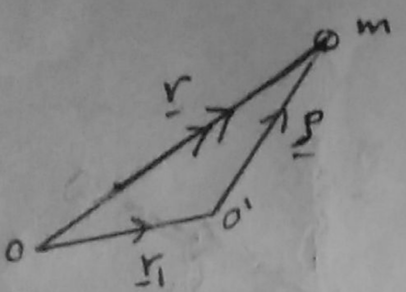
صفحة ١٠١

كمية الحركة الزاوية للجيروستات هي

$$\underline{K} = \underline{G} + \underline{h}$$

حيث $\underline{G} = \underline{\omega} I , \quad \underline{h} = J \dot{x} \underline{e}$

البرهان:



$$\therefore \underline{K} = \sum \underline{r} \wedge m \underline{v} , \quad \underline{v} = \underline{\omega} \wedge \underline{r}$$

$$\therefore \underline{K} = \sum{}' \underline{r} \wedge m (\underline{\omega} \wedge \underline{r}) + \sum{}'' m \underline{r} \wedge (\underline{\omega} \wedge \underline{r} + \dot{x} \underline{e} \wedge \underline{\rho})$$

$$\therefore \underline{K} = \left(\sum{}' + \sum{}''\right)\left[m \underline{r} \wedge (\underline{\omega} \wedge \underline{r})\right] + \sum{}'' m \underline{r} \wedge (\dot{x} \underline{e} \wedge \underline{\rho})$$

$$= \sum m \underline{r} \wedge (\underline{\omega} \wedge \underline{r}) + \dot{x} \sum{}'' m \underline{r} \wedge (\underline{e} \wedge \underline{\rho})$$

$$= \underline{\omega} I + \dot{x} \sum{}'' m (\underline{r}_1 + \underline{\rho}) \wedge (\underline{e} \wedge \underline{\rho})$$

$$= \underline{\omega} I + \cancel{\dot{x} \underline{r}_1 \wedge (\underline{e} \wedge \sum{}'' m \underline{\rho})} + \dot{x} \sum{}'' m \underline{\rho} \wedge (\underline{e} \wedge \underline{\rho})$$

$$= \underline{\omega} I + \dot{x} \underline{e} J$$

$$\underline{K} = \underline{G} + \underline{h} \; ; \quad \underline{G} = \underline{\omega} I , \quad \underline{h} = J \dot{x} \underline{e}$$

$$\dot{\underline{G}} \cdot \underline{r} + (\underline{G} + \underline{k}) \cdot \dot{\underline{r}} = 0$$

$$\Rightarrow \dot{\underline{G}} \cdot \underline{r} + \underline{G} \cdot \dot{\underline{r}} + \underline{k} \cdot \dot{\underline{r}} = 0$$

$$\therefore \frac{d}{dt}(\underline{G} \cdot \underline{r}) + \underline{k} \cdot \dot{\underline{r}} = 0 \qquad (*)$$

وحيث أن $\dot{\underline{k}} = 0$ فإن $\dot{\underline{k}} \cdot \underline{r} = 0$

وبذلك يمكن إضافة الحد $\dot{\underline{k}} \cdot \underline{r} = 0$ إلى المعادلة (*)

$$\therefore \frac{d}{dt}(\underline{G} \cdot \underline{r}) + \underline{k} \cdot \dot{\underline{r}} + \dot{\underline{k}} \cdot \underline{r} = 0$$

$$\therefore \frac{d}{dt}(\underline{G} \cdot \underline{r}) + \frac{d}{dt}(\underline{k} \cdot \underline{r}) = 0$$

$$\Rightarrow \frac{d}{dt}(\underline{G} \cdot \underline{r} + \underline{k} \cdot \underline{r}) = 0$$

$$\therefore \frac{d}{dt}\left[(\underline{G} + \underline{k}) \cdot \underline{r}\right] = 0$$

$$\Longrightarrow (\underline{G} + \underline{k}) \cdot \underline{r} = f \qquad (5)$$

$$\left.\begin{aligned} &\dot{\underline{G}} + \underline{\omega} \wedge (\underline{G} + \underline{k}) = \underline{\gamma} \wedge (Mg\,\underline{r}_0 + \lambda\, \underline{\gamma}\, I) \quad (a) \\ &\dot{\underline{\gamma}} + \underline{\omega} \wedge \underline{\gamma} = \underline{0} \quad (b) \end{aligned}\right\} (3)$$

$\Longrightarrow$

$$\Rightarrow (A\dot{p}, B\dot{q}, C\dot{r}) + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ p & q & r \\ Ap + k_1 & Bq + k_2 & Cr + k_3 \end{vmatrix} =$$

$$Mg \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \gamma_1 & \gamma_2 & \gamma_3 \\ x_0 & y_0 & z_0 \end{vmatrix} + \lambda \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \gamma_1 & \gamma_2 & \gamma_3 \\ \gamma_1 A & \gamma_2 B & \gamma_3 C \end{vmatrix}$$

$\Longrightarrow$

$$A\dot{p} + (C-B)qr + qk_3 - rk_2 = Mg(z_0\gamma_2 - y_0\gamma_3) + \lambda(C-B)\gamma_2\gamma_3,$$

$$B\dot{q} + (A-C)rp + rk_1 - pk_3 = Mg(x_0\gamma_3 - z_0\gamma_1) + \lambda(A-C)\gamma_3\gamma_1,$$

$$C\dot{r} + (B-A)qp + pk_2 - qk_1 = Mg(y_0\gamma_1 - x_0\gamma_2) + \lambda(B-A)\gamma_1\gamma_2.$$

(تعميم حالة لاجرانج) : بوضع $k_1 = k_2 = 0$ ، $x_0 = y_0 = 0$ ، $A = B$ نحصل على

$$A\dot{p} + (C-A)qr + qk_3 = Mg\,z_0\gamma_2 + \lambda(C-A)\gamma_2\gamma_3,$$

$$A\dot{q} + (A-C)rp - pk_3 = -Mg\,z_0\gamma_1 + \lambda(A-C)\gamma_3\gamma_1,$$

$C\dot{r} = 0 \Rightarrow \dot{r} = 0 \quad \therefore \boxed{r = Const.}$ التكامل الرابع في هذه الحالة

(تعميم حالة أويلر) : بوضع $\lambda = 0$ ، $x_0 = y_0 = z_0 = 0$ نحصل على

$$(a) \Rightarrow \dot{\underline{G}} + \underline{\omega} \wedge (\underline{G} + \underline{k}) = \underline{0}$$

ومنها نستنتج بسهولة $\boxed{|\underline{G} + \underline{k}|^2 = Const}$ (يترك للطالب)

وهو التكامل الرابع في هذه الحالة.