نظرية الحاسبات
المحاضرة العاشرة
الزمن: ساعة

Example 43. $S = \{s_0, s_1, s_2\}$ $Y = \{s_1\}$ ← accepted state

	f	
S \ *A*	*a*	*b*
s_0	$\{s_1\}$	$\{s_0\}$
s_1	$\{s_2\}$	$\{s_1, s_2\}$
s_2	$\{s_2\}$	$\{s_2\}$

Contruct a transition diagram for the given nfa and a *dfa* eqivalent to *nfa*.

Solution. The state transition diagram of nfa is shown in Fig. 16.24

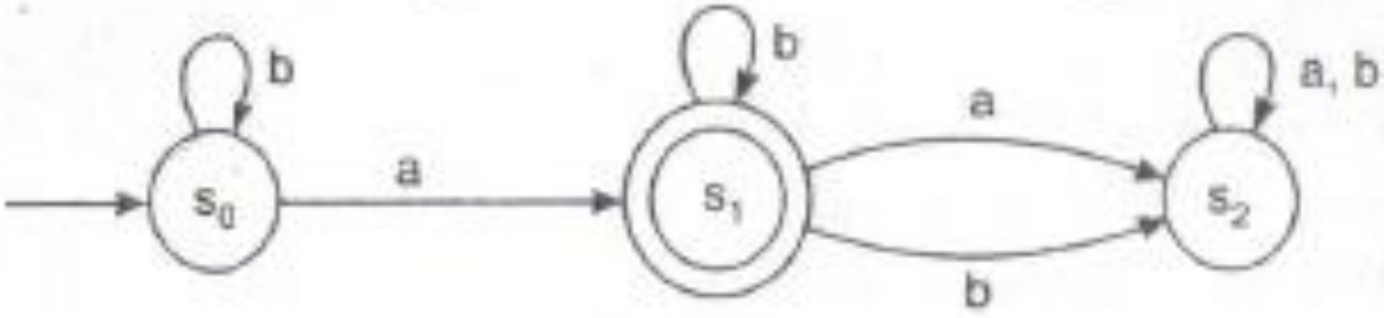


Fig. 16.24

Hence $M = \{A, S, f, Y, s_0\}$

We construct the corresponding dfa

$$M^d = \{A, S^d, f^d, Y^d, s_0^d\}$$

$$S^d = p(S) = \{\phi, \{s_0\}, \{s_1\}, \{s_2\}, \{s_0, s_1\}, \{s_0, s_2\} \{s_1, s_2\}, \{s_0, s_1, s_2\}\}$$

$$Y^d = \{\{s_1\}, \{s_0, s_1\}, \{s_1, s_2\}, \{s_0, s_1, s_2\}\}$$

$$S_0^d = \{s_0\}$$

and $f^d : S^d \times A \rightarrow S^d$ is defined by the following table

S^d \ A	f^d	
	a	b
ϕ	ϕ	ϕ
$\{s_0\}$	$\{s_1\}$	$\{s_0\}$
$\{s_1\}$	$\{s_2\}$	$\{s_1, s_2\}$
$\{s_2\}$	$\{s_2\}$	$\{s_2\}$
$\{s_0, s_1\}$	$\{s_1, s_2\}$	$\{s_0, s_1, s_2\}$
$\{s_1, s_2\}$	$\{s_2\}$	$\{s_1, s_2\}$
$\{s_0, s_2\}$	$\{s_1, s_2\}$	$\{s_0, s_2\}$
$\{s_0, s_1, s_2\}$	$\{s_1, s_2\}$	$\{s_0, s_1, s_2\}$

The set of final states which constains the final state s_1 in $M = \{\{s_1\},\{s_0, s_1\}\{s_1, s_2\}\ \{s_0, s_1, s_2\}\}$.
The state transition diagram of M^d is

