Lecture 8

Lec 8

Recapitulate

- We discussed some examples involving the use of Lorentz Transformation.
- We discussed the problem of $\mu-$ meson shower and explained how time dilation helped us undertand it.

Velocity Transformation

We know the components of velocity a particle in and want to find the same in S' for the same particle .

Notations

$\vec{V}$ Relative velocity between frames. Constant as a function of time.

$\vec{u}$ Instantaneous velocity of particle in S . Need not be constant .

$\vec{u}'$ Instantaneous velocity of particle in S' . Need not be constant

Events Related to Displacement:

Imagine that a particle is moving in x- direction in a frame S.

E1 : Particle found at x_1 at t_1

E2 : Particle found at x_2 at t_2

Even if the velocity of particle is not constant:

$$\frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

In the limit Δt tending to zero , would give the instantaneous velocity of particle in S .

If the motion is in three dimension , in general

$$u_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

$$u_y = \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}$$

$$u_z = \lim_{\Delta t \to 0} \frac{\Delta z}{\Delta t}$$

Similarly looking at the same particle in S' , we can define

$$u'_x = \lim_{\Delta t' \to 0} \frac{\Delta x'}{\Delta t'}$$

$$u'_y = \lim_{\Delta t' \to 0} \frac{\Delta y'}{\Delta t'}$$

$$u'_z = \lim_{\Delta t' \to 0} \frac{\Delta z'}{\Delta t'}$$

Note that like displacement , the time difference has also to be measured in one's own frame .

Lorentz Transformation in differential form:

$$\Delta x' = \gamma(\Delta x - v\,\Delta t)$$
$$\Delta y' = \Delta y$$
$$\Delta z' = \Delta z$$
$$\Delta t' = \gamma(\Delta t - \frac{v\,\Delta x}{c^2})$$

$$\frac{\Delta x'}{\Delta t'} = \frac{\Delta x - v\,\Delta t}{\Delta t - \frac{v\,\Delta x}{c^2}} = \frac{\frac{\Delta x}{\Delta t} - v}{1 - \frac{v}{c^2}\frac{\Delta x}{\Delta t}}$$

$$u'_x = \frac{u_x - v}{1 - \frac{v u_x}{c^2}}$$

$$\frac{\Delta y'}{\Delta t'} = \frac{\Delta y}{\gamma(\Delta t - \frac{v\,\Delta x}{c^2})} = \frac{\frac{\Delta y}{\Delta t}}{\gamma(1 - \frac{v}{c^2}\frac{\Delta x}{\Delta t})}$$

$$u'_y = \frac{u_y}{\gamma(1 - \frac{v u_x}{c^2})}$$

$$\frac{\Delta z'}{\Delta t'} = \frac{\Delta z}{\gamma(\Delta t - \frac{v\,\Delta x}{c^2})} = \frac{\frac{\Delta z}{\Delta t}}{\gamma(1 - \frac{v}{c^2}\frac{\Delta x}{\Delta t})}$$

$$u_z' = \frac{u_z}{\gamma(1 - \frac{vu_x}{c^2})}$$

Velocity Transformation Equations :

$$u_x' = \frac{u_x - v}{1 - \frac{vu_x}{c^2}}$$

$$u_y' = \frac{u_y}{\gamma(1 - \frac{vu_x}{c^2})}$$

$$u_z' = \frac{u_z}{\gamma(1 - \frac{vu_x}{c^2})}$$

Inverse Velocity Transformation

$$u_x = \frac{u_x' + v}{1 + \frac{vu_x'}{c^2}}$$

$$u_y = \frac{u_y'}{\gamma(1 + \frac{vu_x'}{c^2})}$$

$$u_z = \frac{u_z'}{\gamma(1 + \frac{vu_x'}{c^2})}$$

Comment:

One can show that :

- If $u \prec c$ in S, $u \prec c \; in \; S'$ also irrespective of v
- If $u = c$ in S, $u = c \; in \; S'$ also irrespective of v

Example

$v = 0.8c\ ; u_x = -0.9c$
$u_y = 0\ ; u_z = 0$

Then

$$u'_x = \frac{-(0.9c + 0.8c)}{1 + \frac{0.9c \times 0.8c}{c^2}} = -\frac{1.7}{1.72}c = -0.988c$$

$u'_y = u'_z = 0$

Example2

A rod of proper length 1 meter is moving with a speed of 0.6c in +x direction as seen in frame S . In the same frame a particle is found to move in -x direction with speed of 0.8c . Find the time taken for the particle to cross the rod in : (a) rod frame S' . (b) in frame S'' of particle and (c) in S frame .

Events :

E1 : particle reaching end B of the rod.

E2 : particle reaching end A of the rod.

Use of formula:

- Is there a frame in which time interval between the two events is proper ?
- Is there a frame in which length of the rod is proper ?

Time determination:

We can go from S'' to S' or from S'' to S by use of time dilation formula.

However , we cannot go from S to S' directly.

How to go about it:

We need to find out time interval in one frame then we can find in any other by use of formula being careful of the flow direction.

Try in S':

In this frame length is proper , but speed of the particle is not known . So we have to find that applying velocity transformation

$$v = 0.6c; u_x = -0.8c$$
$$u'_x = \frac{-1.4c}{1+0.48} = -\frac{1.4c}{1.48}$$
$$L' = 1m$$

$$\Delta t' = \frac{L'}{u'_x} = \frac{1}{\frac{1.4c}{1.48}} = \frac{1.48}{1.4c} = 3.52\times10^{-9}s$$

To find time interval in other frames, we must know the γ values.

$$\gamma_{SS'} = \frac{1}{\sqrt{1-(0.6)^2}}$$

$$\gamma_{SS''} = \frac{1}{\sqrt{1-(0.8)^2}} = \frac{5}{3}$$

$$\gamma_{S'S''} = \frac{1}{\sqrt{1-(\frac{1.4}{1.48})^2}} = \frac{1.48}{.48} \approx 3.08$$

$$\Delta t'' = \frac{\Delta t'}{\gamma_{S'S''}} = \frac{1.48}{1.4c}\times\frac{0.48}{1.48} = 1.14\times10^{-9}s$$

$$\Delta t = \gamma_{SS''}\times\Delta t'' = \frac{5}{3}\times\frac{0.48}{1.4c} = 1.90\times10^{-9}s$$

Use of length contraction formula:

We can find length in S and S'' by using the following flow chart

Length in other frames:

$$L'' = \frac{L'}{\gamma_{S'S''}} = \frac{1}{1.48}\times0.48m$$
$$L = \frac{L'}{\gamma_{SS'}} = \frac{1}{1.25} = 0.8m$$

Time in S'':

$$\Delta t'' = \frac{L''}{V_{S'S''}} = \frac{1}{1.48} \times 0.48 \times \frac{1.48}{1.4c} = 1.14 \times 10^{-9} s$$

Time in S:

$$0.6c\,\Delta t + 0.8c\,\Delta t = L = 0.8$$

$$\Delta t = \frac{0.8}{1.4c} = 1.90 \times 10^{-9} s$$

Summary

- We obtained velocity transformation.
- Discussed some examples involving velocity transformation.