# المحاضرة العاشرة

# الفرقة: الثالثة

# الشعبة: الرياضيات

# المادة: نظرية المعادلات التفاضلية

In Exercises 1–12,

(a) Determine all equilibria and classify (node, saddle, spiral, center, stable or unstable).
(b) If an eigenvalue is real, find the eigenvectors.
(c) Graph the phase plane using the phase plane of linearized system.
(d) Graph the phase plane using the direction field from software.
(e) Graph the phase plane using the method of nullclines and part (c).

1. $\frac{dx}{dt} = x + xy,\ \frac{dy}{dt} = 2y - 4xy.$
2. $\frac{dx}{dt} = -x + xy,\ \frac{dy}{dt} = -2y + 8xy.$
3. $\frac{dx}{dt} = 2x - 2xy,\ \frac{dy}{dt} = y - xy.$
4. $\frac{dx}{dt} = 1 - x^2,\ \frac{dy}{dt} = y + 1.$
5. $\frac{dx}{dt} = x - y,\ \frac{dy}{dt} = -2x + 2xy.$
6. $\frac{dx}{dt} = y^3 + 1,\ \frac{dy}{dt} = x^2 + y.$
7. $\frac{dx}{dt} = 1 - y^2,\ \frac{dy}{dt} = 1 - x^2.$
8. $\frac{dx}{dt} = x(1 - y^2),\ \frac{dy}{dt} = x + y.$

9. $\dfrac{dx}{dt} = x - y + x^2$, $\dfrac{dy}{dt} = x + y$.

10. $\dfrac{dx}{dt} = 2x - y - xy$, $\dfrac{dy}{dt} = x + 2y$.

11. $\dfrac{dx}{dt} = -x - 2y$, $\dfrac{dy}{dt} = 2x - y + xy^2$.

12. $\dfrac{dx}{dt} = 2x + y$, $\dfrac{dy}{dt} = -x - 2y + y^3$.

Exercises 13–17 refer to $\frac{dx}{dt} = x - xy + \gamma x^2$, $\frac{dy}{dt} = -y + xy$. In each case find all equilibria and classify (node, saddle, spiral, center, stable or unstable).

13. $\gamma = -8$.

14. $\gamma = -\dfrac{1}{3}$.

15. $\gamma = \dfrac{1}{3}$.

16. $\gamma = 1$.

17. $\gamma = 8$.

1. $\frac{dx}{dt} = x(1+y) = x + xy = f$

$\frac{dy}{dt} = y(2-4x) = 2y - 4xy = g$

(a) Equilibria: $f = 0$ and $g = 0 \Rightarrow$

$x(1+y) = 0 \Rightarrow$Either $x = 0$ or $y = -1$

$y(2-4x) = 0 \Rightarrow$If $x = 0$, then $y = 0$. If $y = -1$, then $x = \frac{1}{2}$.

Thus equilibria are $(0,0)$ and $\left(\frac{1}{2}, -1\right)$.

(b) $A = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{bmatrix} = \begin{bmatrix} 1+y & x \\ -4y & 2-4x \end{bmatrix}$.

For equilibrium $(0,0)$, $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$. The eigenvalue, $\lambda$, satisfies

$\det(A - \lambda I) = 0 \Rightarrow \det \begin{bmatrix} 1-\lambda & 0 \\ 0 & 2-\lambda \end{bmatrix} = 0$

$\Rightarrow (1-\lambda)(2-\lambda) = 0 \Rightarrow \lambda = 1, 2$ (both positive).

So the equilibrium $(0,0)$ is an unstable node.

Eigenvector, $\begin{bmatrix} u \\ v \end{bmatrix}$, satisfies $\begin{bmatrix} 1-\lambda & 0 \\ 0 & 2-\lambda \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

Eigenvector for $\lambda = 1$ : $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow v = 0$.

$u$ is free to choose, say, $u = 1 \Rightarrow \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

Eigenvector for $\lambda = 2$ : $\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow u = 0$.