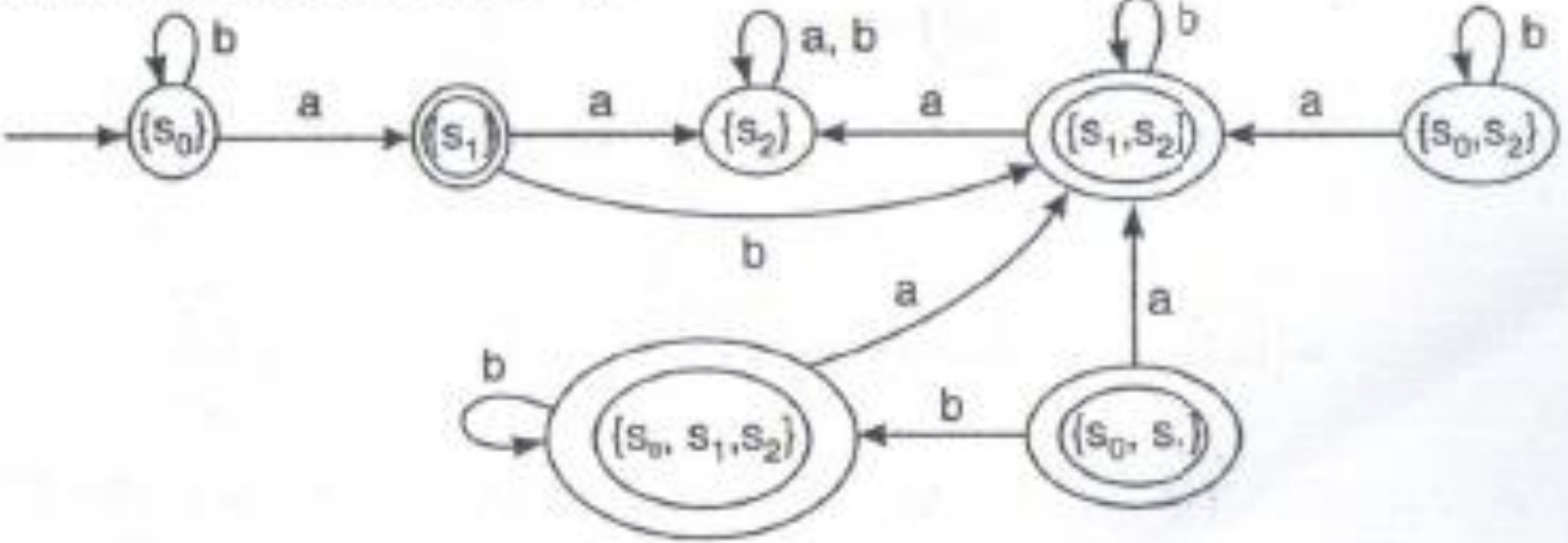


Fig. 16.25 Transition diagram of M^d.

Since the states $\{s_0, s_2\}$, $\{s_0, s_1\}$, $\{s_0, s_1, s_3\}$ can not be reached from the initial state $\{s_0\}$, they can be dropped to yield the simplified *dfa* as shown im Fig. 16.26.

