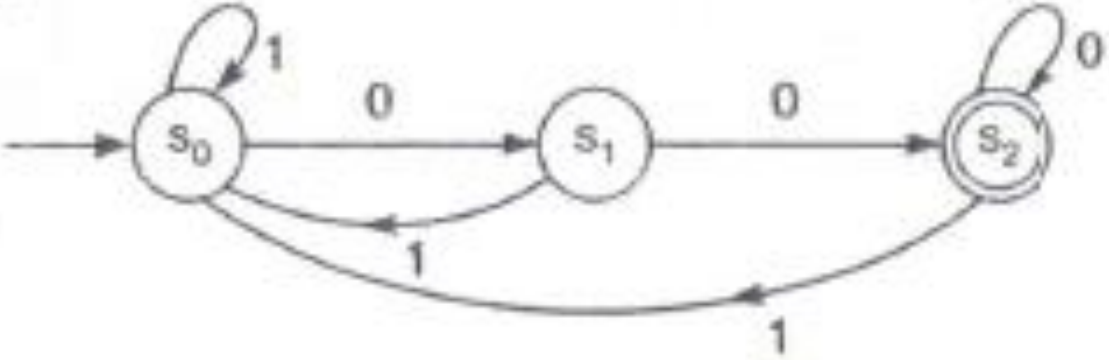
نظرية الحاسبات

المحاضرة الحادية عشر

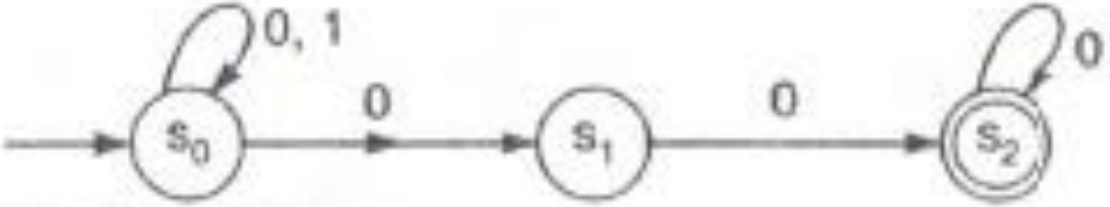
الزمن: ساعة

Example 48. Design *dfa* and nfa which accepts set of all strings ending with 00.

Solution. The required *dfa* is

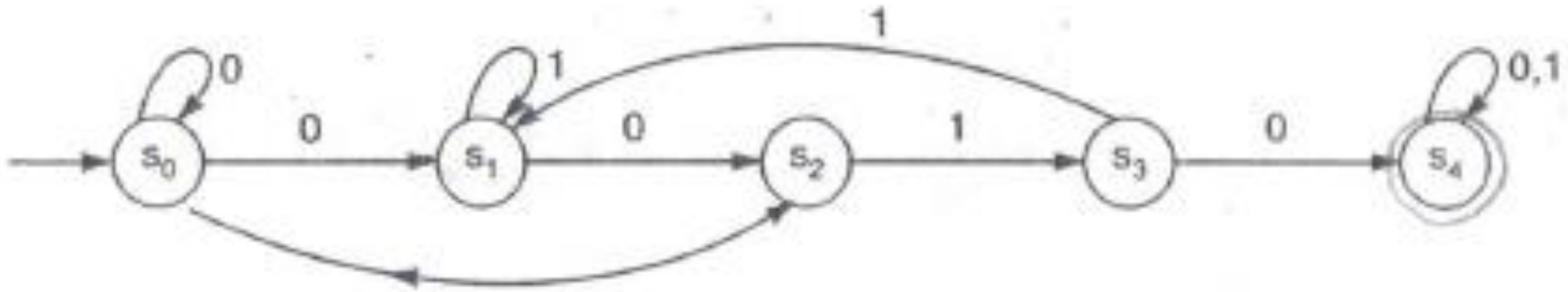


The required *nfa* is

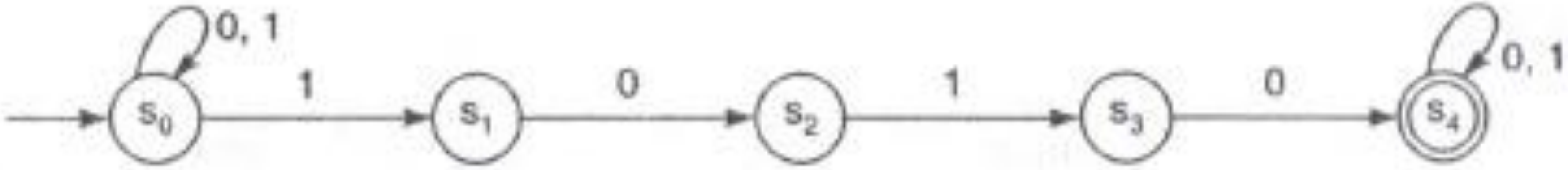


Example 49. Design *dfa* and *nfa* which accepts set of all binary strings containing 1010 as sub string.