$v$ is free to choose, say, $v = 1 \Rightarrow \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

For equilibrium $\left(\frac{1}{2}, -1\right)$, $A = \begin{bmatrix} 0 & \frac{1}{2} \\ 4 & 0 \end{bmatrix}$. The eigenvalue, $\lambda$, satisfies $\det(A - \lambda I) = 0 \Rightarrow \det \begin{bmatrix} -\lambda & \frac{1}{2} \\ 4 & -\lambda \end{bmatrix} = 0 \Rightarrow \lambda^2 - 2 = 0$ $\Rightarrow \lambda = \pm\sqrt{2}$ (one positive, one negative). So the equilibrium $\left(\frac{1}{2}, -1\right)$ is a saddle point.

Eigenvector, $\begin{bmatrix} u \\ v \end{bmatrix}$, satisfies $\begin{bmatrix} -\lambda & \frac{1}{2} \\ 4 & -\lambda \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

Eigenvector for $\lambda = \sqrt{2}$ : $\begin{bmatrix} -\sqrt{2} & \frac{1}{2} \\ 4 & -\sqrt{2} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow -\sqrt{2}u + \frac{1}{2}v = 0$. Let $u = 1$. Then $v = 2\sqrt{2}$.

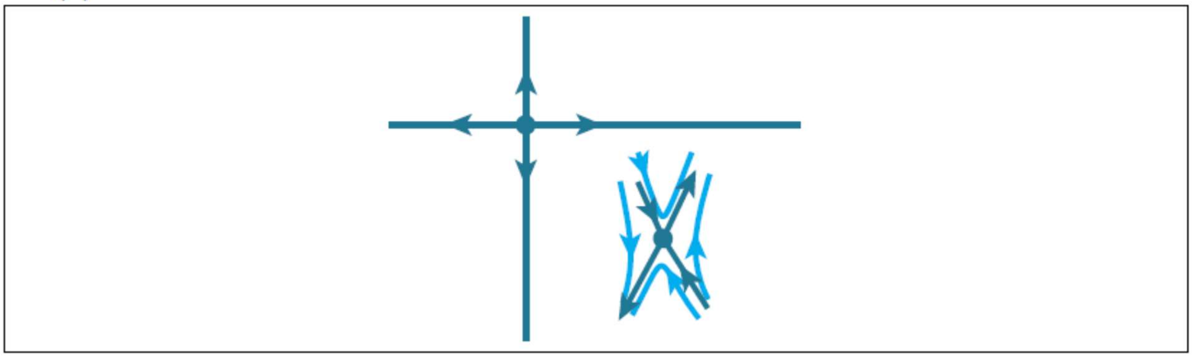
So $\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 1 \\ 2\sqrt{2} \end{bmatrix}$.

Eigenvector for $\lambda = -\sqrt{2}$ : $\begin{bmatrix} \sqrt{2} & \frac{1}{2} \\ 4 & \sqrt{2} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow$

$\sqrt{2}u + \frac{1}{2}v = 0$. Let $u = 1$. Then $v = -2\sqrt{2}$.

So $\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 1 \\ -2\sqrt{2} \end{bmatrix}$.

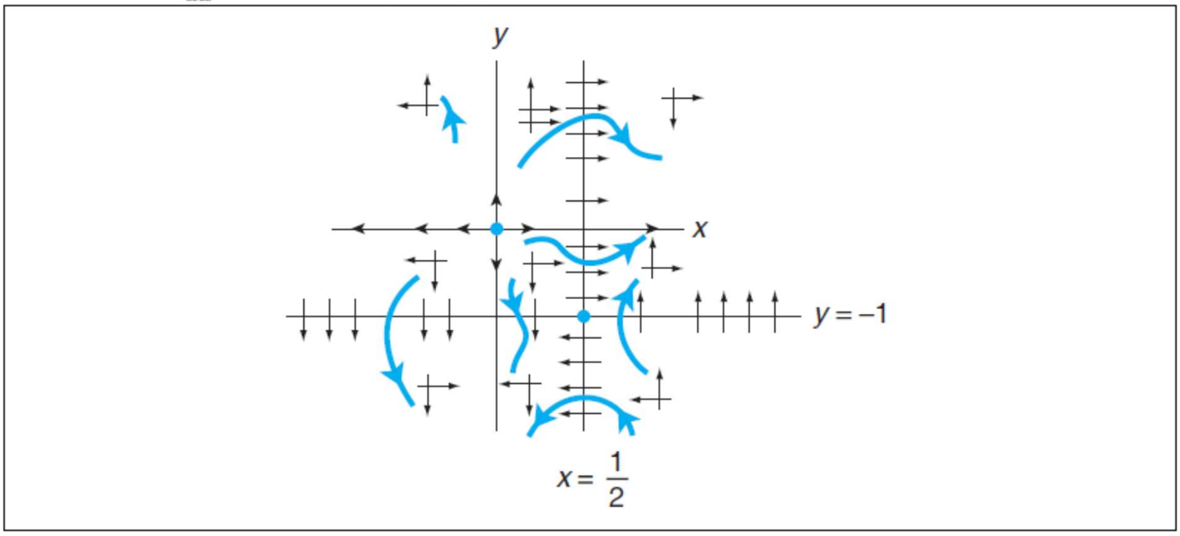
(c)