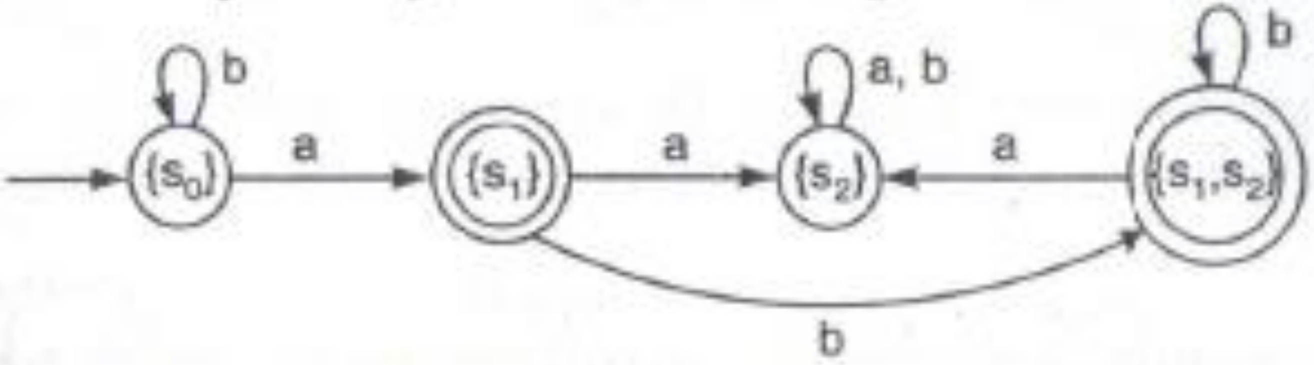


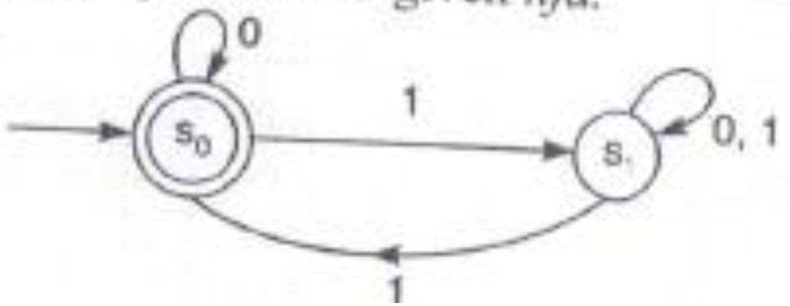
Fig. 16.26 Simplified transition diagram of M^d.

Procedure : *nfa* to *dfa*

If a given *nfa* has *n* states, then the number of states in the equivalent *dfa* will be 2^n with start state corresponding to the subject $\{s_0\}$ and then we try tc find the transition among states of *dfa* for all input symbols. But all the states of *dfa* obtained in this way, may not be reachable from the start state on any possible input sequence, then such a state does not pay any role in deciding, what language is accepted by *dfa*. Inaccessible states can be thrown away as follows:

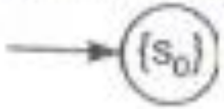
1. Create a diagram D with vertex $\{s_0\}$. Identify this vertex as the initial vertex.
2. Repeat the following steps until no more edges are missing. Take any vertex $\{s_i, s_j, ..., s_k\}$ of D that has no outgoing edge for some input alphabet $a \in A$.
   (*a*) Compute $f(s_i, a) \cup f(s_j, a) \cup ... \cup f(s_k, a)$, let the result be $\{s_1, s_m, ..., s_n\}$
   (*b*) Create a vertex labeled $\{s_1, s_m, ..., s_n\}$ if it does not already exist.
   (*c*) Add to D an edge from $\{s_i, s_j, ..., s_k\}$ to $\{s_1, s_m, ..., s_n\}$ and label it with a.
3. Every state of D whose label contains $s_f \in Y$ is identified as final state (*i.e.*, the accepting states of *dfa* are those sets that include at least one accepting states of *nfa*).

Example 44. Construct a dfa equivalent to given *nfa*.

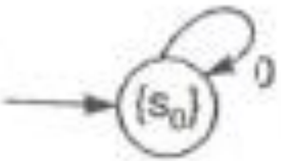


1

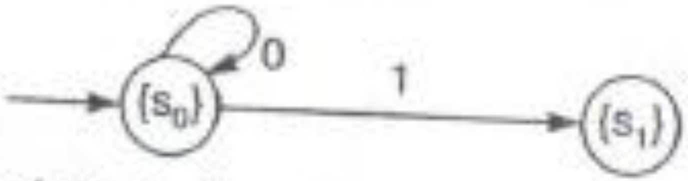
Solution. Initially, we have $\{s_0\}$ as start. We create a vertex $\{s_0\}$.



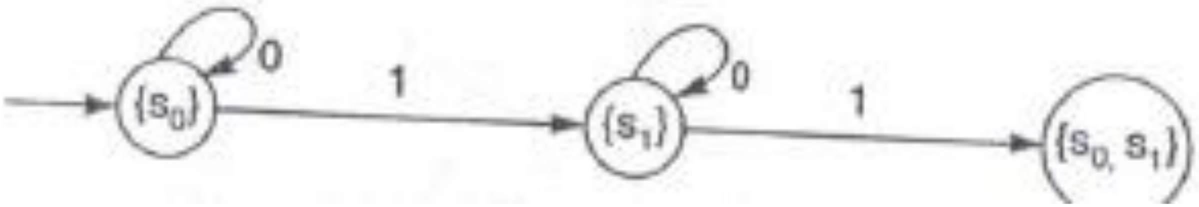
Since, there is only one subset $\{s_0\}$, we get $f^d(\{s_0\}, 0) = \{s_0\}$ which already exist. We show this move as



