_1	0	1

Let $M = \{A, O, S, f, g, s_o\}$ be a finite state machine. The state diagram of M is a finite directed labelled graph in which each vertex represents a state that belongs to S. All the states are represented by circles and the initial state by a circle with an arrow pointing towards it. This arrow does not originate from any vertex. The directed edges indicate the transition of a state and the edges are labelled with input/output. A directed edge (s_1, s_2) exists if there exists an input I (say) with $f(s_1, I) = s_2$. In this case if $g(s_1, I) = 0$, the edge (s_1, s_2) is labelled with I / 0 (input / output). The state diagram of the finite machine of example 25 appears in Fig. 16.6



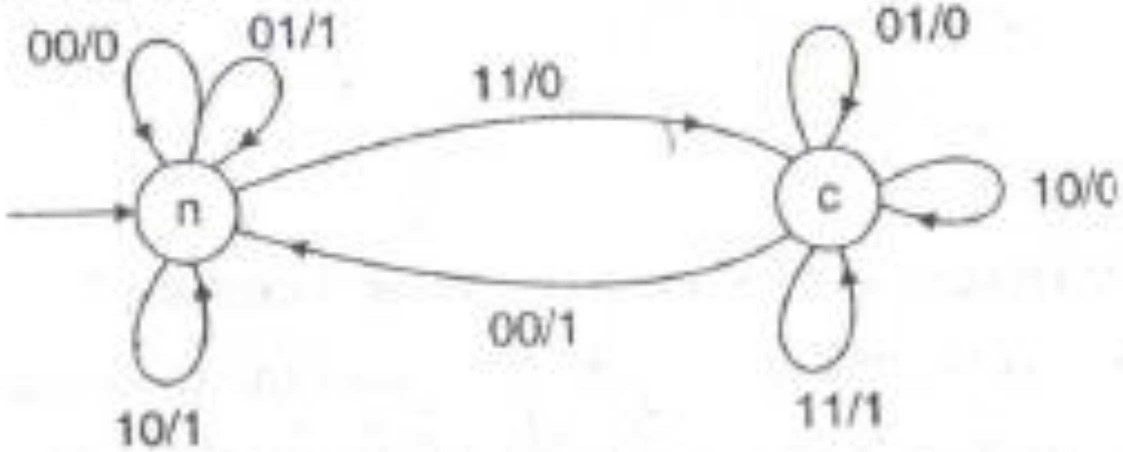
Example 26. Design a finite state machine that adds two binary integers x and y.

Solution. From binary addition, we know 0 + 0 = 0, 0 + 1 = 1, 1 + 0 = 1, 1 + 1 = 10

Let $x = (x_n \ldots\ldots x_1\, x_0)_2$ and $y = (y_n \ldots\ldots y_1\, y_0)_2$ we add the corresponding bits x_i and y_i from right to left. This yields a sum bit z_i and a carry bit c_i. The carry bit c_i is either 0 or 1. Now given an input $x_i\, y_i$ either x_i and y_i are added or x_i, y_i and 1 are added. Thus there are two state carry (c) are non-carry (n). Therefore, $S = \{c, n\}$, A = {00, 01, 10, 11} and output set $o = \{0, 1\}$.

Here n is the initial state and there is no carry prior to the addition of the least significant bits. The state diagram of the machine is shown in Fig. 16.7.

If 00 is input to n, output is 0 and remains in state n. Thus n has a loop labelled 00/0. If 11 is input to n, then there is a carry 1 and output 0 and hence there is a directed graph from n to c, labelled 11/0. If 11 is input to c, then $1 + 1 + 1 = 11$, in this case output is 1 and remains in state c. Thus c has a loop labelled 11/1.



'he state table of the finite state - machine is shown in table 16.3

S \ A	f				g			
	00	01	10	11	00	01	10	11
n	n	n	n	c	0	1	1	0
c	n	c	c	c	1	0	0	1

Example 27. Let M be the finite state machine with state table appearing in Table 16.4

(*a*) Find the input set A, the state set S, the output set O, and initial state of M.

