Electromagnetic Theory

CREDIT HOURS FIRST LEVEL(PHYSICS /PHYSICS AND COMPUTER SCIENCE PROGRAM)

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Chapter 6: Sources of Magnetic Fields

- The Biot-Savart Law.
- Ampere's Law.
- Magnetic Flux.
- Displacement Current and the General Form of Ampere's Law.

B exerts a force on moving charges, also moving charges create a magnetic field.

The two equivalent ways of calculating **B** produced by currents are:

1- Biot-Savart Law: Field of a “current element”, analogous to a point charges in electrostatic.

2-Ampere’s Law: An integral theorem.

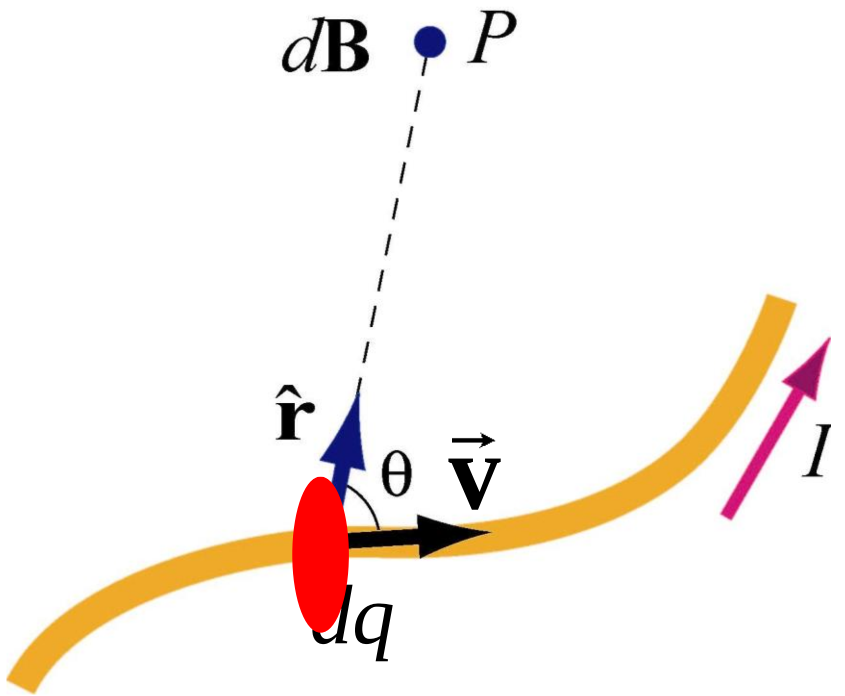
From Charges to Currents?

$$d\vec{\mathbf{B}} \propto dq\,\vec{\mathbf{v}}$$

$$= (\text{charge})\frac{\text{m}}{\text{s}}$$

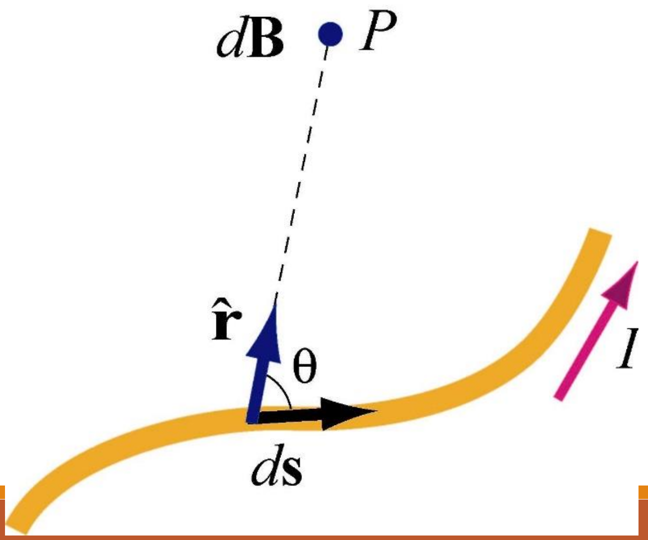
$$= \frac{\text{charge}}{\text{s}}\,\text{m}$$

$$\boxed{d\vec{\mathbf{B}} \propto I d\vec{\mathbf{s}}}$$



The Biot-Savart Law

Shortly after Orested's discovery in 1819 that a compass needle is deflected by a current-carrying conductor, Jean-Baptiste Biot and Felis Savart performed quantitative experiments on the force exerted by an electric current on a nearby magnet. From their experimental results, Biot and Savart arrived at a mathematical expression that gives the magnetic field at some point in space in terms of the current that produces the field. That expression is based on the following experimental observations for the magnetic Field d**B** at point P associated with length element d**s** of a wire carrying a steady current I.



$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I\, d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

Where, $\mu_0 = 4\pi\ x\ 10^{-7}\ \frac{T.m}{A}$

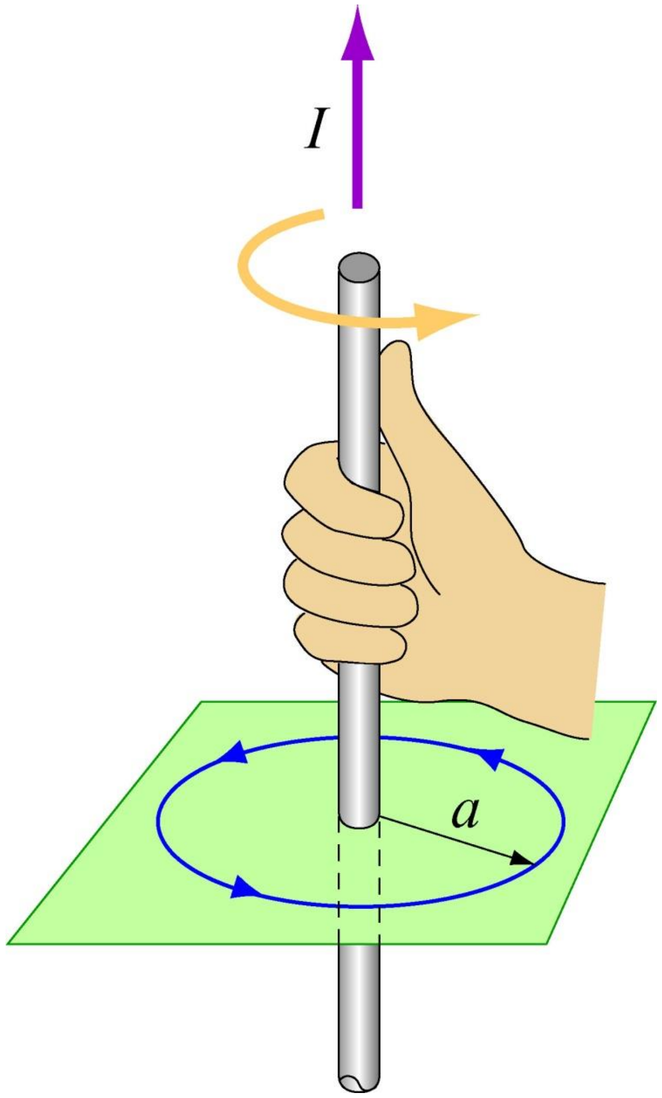
The total magnetic field **B** created at some point by a current of finite size, we must sum up contributions from all current elements I d**s** that make up the current. That is, we must evaluate **B** by integrating the previous equation as the following:

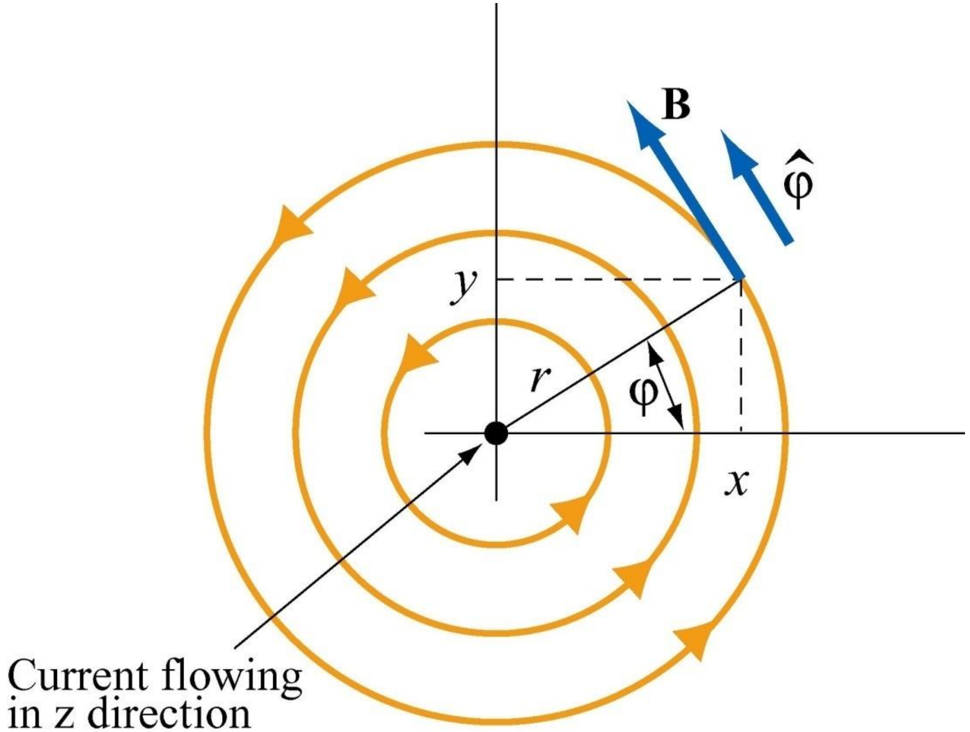
$$B=\frac{\mu_0 I}{4\pi}\int\frac{d\vec{s}\ x\ \hat{r}}{r^2}$$

Where the integral is taken over the entire current distribution.

- The magnitude of the magnetic field varies as the inverse square of the distance from the source, as does the electric field due to a point charge.
- The directions of two fields are quit different, the electric field created by a point charge is radial, but the magnetic field created by a current element is perpendicular to both the length element ds and the unit vector r, as described by the cross product(see lecture no_8).

Remember The Right-Hand Rule

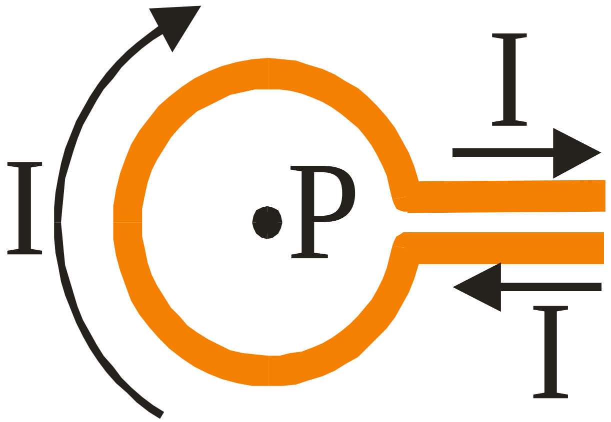




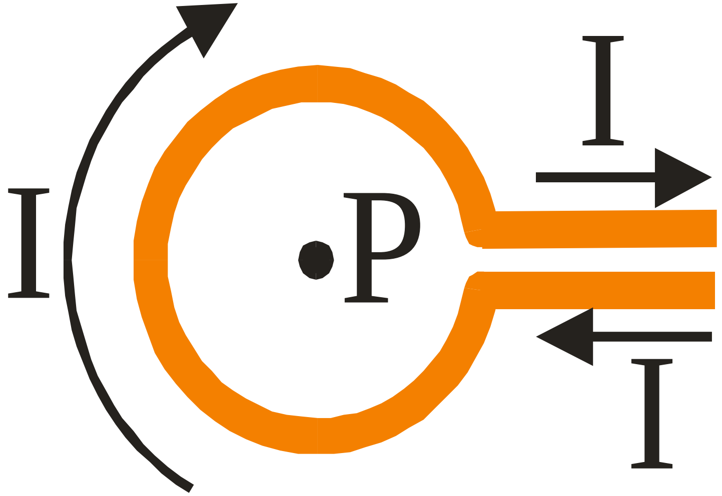
$$\hat{\mathbf{z}} \times \hat{\boldsymbol{\rho}} = \hat{\boldsymbol{\varphi}}$$