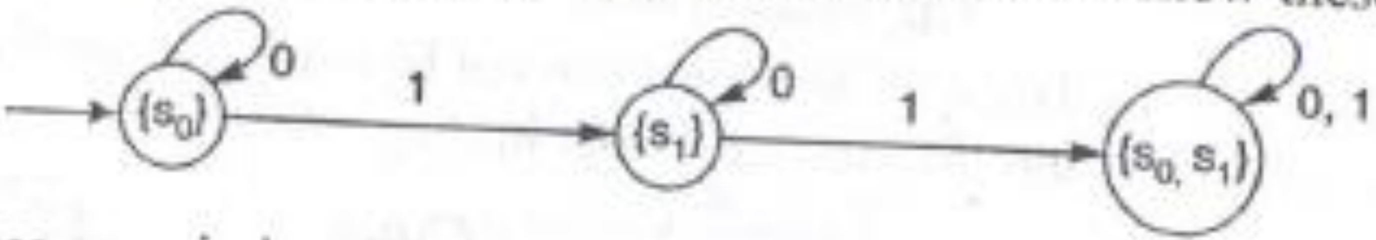
Again, we find $f^d(\{s_0\}, 1) = \{s_1\}$, a new vertex and create this vertex in D and add an edge labeled 1 as



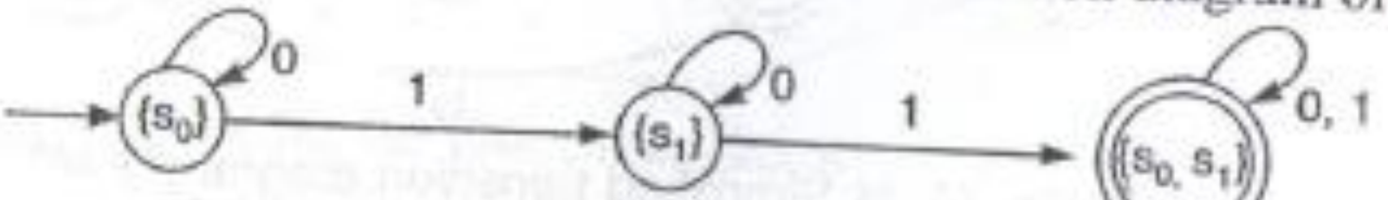
Now $f^d(\{s_1\}, 0) = \{s_1\}$ and $f^d(\{s_1\}, 1) = \{s_0, s_1\}$, we create a new vertex $\{s_0, s_1\}$ and add an edge labeled 0, 1.



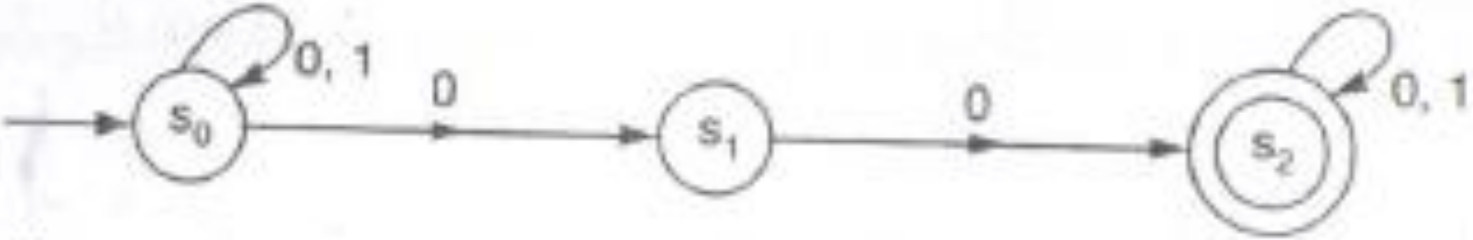
Again $f^d(\{s_0, s_1\}, 0) = f(s_0, 0) \cup f(s_1, 0) = \{s_0\} \cup \{s_1\} = \{s_0, s_1\}$ and $f^d(\{s_0, s_1\}, 1) = f(s_0, 1) \cup f(s_1, 1) = \{s_1\} \cup \{s_0, s_1\} = \{s_0, s_1\}$ which already exist. We show these moves as