(e) Nullclines: $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{y(2-4x)}{x(1+y)}$.

So $\frac{dy}{dx} = 0 \Rightarrow y\,(2-4x) = 0 \Rightarrow y = 0,\ x = \frac{1}{2}$

and $\frac{dy}{dx} = \infty \Rightarrow x\,(1+y) = 0 \Rightarrow x = 0,\ y = -1.$



3. $\frac{dx}{dt} = 2x - 2xy = 2x(1-y) = f$
$\frac{dy}{dt} = y - xy = y(1-x) = g$

(a) Equilibria: $f = 0$ and $g = 0 \Rightarrow$

$2x(1-y) = 0 \Rightarrow$Either $x = 0$ or $y = 1$

$y(1-x) = 0 \Rightarrow$If $x = 0$, then $y = 0$. If $y = 1$, then $x = 1$.

Thus equilibria are $(0,0)$ and $(1,1)$.

(b) $A = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{bmatrix} = \begin{bmatrix} 2-2y & -2x \\ -y & 1-x \end{bmatrix}$.

For equilibrium $(0,0)$, $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$. The eigenvalue, $\lambda$, satisfies

$\det(A - \lambda I) = 0 \Rightarrow \det \begin{bmatrix} 2-\lambda & 0 \\ 0 & 1-\lambda \end{bmatrix} = 0$

$\Rightarrow (1-\lambda)(2-\lambda) = 0 \Rightarrow \lambda = 1, 2$ (both positive).

So the equilibrium $(0,0)$ is an unstable node.

Eigenvector, $\begin{bmatrix} u \\ v \end{bmatrix}$, satisfies $\begin{bmatrix} 2-\lambda & 0 \\ 0 & 1-\lambda \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

Eigenvector for $\lambda = 1$ : $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow u = 0$.

$v$ is free to choose, say, $v = 1 \Rightarrow \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Eigenvector for $\lambda = 2$ : $\begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow v = 0$.

$u$ is free to choose, say, $u = 1 \Rightarrow \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

For equilibrium $(1,1)$, $A = \begin{bmatrix} 0 & -2 \\ -1 & 0 \end{bmatrix}$. The eigenvalue, $\lambda$, satisfies

$\det(A - \lambda I) = 0 \Rightarrow \det \begin{bmatrix} -\lambda & -2 \\ -1 & -\lambda \end{bmatrix} = 0 \Rightarrow \lambda^2 - 2 = 0$

$\Rightarrow \lambda = \pm\sqrt{2}$ (one positive, one negative).

So the equilibrium $(1,1)$ is a saddle point.

Eigenvector, $\begin{bmatrix} u \\ v \end{bmatrix}$, satisfies $\begin{bmatrix} -\lambda & -2 \\ -1 & -\lambda \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

Eigenvector for $\lambda = \sqrt{2}$ : $\begin{bmatrix} -\sqrt{2} & -2 \\ -1 & -\sqrt{2} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