Solution. The required *dfa* is



The required *nfa* is



6.8. Moore and Mealy Machine

These machines are basically DFAs, except that they are assoicated with an output symbol with each state or with each transition. However, there are no final states, because there is no acceptance or rejection involved. They are not language recogniser but output producer.

Moore Machine

A Moore machine is a finite state automation, when the outputs are determined by the current state alone. A Moore machine M_0 is represented by 6-tuples

$$M_0 = (Q, \Sigma, \delta, q_0, O, f)$$

where

(*i*) Q is a finite non-empty set cf states

(*ii*) Σ is a finite set of input symbols

(*iii*) $\delta : Q \times \Sigma \rightarrow Q$ is transition function

(*iv*) $q_0 \in Q$ is the initial state

(*v*) O is a finite set of output symbols

(*vi*) $f : Q \rightarrow O$ is the output function.

Note: The output of a Moore machine is one character longer then its input ($n + 1$, when n is the number of characters in an input string)

Representation of Moore Machine

Moore machine can be represented by transition table as well as transition diagram same as finite automata. For example, the following table gives a transition table of a Moore machine.

Present state	Next state at input		Output
	a	b	
→ q_0	q_1	q_3	1
q_1	q_3	q_1	0
q_2	q_0	q_3	0
q_3	q_3	q_2	1