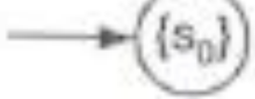
Now no more edges are missing. Finally, the vertex $\{s_0\}$ and $\{s_0, s_1\}$ contain the final state s_0 of *nfa*. So these are the final states in equivalent *dfa*. Thus the transiton diagram of equivalent *dfa* is



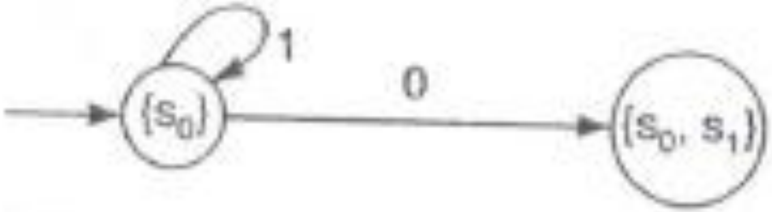
Example 45. Construct a dfa equivalent to the given *nfa*.



Solution. Initially, we have $\{s_0\}$ as a start state. We create a vertex $\{s_0\}$ as a start state.



Since $f^d(\{s_0\}, 0) = \{s_0, s_1\}$ and $f^d(\{s_0\}, 1) = \{s_0\}$, we create a new vertex $\{s_0, s_1\}$ and add an edge labeled 0 between $\{s_0\}$ and $\{s_0, s_1\}$



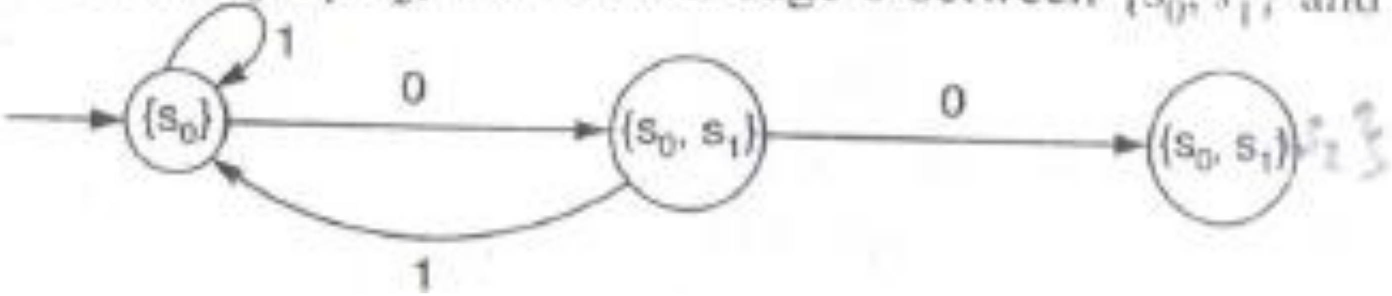
Again $f^d(\{s_0, s_1\}, 0) = f(s_0, 0) \cup f(s_1, 0)$

$$= \{s_0, s_1\} \cup \{s_2\} \cup \{s_2\} = \{s_0, s_1, s_2\}$$

and $f^d(\{s_0, s_1\}, 1) = f(s_0, 1) \cup f(s_1, 1)$

$$= \{s_0\} \cup \phi = \{s_0\}$$

We create a new vertex $\{s_0, s_1, s_2\}$ and add an edge 0 between $\{s_0, s_1\}$ and $\{s_0, s_1, s_2\}$