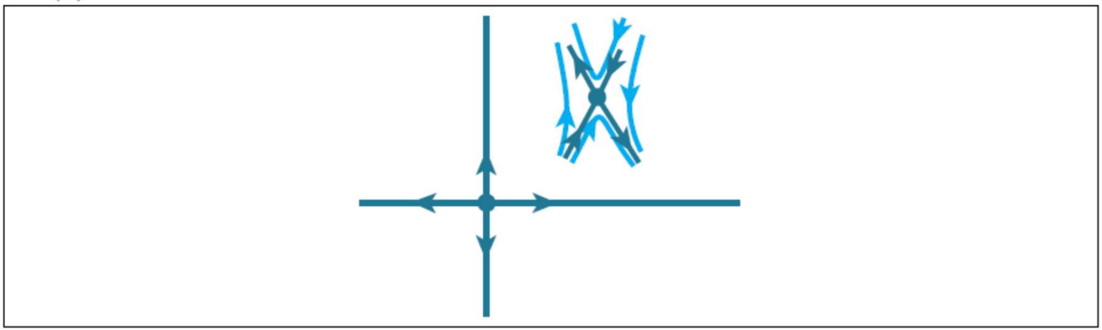
$\Rightarrow -\sqrt{2}u - 2v = 0$. Let $u = \sqrt{2}$. Then $v = -1$.

So $\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sqrt{2} \\ -1 \end{bmatrix}$ is unstable direction.

Eigenvector for $\lambda = -\sqrt{2}$ : $\begin{bmatrix} \sqrt{2} & -2 \\ -1 & \sqrt{2} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow$

$\sqrt{2}u - 2v = 0$. Let $u = \sqrt{2}$. Then $v = 1$. So $\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sqrt{2} \\ 1 \end{bmatrix}$ is stable direction.

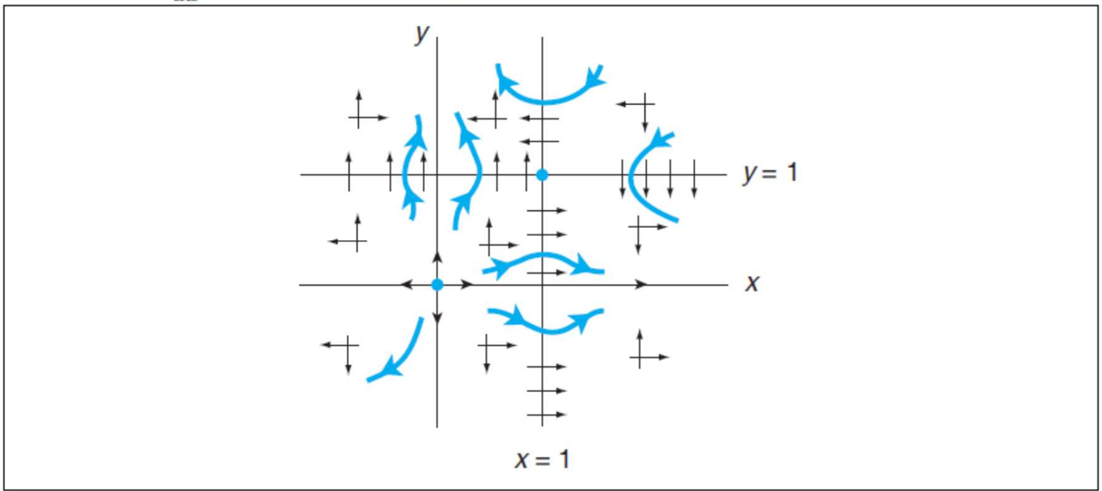
(c)



(e) Nullclines: $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{y(1-x)}{2x(1-y)}$.

So $\frac{dy}{dx} = 0 \Rightarrow y(1-x) = 0 \Rightarrow y = 0,\ x = 1$

and $\frac{dy}{dx} = \infty \Rightarrow 2x(1-y) = 0 \Rightarrow x = 0,\ y = 1$.



7. $\frac{dx}{dt} = 1 - y^2 = f$

$\frac{dy}{dt} = 1 - x^2 = g$

(a) Equilibria: $f = 0$ and $g = 0 \Rightarrow$

$1 - y^2 = 0 \Rightarrow y = \pm 1$

$1 - x^2 = 0 \Rightarrow x = \pm 1.$

Thus equilibria are $(1, 1)$, $(-1, -1)$, $(1, -1)$ and $(-1, 1)$.

