Special Relativity

Lec 9

Three Event Problem

Recapitulate:

- We worked out velocity transformation.
- We discussed examples of relative velocity and those involving transformations between three frames of references.

Example 1

A person gets sick while moving in a train , the speed of which is 0.6c in the positive x- direction (Call this frame P) .He shines light asking for help when he is 30 Km away from the origin in +x direction .A hospital is located at the origin (Called the H frame). As soon as the light reaches the hospital , an ambulance (called A frame)is found in front , which is moving in positive x- direction with a speed of 0.8c .The person in the hospital instructs the ambulance to go and help the sick man. Assume no time delay between receiving the signal and giving the instruction and no slowing down of car and the train.

1) What are the positions and times of the peron ending the ignal in H and A frames ?
2) Find the position of the person in the H frame at t=0 and in A frame at $t'=0$
3) Find the time in H frame and A frames when the ambulance reaches the person .

Identify Events:

E1: Sick person sending the distress signal.

E2 : Hospital receiving the signal .

E3 : Ambulance reaching the sick person .

H Frame :

E1 : $x_1 = 3\times10^4 m, t_1 = ?$

Assume signal is received at hospital time equal to 0

$$t_1 = -\frac{3\times10^{+4}}{c} = -1\times10^{-4} s$$

E2 : $x_2 = 0, t_2 = 0$

Given that hospital is at the origin .

E3: $(?,?)$

The position of the person at t=0

$$3\times 10^4 + \frac{3\times 10^4}{c}\times 0.6c = 4.8\times 10^4 m$$

If t_3 is the time when ambulance reaches the sick person

$$4.8\times 10^4 + 0.6c\times t_3 = 0.8c\times t_3$$

$$t_3 = \frac{4.8\times 10^4}{0.2c} = 8\times 10^{-4} s$$

Coordinate of the person at E3.

$$x_3 = 0.8c\times 8\times 10^{-4} = 19.2\times 10^4 m$$

Event Table in H:

$E1:(3\times 10^4 m, -1\times 10^{-4} s)$

$E2:(0,0)$

$E3:(19.2\times 10^4, 8\times 10^{-4} s)$

A Frame:

Make the frame H and A satisfy the criteria to use Lorentz transformation directly ?

$$\gamma = \frac{1}{\sqrt{1-(0.8)^2}} = \frac{5}{3}$$

$E1:$

$$x_1' = \frac{5}{3}(3\times 10^4 + 0.8c\times 10^{-4}) = 9\times 10^4 m$$

$$t_1' = \frac{5}{3}(-1\times 10^{-4} - \frac{0.8c\times 3\times 10^4}{c^2}) = -3\times 10^{-4} s$$

$E2: x_2' = 0, t_2' = 0$

$E3:$

$$x_3' = \frac{5}{3}(19.2\times 10^4 - 0.8c\times 8\times 10^{-4}) = 0m$$

$$t_3' = \frac{5}{3}(8\times 10^{-4} - \frac{0.8c\times 19.2\times 10^4}{c^2}) = 4.8\times 10^{-4} s$$

We could have found out coordinates of E3 , without using transformation also .

The x has to be zero because it occurs at the origin of A frame .

We can use time dilation for time part . $(t_3' - t_2')$ being proper time interval between E2 and E3.

$$\frac{t_3 - t_2}{t_3' - t_2'} = \gamma$$

$$t_3' = 0 + \frac{8\times10^{-4} - 0}{\gamma} = 4.8\times10^{-4}s$$

Event Table in A:

Are all questions answered ?

What are the positions and times of the person sending the signal in H and A frames?

Find the position of the person in the H frame at t=0 and in A frame at $t' = 0$ (48km and ?)

According to A , E1 occurred when the person was 90 km away and at time $t' = -3\times10^{-4}s$. A reaches origin at $t' = 0$. During this time the person was travelling with a relative speed u_x' .

$$u_x' = \frac{0.6c - 0.8c}{1 - 0.6\times0.8} = -\frac{0.2}{0.52}$$

The distance travelled by person in A frame in $3\times10^{-4}s$ is:

$$\frac{0.2c}{0.52}\times3\times10^{-4} \approx 3.46\times10^4 m$$

The distance of person in A frame at $t' = 0$ is:

$$9\times10^4 - 3.46\times10^4 = 5.54\times10^4 m$$

If we have to find absolute time in P frame , watches have to be resynchronized . This can become cumbersome . However , we can use different form .

Suppose the problem also asks us to find out the time the ambulance took to reach P in P's frame after sending the distress signal .

We realize that E1 and E2 took place at same value of x_P in P's frame . Hence this time interval is dilated in H and A frames.

$$\frac{t_3 - t_1}{t_3'' - t_2''} = \gamma_{PH}$$

$$t_3'' - t_2'' = \frac{9 \times 10^{-4}}{1.25} = 7.2 \times 10^{-4} s$$

Just to confirm we can find the time interval in A frame using this proper time interval between E3 and E1 in P frame .

$$\frac{t_3' - t_1'}{t_3'' - t_2''} = \gamma_{PA}$$

$$\gamma_{PA} = \frac{1}{\sqrt{1 - (\frac{0.2}{0.52})^2}} = \frac{13}{12}$$

$$t_3' - t_2' = 7.2 \times 10^{-4} \times \frac{13}{12} = 7.8 \times 10^{-4} s$$

This matches with what we had evaluated using Lorentz transformation.