Example 1 : the magnetic field on Coil of Radius *R*

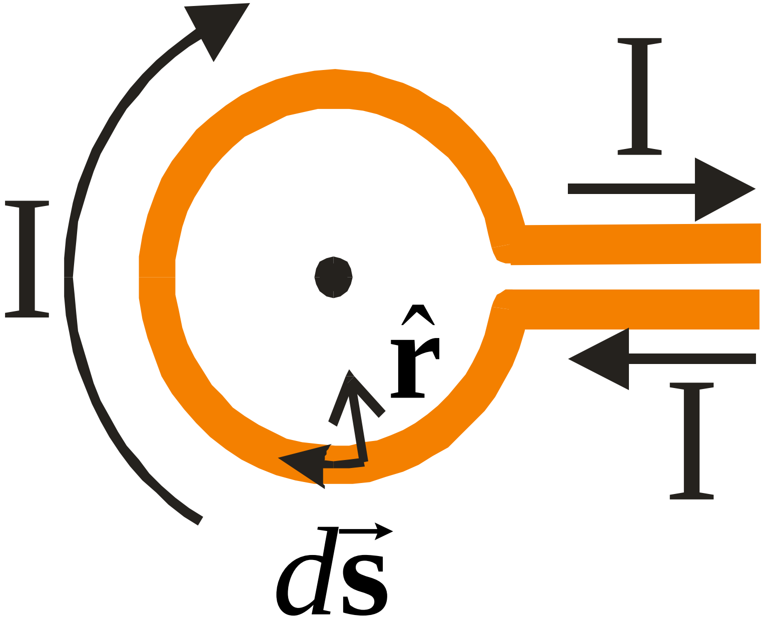
Consider a coil with radius *R* and current *I*



Find the magnetic field B at the center (P)



1) Think about it:
   - Legs contribute nothing / parallel to *r*
   - Ring makes field into page
2) Choose a ds
3) Pick your coordinates
4) Write Biot-Savart



In the circular part of the coil…

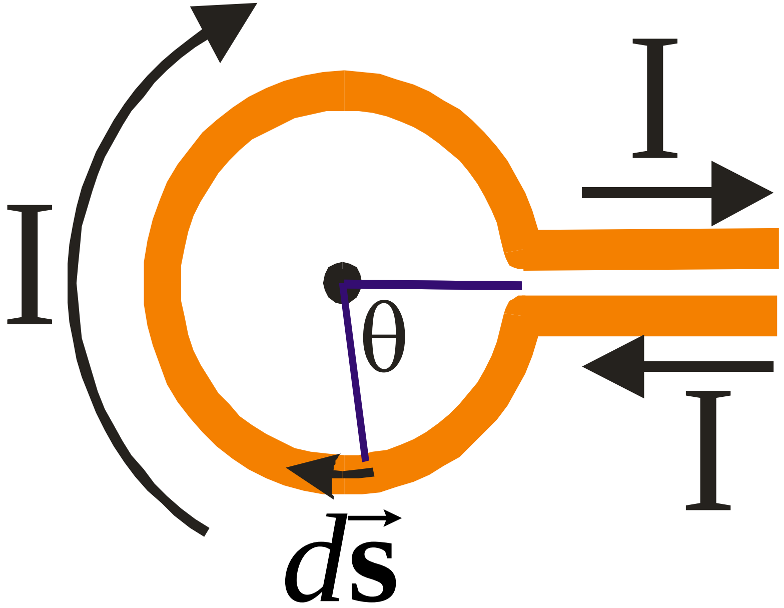
$$d\vec{\mathbf{s}} \perp \hat{\mathbf{r}} \quad \rightarrow \quad |d\vec{\mathbf{s}} \times \hat{\mathbf{r}}| = ds$$

Biot-Savart:

$$dB = \frac{\mu_0 I}{4\pi}\frac{\left|d\vec{\mathbf{s}}\times\hat{\mathbf{r}}\right|}{r^2} = \frac{\mu_0 I}{4\pi}\frac{ds}{r^2}$$

$$= \frac{\mu_0 I}{4\pi}\frac{R\,d\theta}{R^2}$$

$$= \frac{\mu_0 I}{4\pi}\frac{d\theta}{R}$$



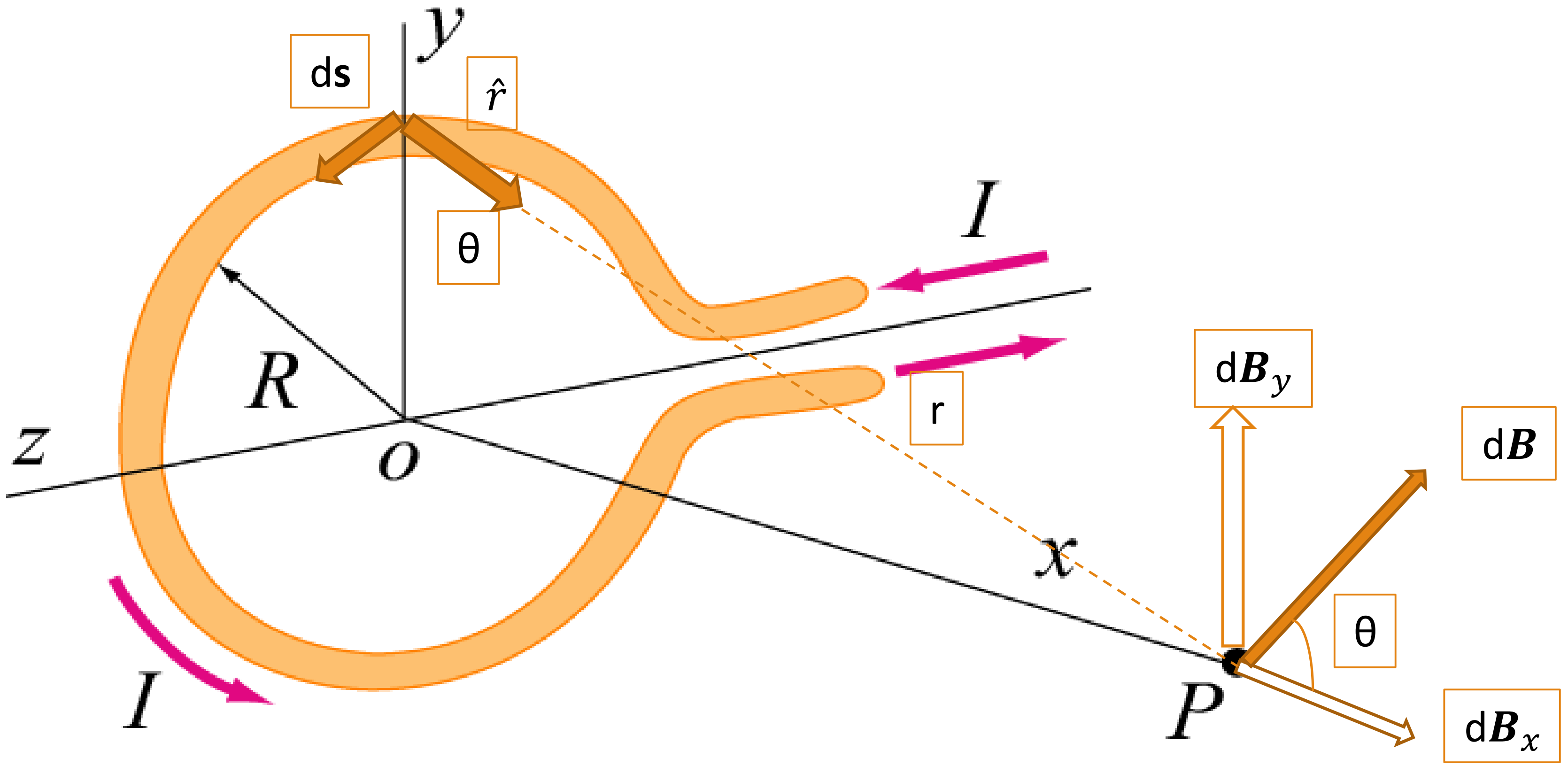
$$B = \int dB = \int_0^{2\pi}\frac{\mu_0 I}{4\pi}\frac{d\theta}{R} \qquad = \frac{\mu_0 I}{4\pi R}\int_0^{2\pi} d\theta = \frac{\mu_0 I}{4\pi R}(2\pi)$$