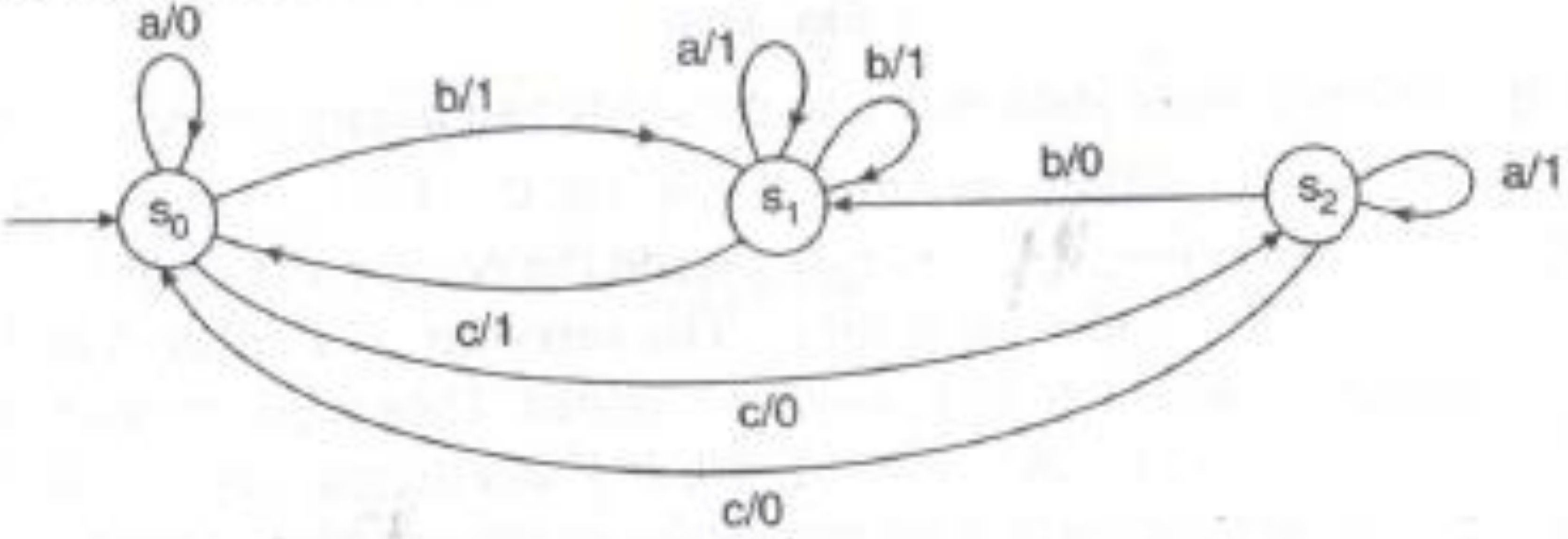
(*b*) Draw the state diagram of M

(*c*) Find the output string for the input string *aabbcc*.

	f			*g*		
S \ A	*a*	*b*	*c*	*a*	*b*	*c*
s_0	s_0	s_1	s_2	0	1	0
s_1	s_1	s_1	s_0	1	1	1
s_2	s_2	s_1	s_0	1	0	0

Solution. (*a*) The input set A = $\{a, b, c\}$, the state set S = $\{s_0, s_1, s_2\}$ output set O = $\{0, 1\}$ and s_0 is the initial state.

(*b*) The state diagram of M is shown in Fig. 16.8.



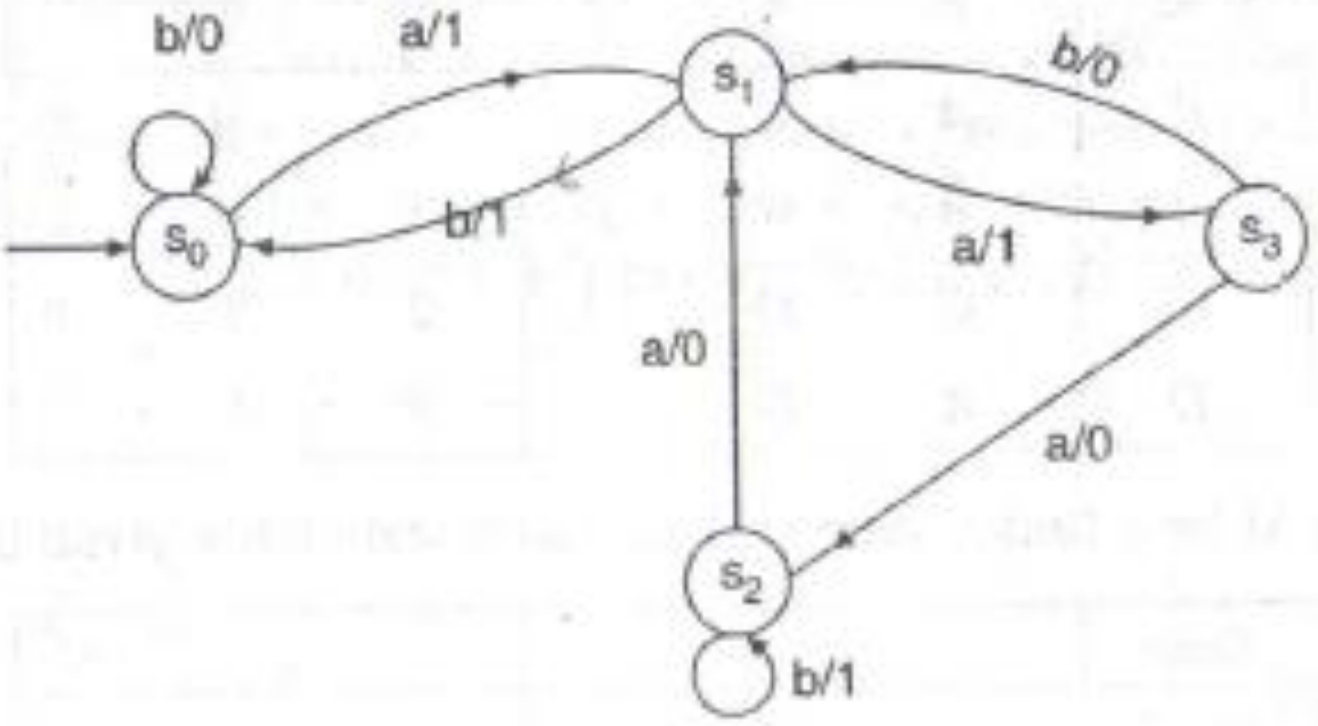
(*c*) Starting with initial state s_0, the input string aabbcc from state to state can be represented by tabled arrows as