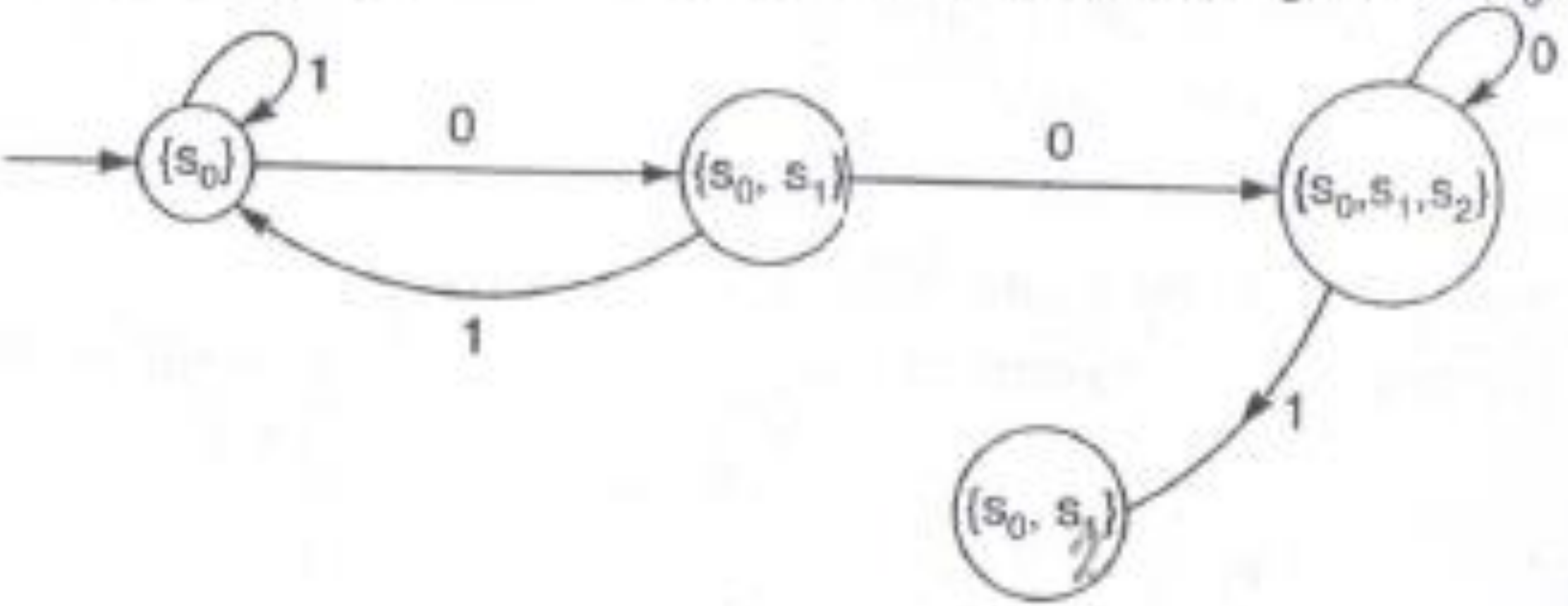
Again $f^d(\{s_0, s_1, s_2\}, 0) = f(s_0, 0) \cup f(s_1, 0) \cup f(s_2, 0)$

$$= \{s_0, s_1\} \cup \{s_2\} \cup \{s_2\} = \{s_0, s_1, s_2\}$$

and $f^d(\{s_0, s_1, s_2\}, 1) = f(s_0, 1) \cup f(s_1, 1) \cup f(s_2, 1)$

$$= \{s_0\} \cup \phi \cup \{s_2\} = \{s_0, s_2\}$$

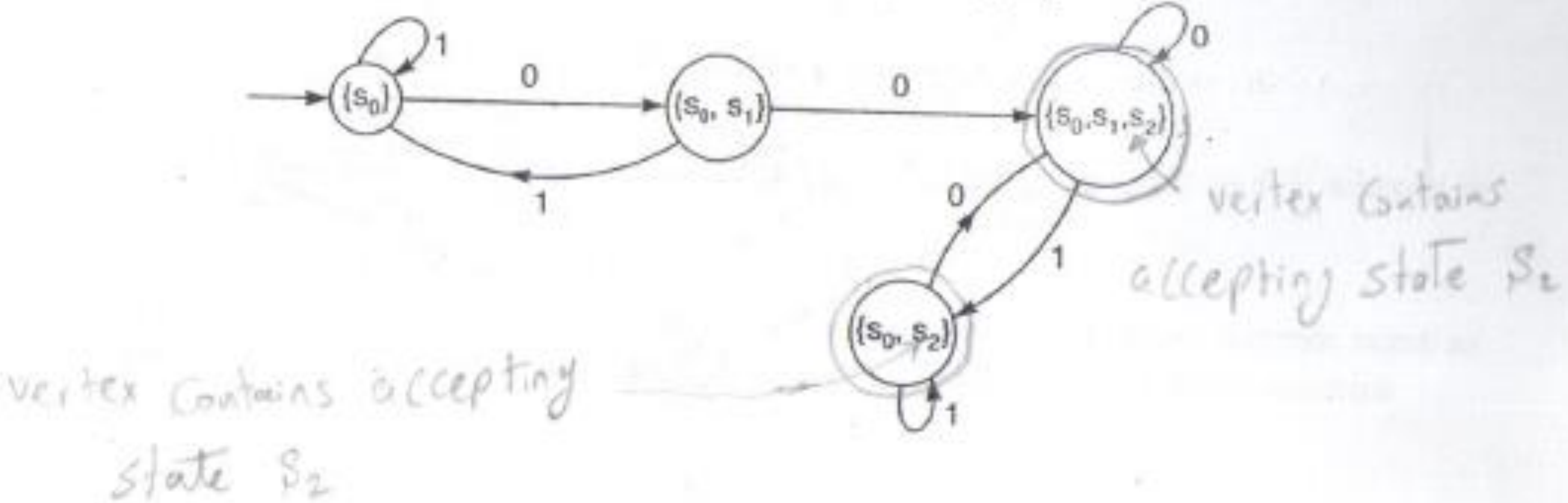
We add a new vertex $\{s_0, s_2\}$ with an edge 1 between $\{s_0, s_1, s_2\}$ and $\{s_0, s_2\}$