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{2R}\text{ into page}$$

Magnetic Field on The Axis of a Circular current loop of Radius R.

Consider a coil with radius R and carrying a current I

What is B at point P?



In this situation, every length element ds is perpendicular to the vector $\hat{r}$ at the location of the element. Thus, for any element, $|\boldsymbol{ds}\ x\ \hat{\boldsymbol{r}}|$ =(ds)(1) sin 90=ds. Furthermore, all length elements around the loop are at the same distance r from P, where $r^2 = x^2 + R^2$.

d**B**=$\frac{\mu_0\, I}{4\pi}\int \frac{|\boldsymbol{ds}\ x\ \hat{\boldsymbol{r}}|}{r^2} = \frac{\mu_0\, I}{4\pi}\ \frac{ds}{x^2+R^2}$

When the components dB_y are summed over all elements around the loop, the resultant component is zero.

That is, by symmetry the current in any element on one side of the loop sets up perpendicular component of dB that cancels the perpendicular component set up by the current through the element diametrically opposite it.

Therefore, the resultant field at P must be along the x axis and we can find it by integrating the components dB_x=dB cos θ **as the following:**

$$\boldsymbol{B}_x = \oint dB\cos\theta = \frac{\mu_0\, I}{4\pi}\oint \frac{ds\cos\theta}{x^2+R^2},\ \cos\theta = \frac{R}{\left(x^2+R^2\right)^{1/2}}$$

Where R, θ and x are constants, we obtain

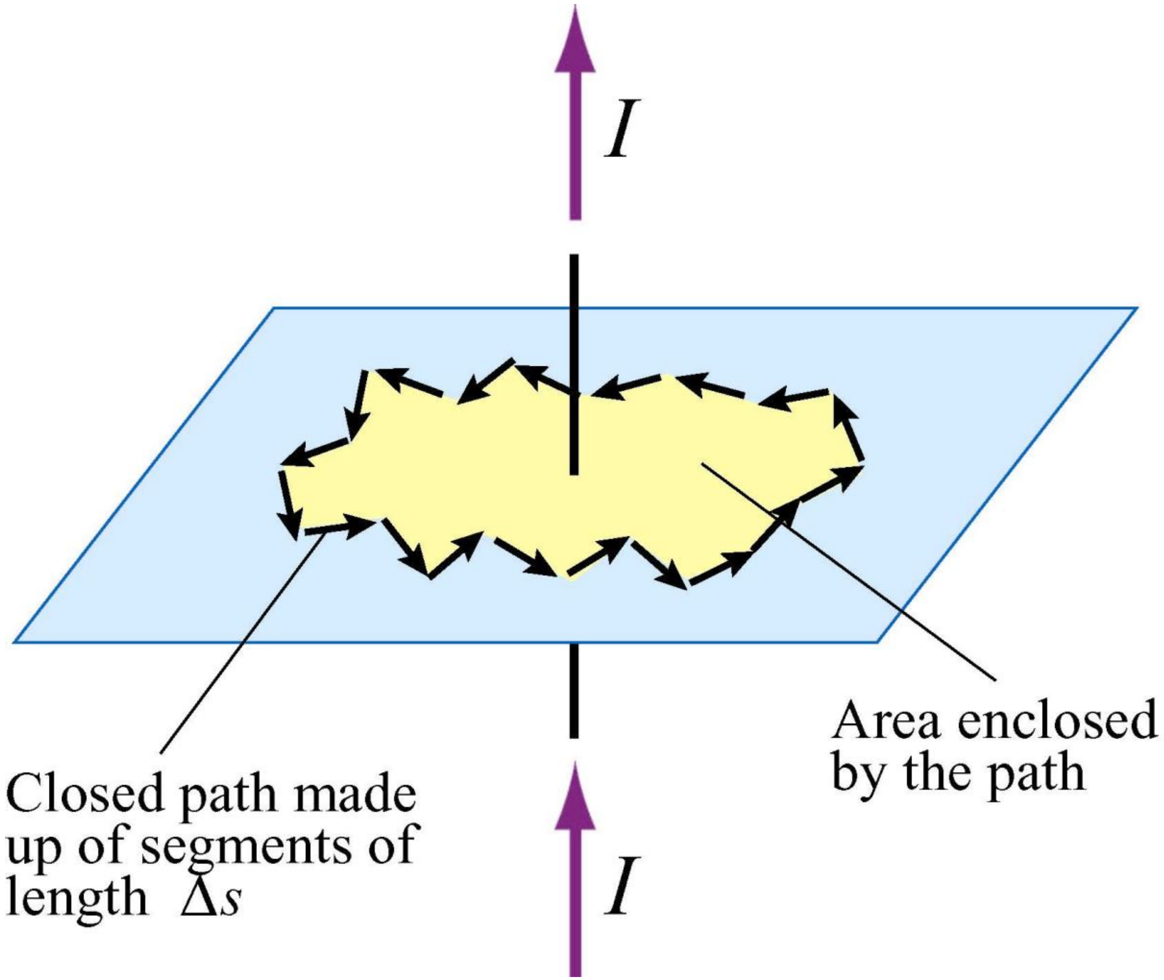
$$\boldsymbol{B}_x = \frac{\mu_0\, I\, R}{4\pi(x^2+R^2)^{3/2}}\oint ds = \frac{\mu_0\, I\, R}{4\pi(x^2+R^2)^{3/2}}.\,2\pi R = \frac{\mu_0\, I\, R^2}{2(x^2+R^2)^{3/2}}$$