$$\longrightarrow s_0 \xrightarrow{\ a\ } s_0 \xrightarrow{\ a\ } s_0 \xrightarrow{\ b\ } s_1 \xrightarrow{\ b\ } s_1 \xrightarrow{\ c\ } s_0 \xrightarrow{\ c\ } s_2$$

This yields the output string

0 0 1 1 1 0

Example 28. Given the state diagram of a finite state machine M, find (*a*) the input set A, the output set 0, the state set S and the initial state (*b*) the state table of M.

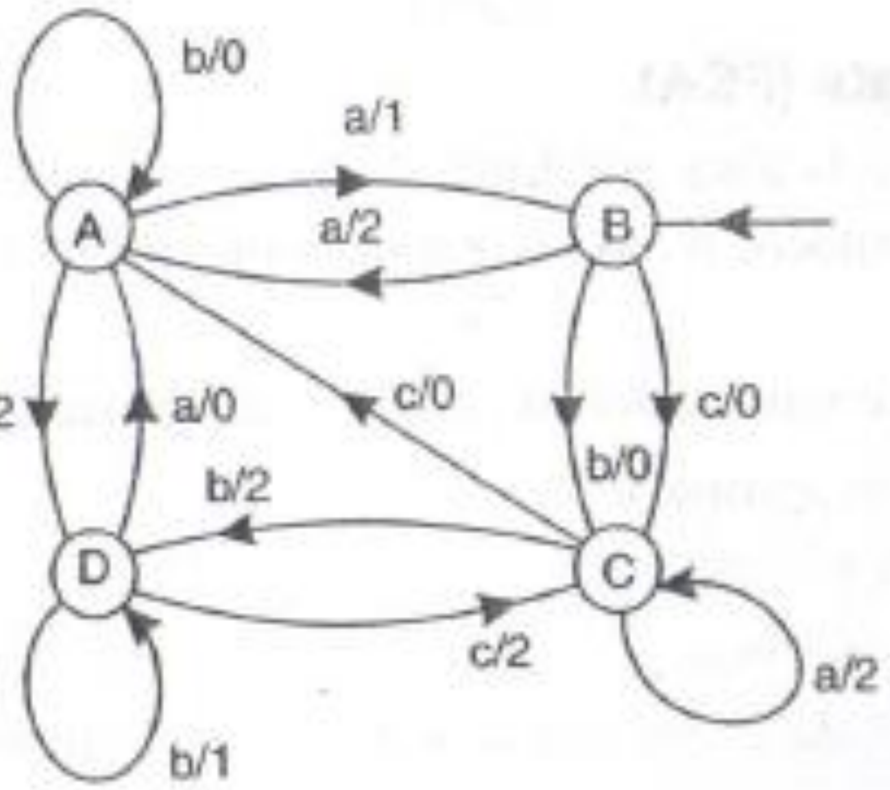


Solution. (*a*) The input set A = {*a*, *b*}, the output set O = {0, 1}, the state set S = {s_0, s_1, s_2, s_3} with initial state s_0.

(*b*) the state table of M is given in Table 16.5.

S \ A	*f*		*g*	
	a	*b*	*a*	*b*
s_0	s_1	s_0	1	0
s_1	s_3	s_0	1	1
s_2	s_1	s_2	0	1
s_3	s_2	s_1	0	0

Example 29. Find the set I, O, S the initial state and the table defining the next state and out put function for each finite-state machine.



Solution. The input set is I = { a, b, c} since a, b and c are the input labels of the directed arrows.

The output set is O = {0,1,2} since 0, 1 and 2 are the output lables of the directed arrows.

The state set S = {A, B,C, D} since A, B C and D are labelled by circles.

The initial state of S is B since the unlabeled arrow points to B.

The next -state table is given below.

	f			g		
S \ A	a	b	c	a	b	c
B	A	C	C	2	0	0
A	B	A	D	1	0	2
C	C	D	A	2	2	0
D	A	D	C	0	1	2

Example 30. Let M be a finite- state machine with state table given below

State	f			g		
S \ I	a	b	c	a	b	c
s_0	s_0	s_3	s_2	0	1	1
s_1	s_1	s_1	s_3	0	0	1
s_2	s_1	s_2	s_3	1	1	0
s_3	s_2	s_3	s_0	1	0	1

Draw the state diagrams M.

Solution. Here the input set I = $\{a, b, c\}$, the output set O = $\{0, 1\}$ and the state set S = $\{s_0, s_1, s_2, s_3\}$. The initial state is s_0. The state diagrams M is shown below.