(b) $A = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{bmatrix} = \begin{bmatrix} 0 & -2y \\ -2x & 0 \end{bmatrix}$.

For equilibrium $(1, 1)$, $A = \begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}$. The eigenvalue, $\lambda$, satisfies

$\det(A - \lambda I) = 0 \Rightarrow \det \begin{bmatrix} -\lambda & -2 \\ -2 & -\lambda \end{bmatrix} = 0 \Rightarrow \lambda^2 - 4 = 0 \Rightarrow \lambda^2 = 4$

$\Rightarrow \lambda = -2, 2$ (one positive, one negative).

So the equilibrium $(1, 1)$ is a saddle point.

Eigenvector, $\begin{bmatrix} u \\ v \end{bmatrix}$, satisfies $\begin{bmatrix} -\lambda & -2 \\ -2 & -\lambda \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

Eigenvector for $\lambda = 2$ : $\begin{bmatrix} -2 & -2 \\ -2 & -2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow$

$-2u - 2v = 0$. Let $u = 1$. Then $v = -1 \Rightarrow \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

is unstable direction.

Eigenvector for $\lambda = -2$ : $\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow$

$2u - 2v = 0$. Let $u = 1$. Then $v = 1 \Rightarrow \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

is stable direction.

For equilibrium $(-1, -1)$, $A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$. The eigenvalue, $\lambda$, satisfies

$\det(A - \lambda I) = 0 \Rightarrow \det \begin{bmatrix} -\lambda & 2 \\ 2 & -\lambda \end{bmatrix} = 0 \Rightarrow \lambda^2 - 4 = 0 \Rightarrow \lambda^2 = 4$

$\Rightarrow \lambda = -2, 2$ (one positive, one negative).

So the equilibrium $(-1, -1)$ is a saddle point.

Eigenvector, $\begin{bmatrix} u \\ v \end{bmatrix}$, satisfies $\begin{bmatrix} -\lambda & 2 \\ 2 & -\lambda \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

Eigenvector for $\lambda = 2$ : $\begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow -2u + 2v = 0$. Let $u = 1$. Then $v = 1 \Rightarrow \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is unstable direction.

Eigenvector for $\lambda = -2$ : $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow$

$2u + 2v = 0$. Let $u = 1$. Then $v = -1 \Rightarrow \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is stable direction.

For equilibrium $(1, -1)$, $A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$. The eigenvalue, $\lambda$, satisfies

$\det(A - \lambda I) = 0 \Rightarrow \det \begin{bmatrix} -\lambda & 2 \\ -2 & -\lambda \end{bmatrix} = 0 \Rightarrow \lambda^2 + 4 = 0$

$\Rightarrow \lambda = \pm 2i$ (complex).

So the equilibrium $(1, -1)$ is a center (nonlinear may be different).

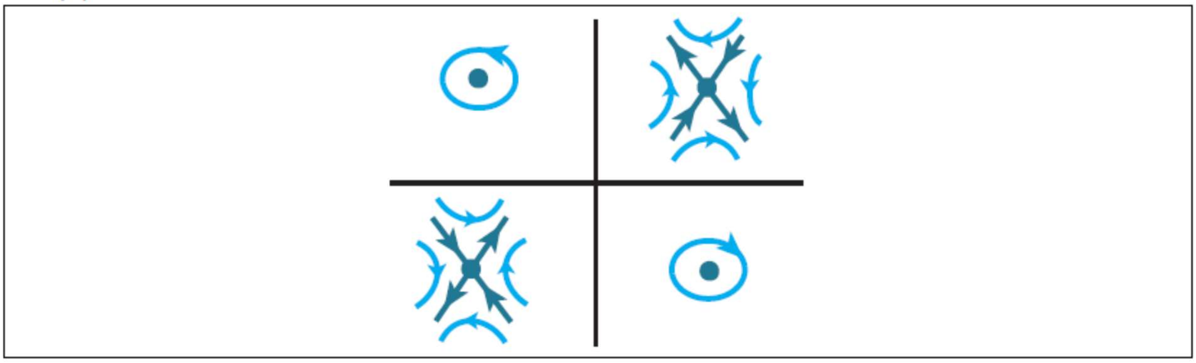
For equilibrium $(-1, 1)$, $A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$. The eigenvalue, $\lambda$, satisfies

$\det(A - \lambda I) = 0 \Rightarrow \det \begin{bmatrix} -\lambda & -2 \\ 2 & -\lambda \end{bmatrix} = 0 \Rightarrow \lambda^2 + 4 = 0$

$\Rightarrow \lambda = \pm 2i$ (complex).