Here $\Sigma = \{a, b\}$, $O = \{0, 1\}$ and $Q = \{q_0, q_1, q_2, q_3\}$

The transition diagram of this machine is shown in Fig. 16.27

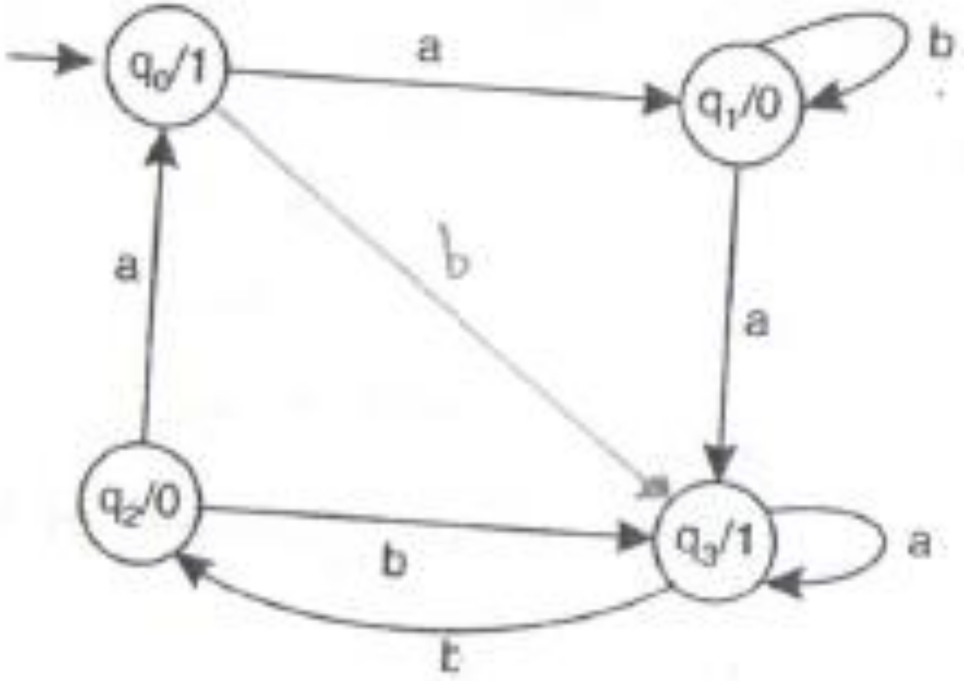


Fig. 16.27

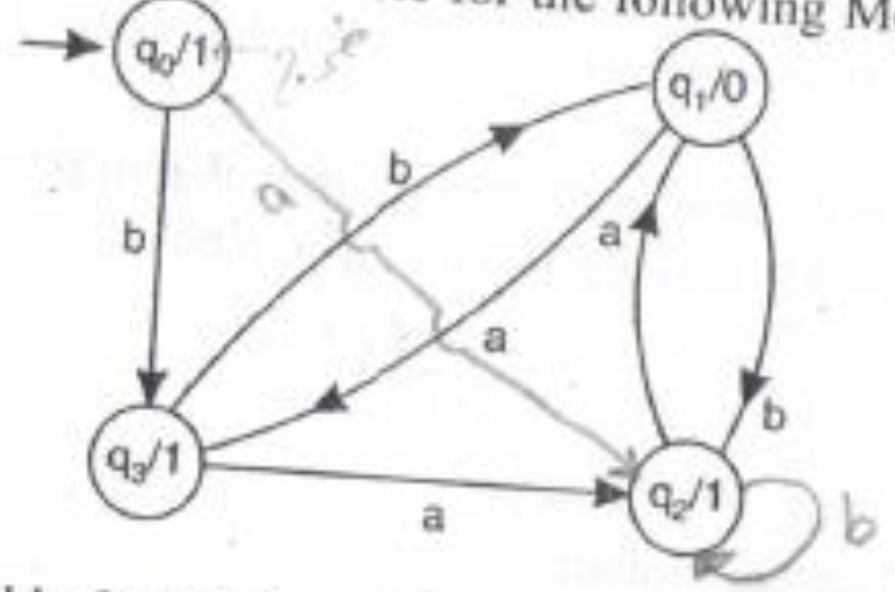
If w = *abab* is the input string, the processing of the string by the Moore machine can be represented by

$$\text{Start} \longrightarrow \overset{1}{q_0} \xrightarrow[\text{input}]{a} \overset{0}{q_1} \xrightarrow[\text{input}]{b} \overset{0}{q_1} \xrightarrow[\text{input}]{a} \overset{1}{q_3} \xrightarrow[\text{input}]{b} \overset{0}{q_2}$$

Thus, output string $w' = 10010$

Note: In the transition diagram, if the output associated with a state q_0 is x, then it is written as q_0/x inside the circle.

Example 50. Construct state transition table for the following Moore machine.



Solution. The transition table for the given Moore machine is

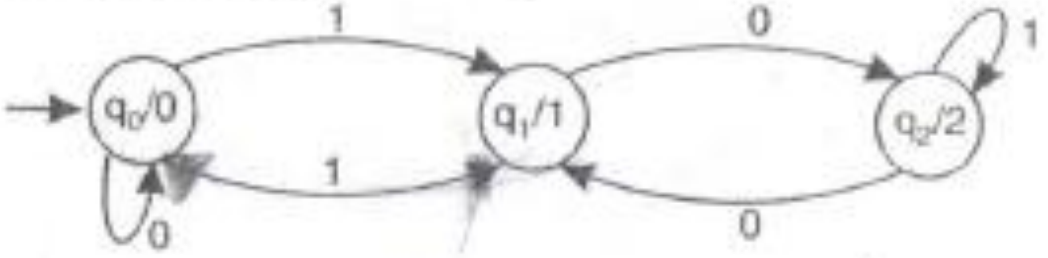
Present state	*Next state at input*		*Output*
	a	*b*	
→ q_0	q_2	q_3	1
q_1	q_3	q_2	0
q_2	q_1	q_2	1
q_3	q_2	q_1	1

Example 51. Design a Moore machine to determine the residue mod 3 for each binary string treated as binary integer.

Solution. Here $\Sigma = \{0, 1\}$ is given, so binary string is a combination of 0 and 1. Residue mod means remainder when decimal number is divided by 3. So, $O = \{0, 1, 2\}$ and hence we need three states.

Let $Q = \{q_0, q_1, q_2\}$. Define $f(q_j) = j$ for $j = 0, 1, 2$.

The transsition diagram is given by



On input 1010 (decimal equivalent is 10), the sequence of states entered as $q_0 \to q_1 \to q_2 \to q_2 \to q_1$ gives output 01221. M_0 ends at q_1 and $f(q_1) = 1$ which is the remainder of 10/3.

Mealy Machine

A Mealy machine is a finite state machine, where the outputs are determined by the current ate and input. The Mealy machine M_e is represented by 6-tuples.