At the center of the loop x=0 in the pervious equation we obtain:

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{2R}\ \text{into page}$$

An this as the pervious example.

Ampere’s Law: The Idea



Oersted’s 1819 discovery about deflected compass needles magnetic field, several compass needles are placed in a horizontal plane near a long vertical wire. When no current is present in the wire. In order to have a B field around a loop, there must be current punching through the loop.

$$\oint B.ds = B\oint ds = \frac{\mu_0 I}{2\pi r} 2\pi r = \mu_0 I,$$

where B surrounding a thin, straight conductor= $\frac{\mu_0 I}{2\pi r}$

This law (Ampere's Law) states that the integral of magnetic field density (B) along an imaginary closed path is equal to the product of current enclosed by the path and permeability of the medium.

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}$$

The line integral is around any closed contour bounding an open surface *S*. I_{enc} is current **through** *S* (called Amperian loop):

$$I_{enc} = \iint_S \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}}$$

Where J is the current density--→ J=I/A (SI unit $\frac{A}{m^2}$)

Biot-Savart vs. Ampere

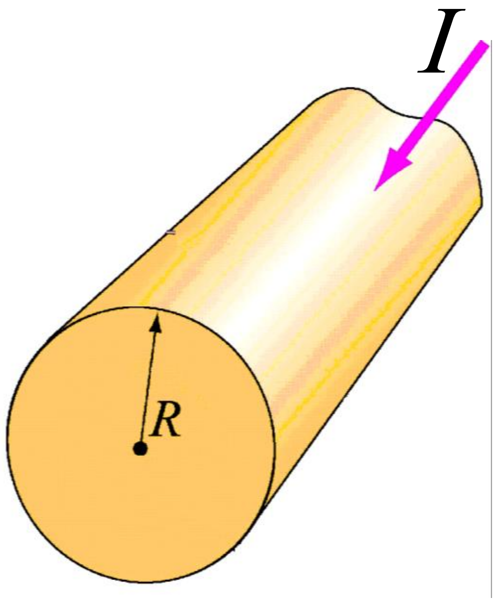
Biot-Savart Law	$\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2}$	general current source ex: finite wire wire loop
Ampere's law	$\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}$	symmetric current source ex: infinite wire infinite current sheet

Applying Ampere's Law

1. Identify regions in which to calculate B field
   Get B direction by right hand rule
2. Choose Amperian Loops S: Symmetry
   B is 0 or constant on the loop!
3. Calculate $\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}}$
4. Calculate current enclosed by loop S
5. Apply Ampere's Law to solve for B

$$\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}$$

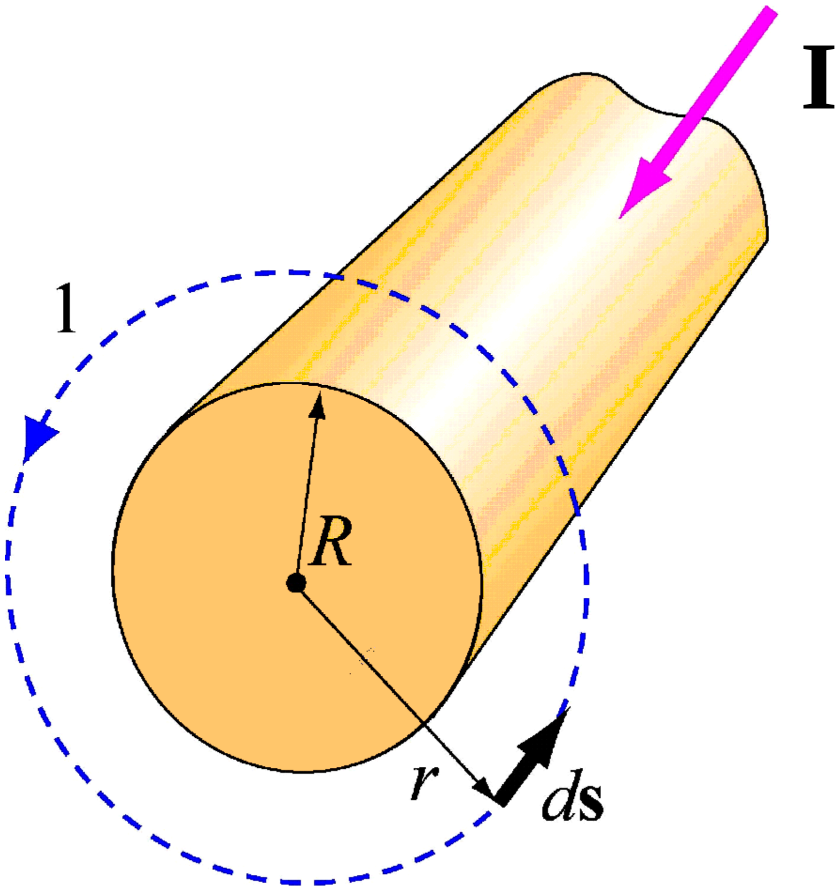
Example: the magnetic field created by a long current-carrying (Infinite Wire)