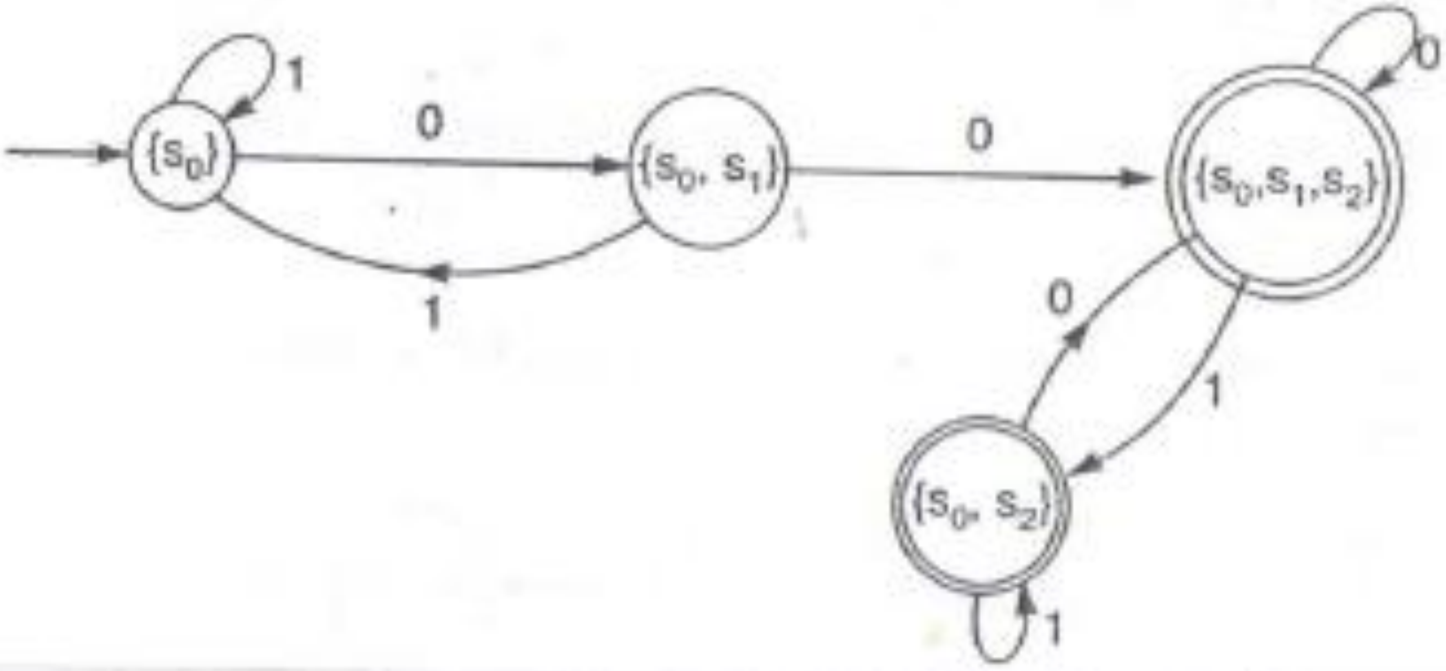
Now $f^d(\{s_0, s_2\}, 0) = f(s_0, 0) \cup f(s_2, 0)$