$$M_e = (Q, \Sigma, \delta, q_0, O, f)$$

where

(*i*) Q is a finite non-empty set of states

(*ii*) Σ is a finite set of input symbols

(*iii*) $\delta : Q \times \Sigma \rightarrow Q$ is the transition function

(*iv*) $q_0 \in Q$ is the initial state

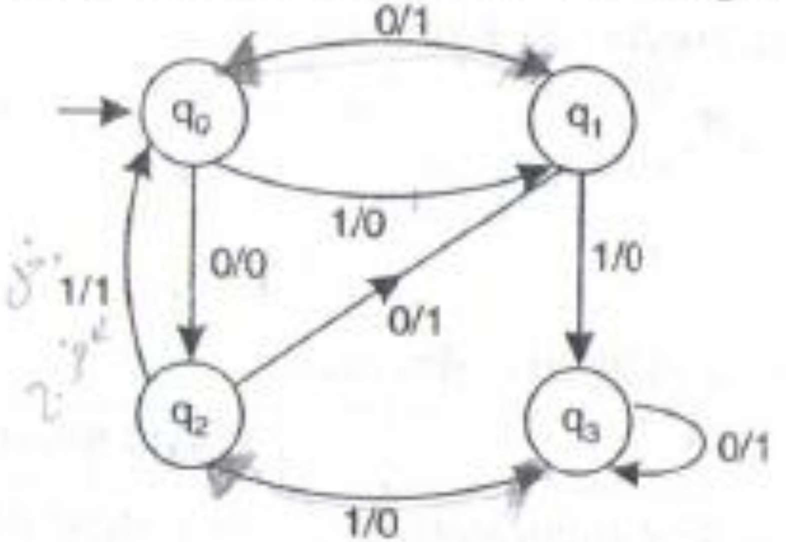
(*v*) O is the finite set of output symbols

(*vi*) $f : Q \times \Sigma \rightarrow O$ is the output function. $f : Q \times \Sigma \rightarrow O$

Representation of Mealy Machine

Mealy machine can also be represented by transition table, as well as transitive diagram, same Moore machine and finite automata.

Example 52. A transition diagram of a Mealy machine is given in Fig. 16.28.



In figure each edge is labeled *i*/0, where *i* is an input symbol, O is the output symbol. The transition table for Mealy machine of the figure shown is below.

Present state	*Next state*			
	Input a = 0		Input *a* = 1	
	State	*Output*	*State*	*Output*
$\rightarrow q_0$	q_2	0	q_1	0
q_1	q_0	1	q_3	0
q_2	q_1	1	q_0	1
q_3	q_3	1	q_2	0

For the input string 01100, the processing steps are shown

$\delta(q_0, 0) = q_2$

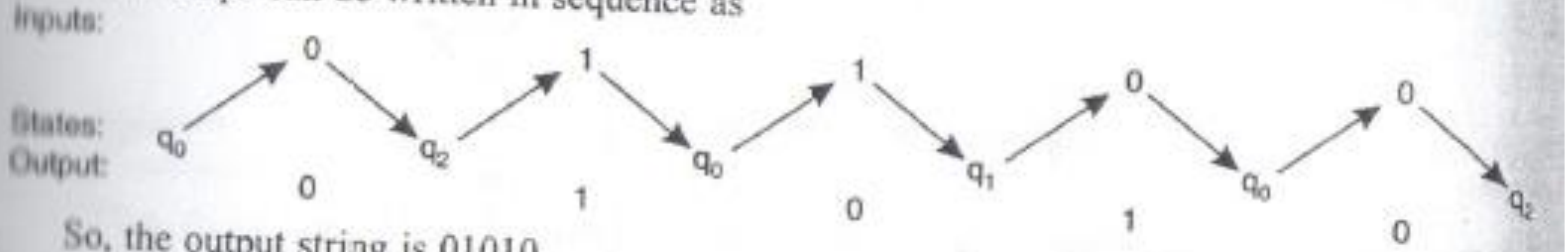
$\delta(q_0, 01) = \delta(\delta(q_0, 0), 1) = \delta(q_2, 1) = q_0$

$\delta(q_0, 011) = \delta(\delta(q_0, 01), 1) = \delta(q_0, 1) = q_1$

$\delta(q_0, 0110) = \delta(\delta(q_0, 011), 0) = \delta(q_1, 0) = q_0$

$\delta(q_0, 01100) = \delta(\delta(q_0, 0110), 0) = \delta(q_0, 0) = q_2$

These steps can be written in sequence as



So, the output string is 01010

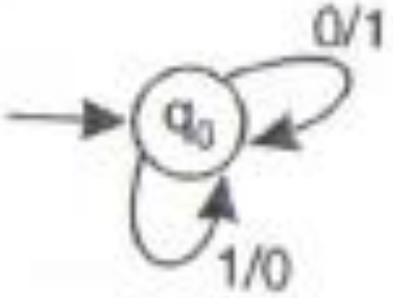
Example 53. Design a Mealy machine which prints 1's complement of input bit string over alphabet $\Sigma = \{0, 1\}$

Solution: Here $\Sigma = O = \{0, 1\}$, $Q = \{q_0\}$. The machine has to produce an output 0 each time it finds an input 1 and vice versa. The transition table and transition diagram is shown below

Transition table

Present state	Next state			
	Input a = 0		Input *a* = 1	
	State	*Output*	*State*	*Output*
→ q_0	q_0	1	q_0	0

Transition diagram



For the input string 011, the processing sequence are