So the equilibrium $(-1, 1)$ is a center (nonlinear may be different).

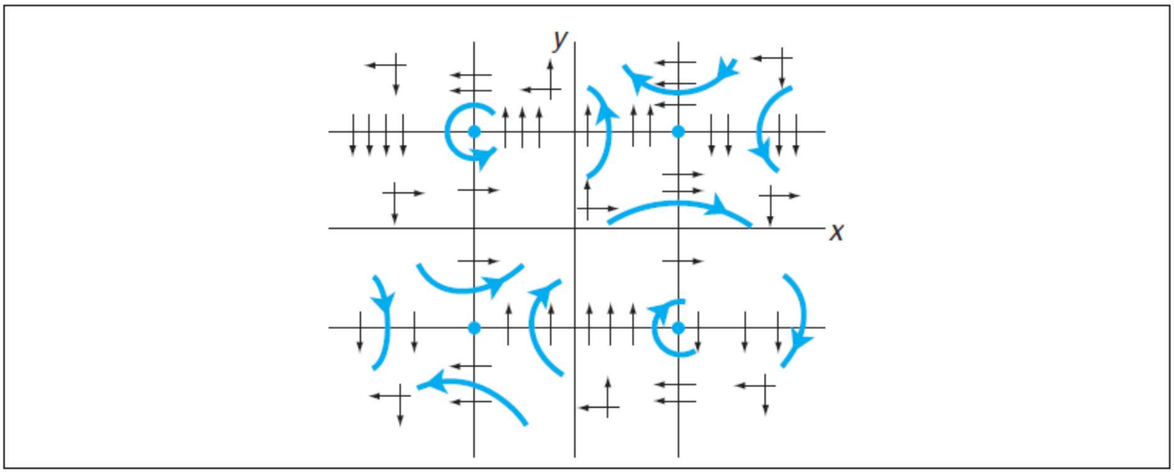
(c)



(e) Nullclines: $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1-x^2}{1-y^2}$.

So $\frac{dy}{dx} = 0 \Rightarrow 1 - x^2 = 0 \Rightarrow x = \pm 1$

and $\frac{dy}{dx} = \infty \Rightarrow 1 - y^2 = 0 \Rightarrow y = \pm 1$.



13. $\frac{dx}{dt} = x - xy + \gamma x^2 = x - xy - 8x^2 = f$
$\frac{dy}{dt} = -y + xy = g$

(a) Equilibria: $g = 0 \Rightarrow -y(1-x) = 0 \Rightarrow y = 0$ or $x = 1$.
$f = 0 \Rightarrow x - xy - 8x^2 = 0$. If $y = 0$, then $x - 8x^2 = 0$
$\Rightarrow x(1-8x) = 0 \Rightarrow x = 0, \frac{1}{8}$.
If $x = 1$, then $1 - y - 8 = 0 \Rightarrow y = -7$.
Thus equilibria is $(0,0)$, $\left(\frac{1}{8}, 0\right)$ and $(1,-7)$.

(b) $A = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{bmatrix} = \begin{bmatrix} 1 - y - 16x & -x \\ y & -1 + x \end{bmatrix}$.

For equilibrium $(0,0)$, $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. The eigenvalue, $\lambda$, satisfies
$\det(A - \lambda I) = 0 \Rightarrow \det \begin{bmatrix} 1-\lambda & 0 \\ 0 & -1-\lambda \end{bmatrix} = 0$
$\Rightarrow -(1-\lambda)(1+\lambda) = 0 \Rightarrow \lambda = 1, -1$ (one positive, one negative).
So the equilibrium $(0,0)$ is a saddle point.