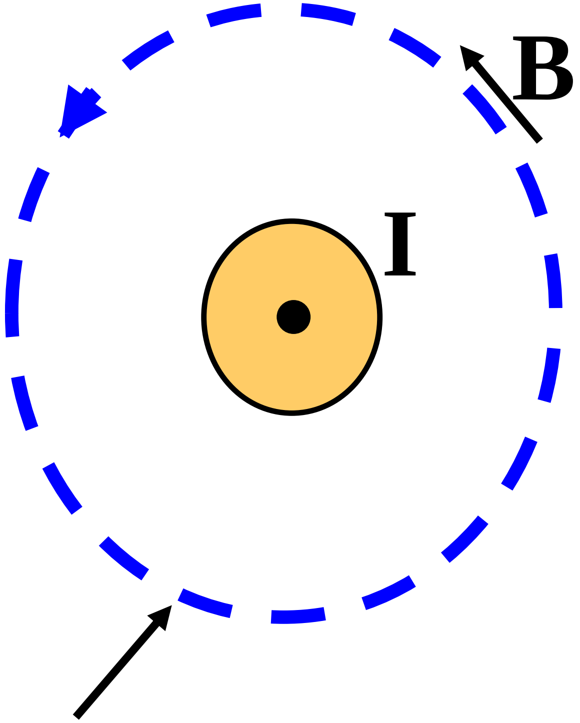


A long, straight wire of radius R carries a steady current I that is uniformly distributed through the cylindrical cross section of the wire.

Calculate the magnetic field at a distance r from the center of the wire in the regions

(1) outside wire ($r \geq R$), (2) inside wire ($r < R$)





<u>Amperian Loop</u>: B is Constant & Parallel

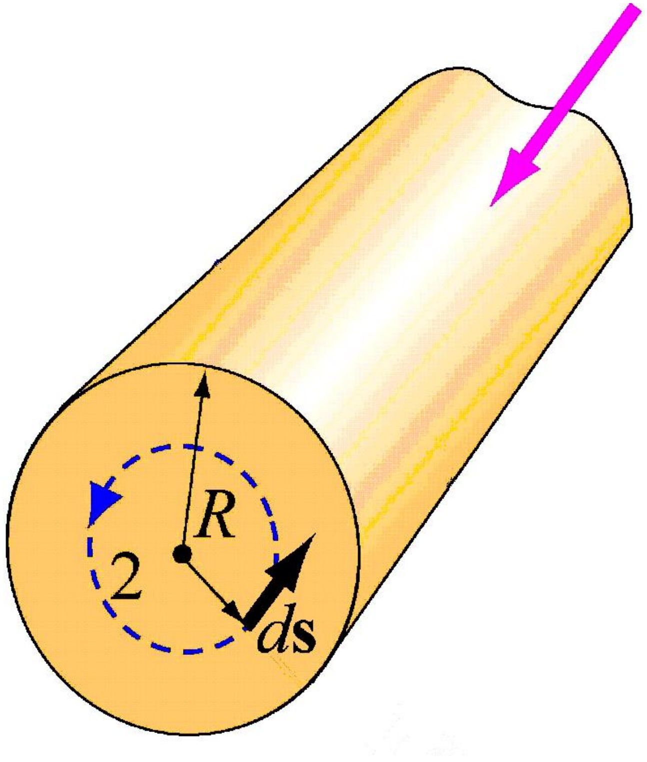
I Penetrates.

Solution: By using the Ampere's law to find B as the following:

$\oint B.ds = B \oint ds = B2\pi r = \mu_0 \, I$ → B= $\frac{\mu_0 \, I}{2\pi r}$, r ≥ R,

anticounterclockwise

Region 2: Inside wire (r < R)



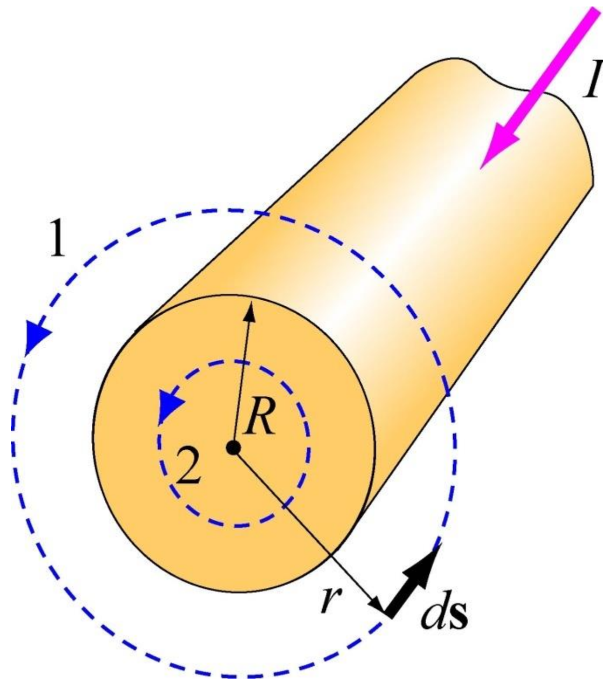
$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B\oint ds = B(2\pi r)$$

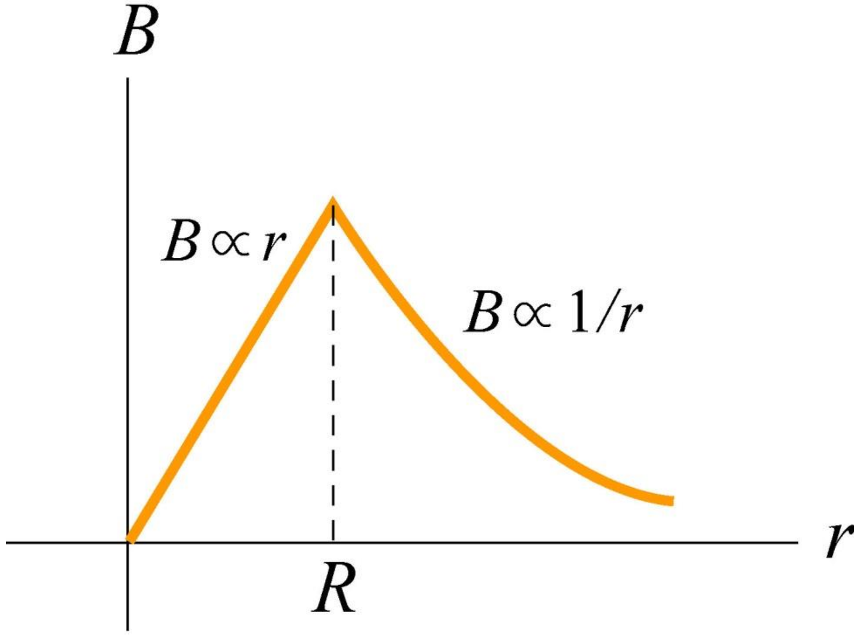
$$= \mu_0 I_{enc} = \mu_0 I\left(\frac{\pi r^2}{\pi R^2}\right)$$

$$\vec{\mathbf{B}} = \frac{\mu_0 I r}{2\pi R^2} \text{counterclockwise}$$

Could also say:

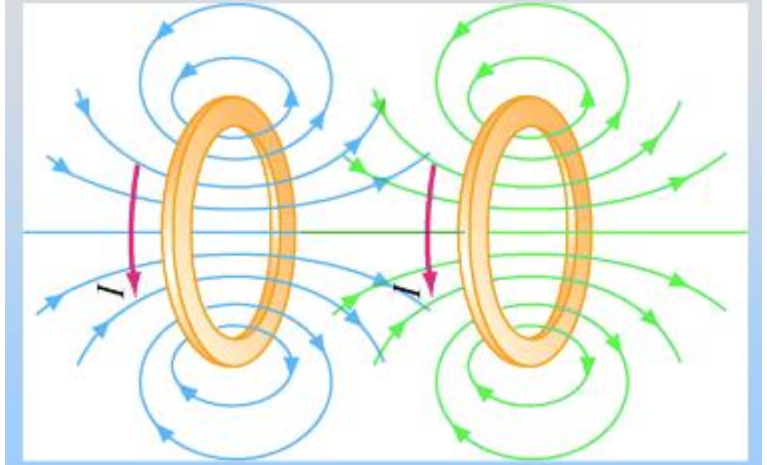
$$J = \frac{I}{A} = \frac{I}{\pi R^2};\ I_{enc} = JA_{enc} = \frac{I}{\pi R^2}\left(\pi r^2\right)$$



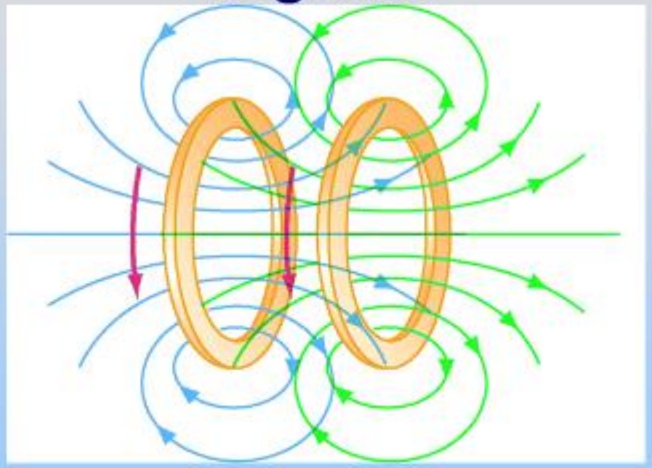


$$B_{in} = \frac{\mu_0 I r}{2\pi R^2} \qquad B_{out} = \frac{\mu_0 I}{2\pi r}$$

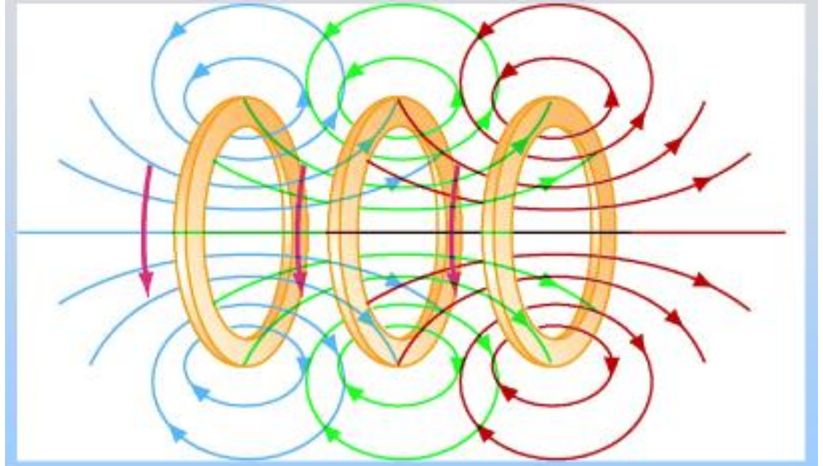
Two Loops



Two Loops Moved Closer Together

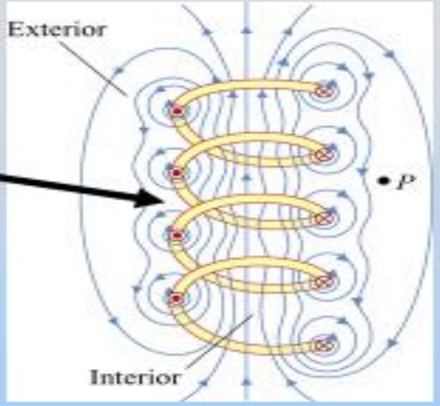


Multiple Wire Loops

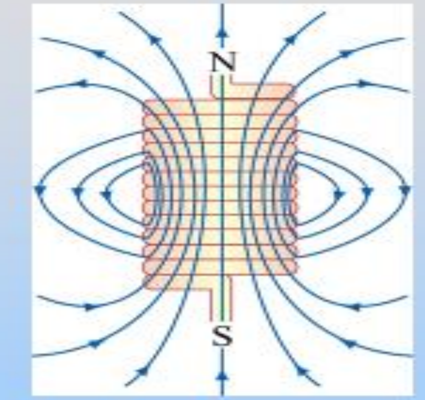


Magnetic Field of Solenoid

Horiz. comp. cancel



loosely wound



tightly wound

For ideal solenoid, B is uniform inside & zero outside

Maxwell's Equations (So Far)

Gauss's Law: $$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0}$$

Electric charges make diverging Electric Fields

Magnetic Gauss's Law: $$\oiint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

No Magnetic Monopoles! (No diverging B Fields)

Ampere's Law: $$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}$$

Currents make curling Magnetic Fields

Magnetic Flux:

In electromagnetism, a sub-discipline of physics, the magnetic flux through a surface is the surface integral of the normal component of the magnetic field (B) passing through that surface. Denoted by Φ or Φ_B.