$$= \{s_0, s_1\} \cup \{s_2\} = \{s_0, s_1, s_2\}$$

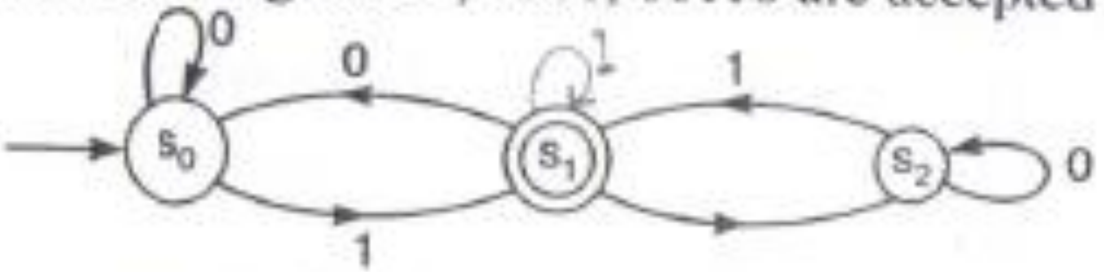
And $f^d(\{s_0, s_2\}, 1) = f(s_0, 1) \cup f(s_2, 1) = \{s_0\} \cup \{s_2\} = \{s_0, s_2\}$



Now no more edges are missing. Finally, the vertex $\{s_0, s_2\}$ and $\{s_0, s_1, s_2\}$ contain the final state s_2 of *nfa*. So these are the final states in equivalent *dfa*. Then the transition diagram of equivalent *dfa* is



Example 46. Which of the strings 0001, 0101, 00110 are accepted by the following *dfa*



Solution. Using the mapping function f, the sequence of the steps can be written as $f(s_0, 0001) = f(s_0, 001) = f(s_0, 01) = f(s_0, 1) = s_1$ (final state).

Hence, the string 0001 is accepted by the *dfa*.

Again, $f(s_0, 0101) = f(s_0, 101) = f(s_1, 01) = f(s_0, 1) = s_1$(final state).

Hence the string 0101 is accepted by the *dfa*.

For the string 00110, we have

$$f(s_0, 00110) = f(s_0, 0110) = f(s_0, 110) = f(s_1, 10)$$
$$= f(s_1, 0) = s_0 \text{ (non final state)}$$

Hence, the string is not accepted by the *dfa*.

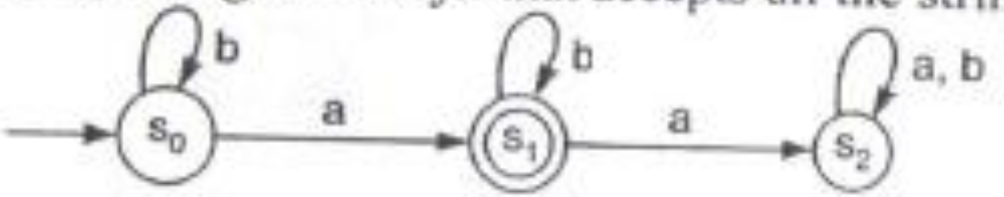
Example 47. For $\Sigma = \{a, b\}$, design dfas that accept the sets consisting of

(*a*) all the strings with exactly one *a*

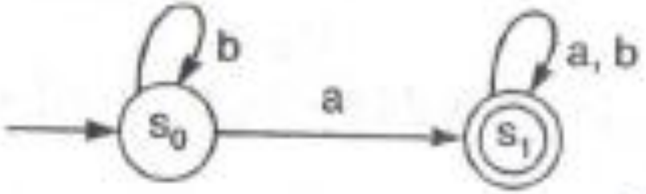
(*b*) all the strings with at least one *a*

(*c*) all strings with at least one a and followed by exactly two *b*'s.

Solution. (*a*) The transition diagram of *dfa* that accepts all the string with exactly one a is



(*b*) All the string with atleast one *a*



(*c*) All the strings with atleast one *a* followed by exactly two *b*'s.