For equilibrium $\left(\frac{1}{8}, 0\right)$, $A = \begin{bmatrix} -1 & -\frac{1}{8} \\ 0 & -\frac{7}{8} \end{bmatrix}$. The eigenvalue, $\lambda$,
satisfies $\det(A - \lambda I) = 0 \Rightarrow \det \begin{bmatrix} -1-\lambda & -\frac{1}{8} \\ 0 & -\frac{7}{8}-\lambda \end{bmatrix} = 0$
$\Rightarrow (-1-\lambda)\left(-\frac{7}{8}-\lambda\right) = 0 \Rightarrow \lambda = -1, -\frac{7}{8}$ (both negative).
So the equilibrium $\left(\frac{1}{8}, 0\right)$ is a stable node.
For equilibrium $(1,-7)$, $A = \begin{bmatrix} -8 & -1 \\ -7 & 0 \end{bmatrix}$. The eigenvalue, $\lambda$,
satisfies $\det(A - \lambda I) = 0 \Rightarrow \det \begin{bmatrix} -8-\lambda & -1 \\ -7 & -\lambda \end{bmatrix} = 0$
$\Rightarrow \lambda^2 + 8\lambda - 7 = 0 \Rightarrow \lambda = \frac{-8 \pm \sqrt{64+28}}{2}$
$\Rightarrow \lambda = -4 \pm \sqrt{23}$ (one positive, one negative).
So the equilibrium $(1,-7)$ is a saddle point.

17. $\frac{dx}{dt} = x - xy + \gamma x^2 = x - xy + 8x^2 = f$
$\frac{dy}{dt} = -y + xy = g$

(a) Equilibria: $g = 0 \Rightarrow -y(1-x) = 0 \Rightarrow y = 0$ or $x = 1$.
$f = 0 \Rightarrow x - xy + 8x^2 = 0$. If $y = 0$, then $x + 8x^2 = 0$
$\Rightarrow x(1 + 8x) = 0 \Rightarrow x = 0,\ -\frac{1}{8}$.
If $x = 1$, then $1 - y + 8 = 0 \Rightarrow y = 9$.
Thus equilibria is $(0,0)$, $\left(-\frac{1}{8}, 0\right)$ and $(1,9)$.

(b) $A = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{bmatrix} = \begin{bmatrix} 1 - y + 16x & -x \\ y & -1 + x \end{bmatrix}$.

For equilibrium $(0,0)$, $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. The eigenvalue, $\lambda$, satisfies
$\det(A - \lambda I) = 0 \Rightarrow \det \begin{bmatrix} 1-\lambda & 0 \\ 0 & -1-\lambda \end{bmatrix} = 0$
$\Rightarrow -(1-\lambda)(1+\lambda) = 0 \Rightarrow \lambda = 1,\ -1$ (one positive, one negative).
So the equilibrium $(0,0)$ is a saddle point.

For equilibrium $\left(-\frac{1}{8}, 0\right)$, $A = \begin{bmatrix} -1 & \frac{1}{8} \\ 0 & -\frac{9}{8} \end{bmatrix}$. The eigenvalue, $\lambda$,
satisfies $\det(A - \lambda I) = 0 \Rightarrow \det \begin{bmatrix} -1-\lambda & \frac{1}{8} \\ 0 & -\frac{9}{8} - \lambda \end{bmatrix} = 0$
$\Rightarrow (-1-\lambda)\left(-\frac{9}{8} - \lambda\right) = 0 \Rightarrow \lambda = -1,\ -\frac{9}{8}$ (both negative).
So the equilibrium $\left(-\frac{1}{8}, 0\right)$ is a stable node.

For equilibrium $(1,9)$, $A = \begin{bmatrix} 8 & -1 \\ 9 & 0 \end{bmatrix}$. The eigenvalue, $\lambda$, satisfies
$\det(A - \lambda I) = 0 \Rightarrow \det \begin{bmatrix} 8-\lambda & -1 \\ 9 & -\lambda \end{bmatrix} = 0 \Rightarrow \lambda^2 - 8\lambda + 9 = 0$
$\Rightarrow \lambda = \frac{8 \pm \sqrt{64-36}}{2} \Rightarrow \lambda = 4 \pm \sqrt{7}$ (both positive).
So the equilibrium $(1,9)$ is an unstable node.