Magnetic Flux is defined as the number of magnetic field lines passing through a given closed surface. It gives the measurement of the total magnetic field that passes through a given surface area. Here, the area under consideration can be of any size and under any orientation with respect to the direction of the magnetic field.

Magnetic Flux Unit

Magnetic flux is usually measured with a **fluxmeter**. The SI and CGS unit of magnetic flux is given below:

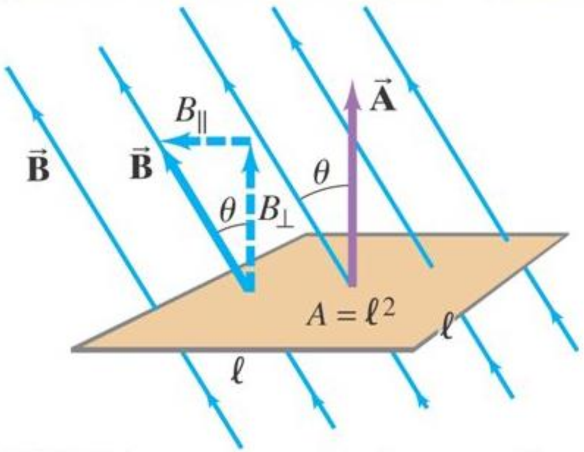
SI unit of magnetic flux is **Weber(Wb)**.

The fundamental unit is **Volt-seconds**.

The CGS unit is **Maxwell**.

$\emptyset_B = \int \boldsymbol{B}.d\boldsymbol{A} = \boldsymbol{BA}\,\boldsymbol{cos}\,\boldsymbol{\theta}$, maximum value at $\boldsymbol{\theta}$=0, but when the magnetic field is parallel to the plane, then $\boldsymbol{\theta}$= 90 º the flux through the plane is zero.

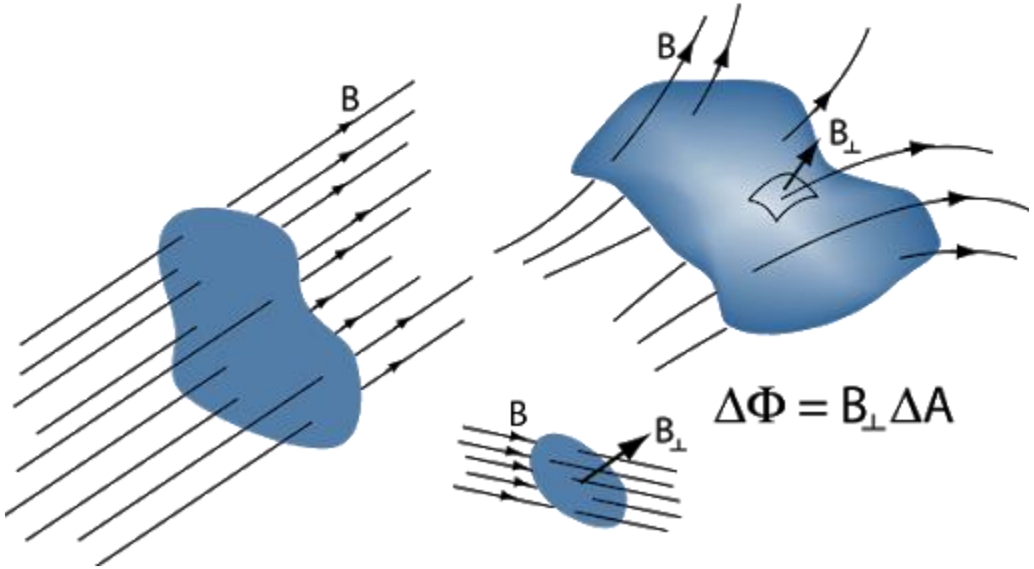
- Out of Faraday's investigations, the development of the concept of ***magnetic flux*** developed (similar to electric flux)



$$\Phi_B = B \bullet A = BA\cos\theta$$

for any surface ***A***, made of infinitely small segments ***dA***, of arbitrary shape:

$$\Phi_B = B \bullet \sum \Delta A = \int \vec{B} \bullet \vec{dA}$$



What is Magnetic Flux Density ?

Magnetic flux density(B) is defined as the force acting per unit current per unit length on a wire placed at right angles to the magnetic field.

•Units of B is Tesla (T) or $\frac{Kg}{A\,sec^2} = \frac{Wb}{m^2}$

•B is a vector quantity

Magnetic Flux Density Unit:

The CGS and SI unit of magnetic flux density is given in the table below.

Units of Magnetic Flux Density (B)	
SI unit	Tesla (abbreviated as T)
CGS unit	Gauss (abbreviated as G or Gs)

Displacement current (D)

In electromagnetism, **displacement current density** is the quantity ∂***D***/∂*t* appearing in Maxwell's equations that is defined in terms of the rate of change of ***D***, the electric displacement field. Displacement current density has the same units as electric current density J($\frac{A}{m^2}$, SI unit), and it is a source of the magnetic field just as actual current is. However it is not an electric current of moving charges, but a time-varying electric field. In physical materials (as opposed to vacuum), there is also a contribution from the slight motion of charges bound in atoms, called dielectric polarization.

The idea was conceived by James Clerk Maxwell in his 1861 paper *On Physical Lines of Force, Part III* in connection with the displacement of electric particles in a dielectric medium. Maxwell added displacement current to the electric current term in Ampère's Circuital Law. In his 1865 paper A Dynamical Theory of the Electromagnetic Field Maxwell used this amended version of Ampère's Circuital Law to derive the electromagnetic wave equation. This derivation is now generally accepted as a historical landmark in physics by virtue of uniting electricity, magnetism and optics into one single unified theory. The displacement current term is now seen as a crucial addition that completed Maxwell's equations and is necessary to explain many phenomena, most particularly the existence of electromagnetic waves.

Maxwell added this term to Ampere's Law $I_d = \in_0 \frac{d\emptyset_E}{dt}$

so the Ampere's Law modification by Maxwell is

$$\oint B.ds = \mu_0(I + I_d) = \mu_0 I + \mu_0 \in_0 \frac{d\emptyset_E}{dt}$$